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15/ENG06/050
ENG 381

1 $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)\frac{d^2y}{dx^2}$

$v = 1-x^2 \qquad u = y^2$

$v' = -2x \qquad u^n = y^{n+2}$

$v^2 = -2 \qquad u^{n-1} = y^{n+1}$

$v^3 = 0 \qquad u^{n-2} = y^n$

Using Leibnitz

$$u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2$$

$$y^{n+2}(1-x^2) + n(y^{n+1})(-2x) + \frac{n(n-1)}{2} y^n = 0$$

at $x = 0$

$$y^{n+2} + -n(n-1)y^n = y^{n+2} \cancel{-(n^2+n)}y^n + (-n^2+n)y^n$$

$-2x\frac{dy}{dx}$

$v = -2x \qquad u = y'$

$v' = -2 \qquad u^n = y^{n+1}$

$v^2 = 0 \qquad u^{n-1} = y^n$

$(y^{n+1})(-2x) + n(y^n)-2$

at $x = 0$

$-2ny^n$

$2y$

$v = 2 \qquad u = y \qquad = 2y^n$

$v' = 0 \qquad u^n = y^n$

$y^{n+2} + (-n^2+n)y^n - 2ny^n + 2y^n$ — recurrence eqn

$y^{n+2} + y^n(-n^2+n-2n+2)$

$y^{n+2} + y^n(-n^2-n+2) = 0$

$y^{n+2} = -y^n(n^2+n-2)$

at $n=0$

$y^2 = -y^0(-2) = +2y^0$

$n=1$ $y^3 = -y_0^1(0) = 0$

$n=2$ $y^4 = -y_0^2(4) = -4y^2$ ie $-4 \times 2y^0 = -8y^0$

$n=3$ $y^5 = -y_0^3(10) = -10y^3$ ie $-10 \times 0 = 0$

$n=4$ $y^6 = -y_0^4(18) = -18y^4$ ie $-18 \times -8y^0 = 144y^0$

$n=5$ $y^7 = -y_0^5(28) = -28y^5$ ie $-28 \times 0 = 0$

Maclaurin

$x + xy_0^1 + \frac{x^2}{2!}y_0^2 + \frac{x^3}{3!}y_0^3 + \frac{x^4}{4!}y_0^4 + \frac{x^5}{5!}y_0^5$

$= x + \frac{x^2}{2!} - 8y^0 + \frac{x^4}{4!} \cdot 144y^0 \ldots\ldots$

Q.) i $3e^{-4t} - 5e^{4t} = \frac{3}{s+4} - \frac{5}{s-4}$

ii $\sin 4t + \cos 4t = \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2}$

iv) $e^{-2t}\cos 5t$

$e^{-2t} = \frac{1}{s+2}$, $\cos 5t = \frac{s}{s^2+5^2}$ $e^{-2t}\cos 5t = \frac{s+2}{(s+2)^2+5}$

v) $t \sin 3t = \dfrac{3}{s^2+9}$

$$-\frac{d}{ds}\left[\frac{3}{s^2+9}\right] \qquad u = 3 \quad v = s^2+9 \qquad du/dv = 0 \quad dv/du = 2s$$

$$\frac{V\, du/dv - U\, dv/du}{V^2} = \frac{(s^2+9)(0) - (3)(2s)}{(s^2+9)^2}$$

$$= \frac{+6s}{(s^2+9)^2}$$

vii) $e^{4t} \cos 2t$

$$e^{4t} = \frac{1}{s-4} \qquad \cos 2t = \frac{s}{s^2+2^2}$$

$$e^{4t}\cos 2t = \frac{s-4}{(s-4)^2+2^2}$$

viii) $t \sin 2t = \dfrac{2}{s^2+4}$

$$-\frac{d}{ds}\left[\frac{2}{s^2+4}\right] \qquad u = 2 \quad v = s^2+4 \qquad du/dv = 0 \quad dv/du = 2s$$

Quotient rule — $\cancel{(3)(0) - (s^2+4)(2s)}\ (3)(2s)$

$$\frac{(s^2+4)(0) - (2)(2s)}{(s^2+4)^2} = \frac{+4s}{(s^2+4)^2}$$

ix $t^2 + 4t^2 + 5 = L(t^3) + 4L(t^2) + L(5)$

$$= \frac{3!}{s^{3+1}} + 4\left(\frac{2!}{s^{2+1}}\right) + 5/s$$

x $\quad e^{3t}(t^2+4)$

$e^{3t} = 1/_{s-3} \qquad t^2+4 = \frac{2!}{s^2+1} + 4/_s$

$e^{3t}(t^2+4) = \frac{2!}{(s-3)^2+1} + \frac{4}{s-3}$

$X_1)$ $\quad t^2 \cos t$

$\cos t = \frac{s}{s^2+1}$

$-\frac{d}{ds}\left(\frac{s}{s^2+1}\right) \qquad u=s \qquad du/_{dv} = 1$

$v = s^2+1 \qquad dv/_{du} = 2s$

$-\frac{(s^2+1)(1)-(s)(2s)}{(s^2+1)^2} = \frac{-s^2+1-2s^2}{s^2+1}$

$\frac{-2s^2-s^2+1}{(s^2+1)^2} \qquad u = -2s^2-s^2+1 \qquad du/_{dv} = -4s-2s$

$v = s^2+1 \qquad dv/_{du} = 2s$

$\frac{(s^2+1)(-4s-2s)+(2s^2-s^2+1)(2s)}{(s^2+1)^2}$

$\frac{(s^2+1)(-6s)+(4s^3-2s^3+2s)}{(s^2+1)^2}$

X_{11} i) $\quad \frac{\sinh 2t}{t} = \frac{2}{s^2-4} \qquad L\left[\frac{\sinh t}{t}\right] = \int_{\sigma=}^{\infty} \frac{2}{\sigma^2-4}\, d\sigma$

let $u = \sigma^2-4$

$du/_{dy} = 2\sigma$

$d\sigma = \frac{d\sigma}{2\sigma}$

$$= \int_{\infty=s}^{\infty} \frac{s}{v} \frac{dv}{2s}$$

$$\frac{1}{s} \int_{s=}^{\infty} \frac{1}{v}\, dv$$

$$\frac{1}{s} \ln v \Big|_{s=s}^{-}$$

$$= \frac{1}{s} \ln(s^2 - 4) \Big|_{s}^{\infty}$$

3. i

$$\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$$

$$s-5 = A(s-4) + B(s-3)$$

$s = 4$ $\quad$ $s = 3$

$B = -1$ $\quad$ $A = 2$

$$\frac{s-5}{(s-3)(s-4)} = \frac{2}{s-3} - \frac{1}{s-4}$$

$$= 2e^{3t} - e^{4t}$$

ii

$$\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$$

$$2s - 6 = A(s-4) + B(s-2)$$

$s = 2$ $\quad$ $s = 4$

$A = 1$ $\quad$ $B = 1$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{1}{s-2} + \frac{B}{s-4}$$

$$= e^{2t} + e^{4t}$$

iii) $$\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$\frac{5s-8}{s(s-4)} = A(s-4) + B(s)$$

$s = 0$ $\quad$ $s = 4$

$A = 2$ $\quad$ $B = 3$

$$\frac{5s-8}{s(s-4)} = \frac{2}{s} + \frac{3}{s-4}$$

$$= 2t + 3e^{4t}$$

iv) $$\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{A}{(s-3)} + \frac{B}{s-1} + \frac{C}{(s-1)^2}$$

$$s^2 - 3s - 4 = A(s-1)^2 + B(s-1)(s-3) + C(s-3)$$

$s = 3$ $\quad$ $s = 1$

$A = -1$ $\quad$ $C = 4$

Comparing coefficient

$1 = A + B$ $\quad$ but $A = -1$

$1 = -1 + B$

$B = 1 + 1$

$B = 2$

$$\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{-1}{s-3} + \frac{2}{s-1} + \frac{4}{(s-1)^2}$$

$$= -e^{3t} + 2e^{t} + 4\cosh t$$