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15/ENG03/071

CIVIL ENGINEERING

ENG 381

ASSIGNMENT 4

1) $(1-x)^2 \frac{d^2y}{dx^2} - 2x\, dy/dx + 2y = 0$

$(1-x^2)y^2 - 2xy' + 2y = 0$

$y^n = u^n v + nu^{n-1}v' + \frac{n(n-1)}{2!} u^{n-2}v^2 + \ldots\ldots$

$\left[y^{(2+n)}.(1-x^2) + ny^{(1+n)}.(-2x) + \frac{n(n-1)}{2!} y^n.(-2)\right]$

$+ \cancel{[y^{(1+n)}. -2x + ny^n]}$

$+ [y^{(1+n)}. -2x + ny^n. -2 + [2y^n] = 0$

$(1-x^2)y^{n+2} - 2x.ny^{n+1} - n(n-1)y^n - 2xy^{n+1} - 2ny^n$

$+ 2y^n = 0$

Let $x = 0$

$y^{n+2} - n(n-1)y^n - 2ny^n + 2y^n = 0$

$y^{n+2} + y^n[-n(n-1) - 2n + 2] = 0$

$y^{n+2} + y^n[-n^2 + n - 2n + 2] = 0$

$y^{n+2} + y^n[-n^2 - n + 2] = 0$

$y^{n+2} = -(y^n)_0[-n^2 - n + 2]$

$n = 0: y^2 = -y^0 . 2 = -2y^0$

$n = 1: y^3 = -y^1.[0] = 0$

$n = 2: y^4 = -y^2.[-4] = 4y^2 = 4(-2y^0) = -8y^0$

$n = 3: y^5 = -y^3.[-10] = 10y^3 = 10.0 = 0$

$n = 4: y^6 = -y^4.[-18] = 10y^4 = 18.4 - 2.y^0$

$n = 5: y^7 = -y^5.[-28] = 28(y^5)^0 = 28.0 = 0$

$y = y^0 + xy^1 + \frac{x^2 y^2}{2!} + \frac{x^3}{3!}y^3 + \ldots\ldots$

$y = y^0 + xy^1 + \frac{x^2}{2!}(-2)y^0 + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(4)(-2)y^0$

$$+ \frac{x^5}{5!}(0) + \frac{x^3}{6!}(18)y(-2)y^0 + \frac{x^7}{7!}(0)$$

$$y = y^0 + xy^1 - x^2y^0 - \frac{x^4}{3\times 1}y^0 = \frac{x^6}{5}y^0$$

$$y = y^0\left[1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5}\right] + y^1_0[x]$$

2i) $3e^{-4t} - 5e^{4t}$

$$= L[3e^{-4t} - 5e^{4t}] \Rightarrow L[3e^{-4t}] - L[5e^{4t}]$$

$$= 3\left[\frac{1}{s-a}\right] - 5\left[\frac{1}{s-a}\right]$$

$$= 3\left[\frac{1}{s+4}\right] - 5\left[\frac{1}{s-4}\right]$$

$$= \frac{3}{s+4} - \frac{5}{s-4}$$

ii) $\sin 4t + \cos 4t$

$$L[\sin 4t + \cos 4t] = L[\sin 4t] + L[\cos 4t]$$

$$= \frac{a}{s^2+a^2} + \frac{s}{s^2+a^2}$$

$$= \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2}$$

$$= \frac{4}{s^2+16} + \frac{s}{s^2+16} = \frac{4+s}{(s^2+16)}$$

iii) $t^3 + 2t^2 - t + 4$

$$t^n = \frac{n!}{s^{n+1}}$$

$$= \frac{3!}{s^{3+1}} + 2\left[\frac{2!}{s^{2+1}}\right] - \left[\frac{1!}{s^{1+1}}\right]$$

$$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

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iv) $e^{-2t} \cos 5t$

$$L[\cos 5t] = \frac{s}{s^2+a^2}$$

$$= \frac{s}{s^2+5^2} = \frac{s}{s^2+25}$$

$$L[e^{-2t} \cos 5t] = \frac{s+2}{[s+2]^2+25}$$

v) $t \sin 3t$

$$L[\sin 3t] = \frac{a}{s^2+a^2}$$

$$= \frac{3}{s^2+3^2}$$

$$= \frac{3}{s^2+9}$$

$$L[t \sin 3t] = -F'(s)$$

$$\frac{V \, du/ds - U \, dv/ds}{V^2}$$

$u = 3$ $\quad V = s^2+9$

$du/ds = 0$ $\quad dv/ds = 2s$

$$= \frac{[s^2+9] \cdot 0 - 3(2s)}{[s^2+9]^2}$$

$$= \frac{-6s}{[s^2+9]^2}$$

$$-F'(s) = -1 \left[\frac{-6s}{[s^2+9]^2}\right]$$

$$= \frac{6s}{[s^2+9]^2}$$

vi) $\dfrac{e^{-t} - e^{-2t}}{t}$

$$L\left[\frac{e^{-t} - e^{-2t}}{t}\right]$$

$$L[f(t)] = e^{-t} - e^{-2t} = \frac{1}{s+a} - \frac{1}{s+a}$$

$$= \frac{1}{s+1} - \frac{1}{s+2}$$

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$$F\sigma = \frac{1}{\sigma+1} - \frac{1}{\sigma+2}$$

$$\int F\sigma \quad L\left[\frac{f(t)}{t}\right] = \int_{\sigma=s}^{\infty} \frac{1}{\sigma+1} - \frac{1}{\sigma+2}\, d\sigma$$

$$= \int_{\sigma=s}^{\infty} \frac{1}{\sigma+1}\, d\sigma - \int_{\sigma=s}^{\infty} \frac{1}{\sigma+2}\, d\sigma$$

$$\ln[\sigma+1] - \ln[(\sigma+2)]\Big]_s^{\infty}$$

$$= \left[\ln(\sigma+1) - \ln(\sigma+2)\right]_s^{\infty}$$

$$= \left[\ln\frac{\sigma+1}{(\sigma+2)}\right]_s^{\infty} = \cancel{\ln\sigma+1}$$

$$= \ln\left[\frac{\alpha+1}{(\sigma+2)} - \frac{s+1}{(s+2)}\right] = -\ln\left[\frac{s+1}{(s+2)}\right] = \ln\left[\frac{(s+2)}{(s+1)}\right]$$

vii) $e^{4t}\cos 2t$

$$L[\cos 2t] = \frac{s}{s^2+2^2} = \frac{s}{s^2+4}$$

$$L[e^{4t}\cos 2t] = \cancel{\frac{s-4}{[(s-4)^2+4]}} \quad \frac{s-4}{[s-4]^2+4}$$

viii) $t\sin 2t$

$$L[t\sin 2t] = \frac{-d}{ds}[F(s)]$$

$$F(s) = L[\sin 2t] = \frac{2}{s^2+2^2}$$

$$F(s) = \frac{2}{s^2+4}$$

$$\frac{V\, du/ds - U\, dv/ds}{V^2} \qquad U = 2 \qquad V = s^2+4$$

$$du/ds = 0 \qquad dv/ds = 2s$$

$$= \frac{(s^2+4)\cdot 0 - 2(2s)}{(s^2+4)^2}$$

$$= \frac{-4s}{(s^2+4)^2}$$

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$$L[t\sin 2t] = f'(s)$$
$$= -1^2 \cdot \left[\frac{-4s}{(s^2+4)^2}\right]$$
$$= \frac{4s}{(s^2+4)^2}$$

ix) $t^3 + 4t^2 + 5$

$$L[t^3 + 4t^2 + 5]$$
$$= \frac{3!}{s^{3+1}} + 4\left[\frac{2!}{s^{2+1}}\right] + \frac{5}{s}$$
$$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$$

x) $e^{3t}(t^3+4)$

let $x = t^2 + 4$

$$L[e^{3t}x]$$
$$L[x] = L[t^3+4]$$
$$= L[t^2] + L[4]$$
$$= \cancel{4} \frac{2!}{s^2+1} + \frac{4}{s}$$
$$= \frac{2!}{s^3} + \frac{4}{s}$$
$$L[e^{3t}] = \frac{2}{[s-3]^3} + \frac{4}{[s-3]}$$

xi) $t^2 \cos t$

$$L[t^2\cos t] = t^2 L(\cos t]$$
$$f(s) = L(\cos t) = \frac{s}{s^2+1^2}$$
$$f(s) = \frac{s}{s^2+1^2}$$

$F'(s)$

$\cancel{d/ds} \quad u = s \qquad v = s^2 + 1^2$

$\cancel{du/ds = 1} \quad du/ds = 1 \qquad \frac{dv}{ds} = 2s$

$$= \frac{[s^2 + 1^2] \cdot 1 - 2s[s]}{[s^2 + 1^2]^2}$$

$$= \frac{s^2 + 1^2 - 2s^2}{[s^2 + 1^n]^2}$$

$$= \frac{-s^2 + 1}{[s^2 + 1^2]^2}$$

$$-f''(s) = \frac{-d}{ds}\left[\frac{s^2 - 1}{[s^2 + 1^2]^2}\right]$$

$u = s^2 - 1 \qquad v = [s^2 + 1]^2$

$\frac{du}{ds} = 2s \qquad dv/ds = 4s[s^2 + 1]$

$$\frac{[s^2 + 1]^2 \cdot 2s - [s^2 - 1][4s^2 + 4s]}{[s^2 + 1]^2}$$

$$= \frac{[2s^5 - 4s^2 + 2s] - [4s^2 - 4s]}{[(s^2 + 1)^2]}$$

$$= \frac{2s^5 - 4s^3 + 2s - 4s^5 + 4s}{(s^2 + 1)^2}$$

$$= \frac{-2s^5 - 4s^3 + 6s}{(s^2 + 1)^2}$$

$$= \frac{-2s^5 - 4s^2 + 6s}{s^4 + 2s^2 + 1}$$

$$f''(s) = -\frac{d}{ds}\left[\frac{-2s^5 - 4s^3 + 6s}{s^4 + 2s^2 + 1}\right]$$

$$f''(s) = \frac{2s^5 + 4s^2 - 6s}{s^4 + 2s^2 + 1}$$

3(i) $\dfrac{s-5}{(s-3)(s-4)}$

$$L^{-1}\left\{\frac{s-5}{(s-3(s-4)}\right\} = \frac{A}{(s-3)} + \frac{B}{(s-4)}$$

$$\frac{s-5}{(s-3)(s-4)} = \frac{A(s-4)+B(s-3)}{(s-3)(s-4)}$$

Assuming $s=4$

$4-5 = A(4-4)+B(4-3)$

$-1 = B(1)$

$B = -1$

Assuming $s=3$

$3-5 = A(3-4)+B(3-3)$

$-2 = A(-1)$

$A = 2$

$$\therefore \frac{s-5}{(s-3)(s-4)} = \frac{2}{s-3} + \frac{-1}{s-4}$$

$$= \frac{2}{s-3} - \frac{1}{s-4}$$

$$= 2\left[\frac{1}{s-3}\right] - \left[\frac{1}{s-4}\right]$$

$$= 2e^{3t} - e^{4t}$$

ii) $\dfrac{2s-6}{(s-2)(s-4)}$

$$L^{-1}\left[\frac{2s-6}{(s-2)(s-4)}\right]$$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{A}{(s-2)} + \frac{B}{(s-4)}$$

$2s-6 = A(s-4)+B(s-2)$

Cont'd of number 3

Assuming $s = 4$

$(2(4) - 6) = A(4-4) + B(4-2)$

$8 - 6 = A(0) + B(2)$

$2 = 2B$

$B = 1$

Assuming $s = 2$

$(2(2) - 6) = A(2-4) + B(2-2)$

$4 - 6 = -2A + 0$

$-2 = -2A$

$A = 1$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{1}{s-2} + \frac{1}{s-4}$$

$$L^{-1}\left[\frac{2s-6}{(s-2)(s-4)}\right]$$

$= e^{2t} + e^{4t}$

)

$$\frac{5s-8}{s(s-4)}$$

$$L^{-1}\left[\frac{5s-8}{s(s-4)}\right] = \frac{A}{s} + \frac{B}{s-4}$$

$5s - 8 = A(s-4) + B(s)$

Assuming $s = 4$

$5(4) - 8 = A(4-4) + B(4)$

$20 - 8 = A(0) + 4B$

$12 = 4B$

$B = 3$

Assuming $s = 0$

$5(0) - 8 = A(0-4) + B(0)$

$-8 = -4A$

$A = 2$

$$L^{-1}\left[\frac{s2-8}{s(s-4)}\right] = \frac{2}{s} + \frac{3}{s-4}$$

$$= \frac{2}{s} + 3\left[\frac{1}{s-4}\right]$$

$$= 2 + 3e^{4t}$$

4) $\frac{s-5}{s^2+4s+20}$

$$L^{-1}\left[\frac{s-5}{s^2+4s+20}\right]$$

$$F(s) = \frac{s-5}{s^2+4s+20} = \frac{s}{\cancel{4}(s+2)^2+16} - \frac{5}{(s+2)^2+16}$$

$$= \frac{s+2-2}{(s+2)^2+16} - \frac{5}{(s+2)^2+16} = \frac{s+2}{(s+2)^2+16} - \frac{2}{(s+2)^2+16}$$

$$= \frac{s+2}{(s+2)^2+\cancel{4}^2} - \frac{7 \times 4/4}{(s+2)^2+4^2}$$

$$= \frac{s+2}{(s+2)^2+4^2} - \frac{7}{4} \cdot \frac{4}{(s+2)^2+4^2}$$

$$f(t) = e^{-2t}\cos 4t - \frac{7}{4}e^{-2t}\sin 4t$$

$$= e^{-2}\left[\cos 4t - 7/_4 \sin 4t\right]$$

14) $\frac{s^2-3s-4}{(s-3)(s-1)^2}$

$$f(s) = \frac{A}{s-3} + \frac{B}{(s-1)^2} + \frac{C}{s-1}$$

$$A = \left.\frac{s^2-3s-4}{(s-1)^2}\right|_{s=3} = \frac{3^2-3(3)-4}{(3-1)^2} = -1$$

$$B = \left.\frac{s^2 - 3s - 4}{s-3}\right|_{s=1} = \frac{1^2 - 3(1) - 4}{1-3} = 3$$

$$C = \frac{d}{ds}\left[\frac{s^2 - 3s - 4}{s-3}\right]_{s=1} = \frac{(s-3)(2s-3) - (s^2 - 3s - 4)}{(s-3)^2}$$

at $s=1$

$$\frac{(1-3)(2(1)-3) - (1^2 - 3(1) - 4)}{(1-3)^2} = 2$$

$$f(s) = \frac{-1}{s-3} + \frac{3}{(s-1)^2} + \frac{2}{s-1}$$

$$f(t) = -e^{-3t} + 3te^{t} + 2e^{t}$$

$$= e^{t}[3t + 2] - e^{3t}$$

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