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15/ENG02/012 - Computer Engr.

i) $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$ $(y)_0$

$(1-x^2)y'' - 2x\frac{dy'}{\cancel{d}} + 2y = 0$ $(y)_0$

$W_1 = (1-x^2)y''$

$u = y''$ $u^n = y^{(n+2)}$

$v = 1-x^2$ $v' = -2x$ $v'' = -2$ $v''' = 0$

$W_1^n = y^{(n+2)}(1-x^2) + ny^{(n+1)} \cdot -2x + \frac{n(n-1)y^n \cdot -\cancel{2}}{\cancel{2}!} + 0$

$W_1^n = y^{(n+2)}(1-x^2) + -2xny^{(n+1)} - n(n-1)y^n$

$W_1^n = (1-x^2)y^{(n+2)} - 2xny^{(n+1)} - (n^2+n)y^n$

$W_2 = -2xy'$

$v = -2x$ $v' = -2$ $v'' = 0$

$u = y'$ $u^n = y^{(n+1)}$

$W_2^n = y^{(n+1)} \cdot -2x + ny^n \cdot -2 + 0$

$W_2^n = -2xy^{(n+1)} - 2ny^n$

$W_3 = 2y$

$v = 2$ $v' = 0$

$u = y$ $u^n = y^n$

$W_3^n = y^n \cdot 2 + 0$

$W_3^n = 2y^n$

$\therefore y^n = y^{(n+2)}(1-x^2) - 2xny^{(n+1)} - (n^2-n)y^n - 2xy^{(n+1)} - 2ny^n + 2y^n$

$y^n = (1-x^2)y^{(n+2)} - 2xny^{(n+1)} - 2xy^{(n+1)} - (n^2-n)y^n - 2ny^n + 2y^n$

$y^n = (1-x^2)y^{(n+2)} - 2xy^{(n+1)}(n+1) - y^n(n^2-n+2n+1)$

$y^n = (1-x^2)y^{(n+2)} - (n+1)2xy^{(n+1)} - (n^2+n+1)y^n$

$\therefore$ at $x = 0$

$$(1-0^2)\,y^{(n+2)} - \cancel{(n+1)\,2(0)\,y^{(n+1)}}^{\,0} - (n^2+n+1)\,y^n$$

$$y^{(n+2)} - (n^2+n+1)\,y^n$$

$$\left(y^{(n+2)}\right)_0 = (n^2+n+1)\,y^n$$

$\therefore$ at $n = 0$

$$(y^2)_0 = (0^2+0+1)(y^0)_0$$

$$(y^2)_0 = (y^0)_0$$

at $n = 1$

$$(y^3)_0 = (1^2+1+1)(y^1)_0 = 3(y^1)_0$$

at $n = 2$

$$(y^4)_0 = (2^2+2+1)[y^2]_0 = 7[y^2]_0 = 7[y^0]_0$$

at $n = 3$

$$[y^5] = (3^2+3+1)[y^3]_0 = 13[y^3]_0 = 13\cdot 3[y^1]_0 = 39[y^1]_0$$

at $n = 4$

$$[y^6] = (4^2+4+1)[y^4]_0 = 21[y^4]_0 = 21\cdot 7[y^0]_0 = 147[y^0]_0$$

at $n = 5$

$$[y^7] = (5^2+5+1)[y^5]_0 = 31[y^5]_0 = 31\cdot 39[y^1]_0 = 1209[y^1]$$

$$\therefore y = (y^0)_0 + x(y^1)_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^4)_0$$

$$+ \frac{x^5}{5!}(y^5)_0 + \frac{x^6}{6!}(y^6)_0 + \frac{x^7}{7!}(y^7)_0 + \cdots$$

$$y = (y^0)_0 + x(y')_0 + \frac{x^2}{2}(y^0)_0 + \frac{x^3 \cdot \cancel{3}}{\cancel{3} \times 2}(y')_0 + \frac{x^4}{4 \times 3 \times 2} \cdot 7(y^0)_0$$

$$+ \frac{x^5}{5 \times 4 \times \cancel{3} \times 2} \cdot \overset{13}{\cancel{39}}(y')_0 + \frac{x^6}{6 \times 5 \times 4 \times \cancel{3} \times 2} \cdot \overset{49}{\cancel{147}}(y^0)_0 + \frac{x^7}{7!} \cdot 1209(y')_0$$

$$y = (y^0)_0 + x(y')_0 + \frac{x^2}{2}(y^0)_0 + \frac{x^3}{2}(y')_0 + \frac{7x^4}{24}(y^0)_0 + \frac{13x^5}{40}(y')_0$$

$$+ \frac{49x^6}{240}(y^0)_0 + \frac{403x^7}{1608}(y')_0$$

$$y = (y^0)_0 + \frac{x^2}{2}(y^0)_0 + \frac{7x^4}{24}(y^0)_0 + \frac{49x^6}{240}(y^0)_0 + x(y')_0$$

$$+ \frac{x^3}{2}(y')_0 + \frac{13x^5}{40}(y')_0 + \frac{403x^7}{1608}(y')_0$$

$$y = (y^0)_0\left[1 + \frac{x^2}{2} + \frac{7x^4}{24} + \frac{49x^6}{240} + \cdots\cdots\right] + (y')_0\left[x + \frac{x^3}{2} + \frac{13x^5}{40} + \frac{403x^7}{1608} + \cdots\cdots\right]$$

2) i) $L[3e^{-4t} - 5e^{4t}]$

$$= L[3e^{-4t}] - L[5e^{4t}]$$

$$= \frac{3}{s+4} - \frac{5}{s-4}$$

ii) $L[\sin 4t + \cos 4t]$

$$= L[\sin 4t] + L[\cos 4t]$$

$$= \frac{4}{s^2+16} + \frac{s}{s^2+16}$$

$$= \frac{4+s}{s^2+16}$$

viii) $t\sin 2t$

$$L[\sin 2t] = \frac{2}{s^2+4}$$

$$L[t\sin 2t] = -1\frac{d}{ds}\left[\frac{2}{s^2+4}\right]$$

$$= -1 \times \frac{-4s}{(s^2+4)^2}$$

$$x(s) = \frac{4s}{(s^2+4)^2}$$

ix) t^3+4t^2+5

$$= L[t^3] + L[4t^2] + L[5]$$

$$= \frac{3!}{s^4} + \frac{4 \cdot 2!}{s^3} + \frac{5}{s}$$

$$x(s) = \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$$

xi) $t^2\cos t$

$$L[\cos t] = \frac{s}{s^2+1}$$

$$L[t^2\cos t] = (-1)^2\frac{d^2}{ds^2}\left[\frac{s}{s^2+1}\right]$$

$$= 1 \times \frac{2s^3-6s}{4s^3+4s}$$

$$= \frac{\cancel{2}}{\cancel{2}}\left[\frac{s^3-3s}{2s^3+2s}\right]$$

$$x(s) = \frac{s^3-3s}{2s^3+2s}$$

xx) $\cancel{t^2}\, e^{3t}(t^2+4)$

$$L[t^2+4] = \frac{2}{s^3} + \frac{4}{s}$$

$$(s) = L[e^{3t}(t^2+4)] = \frac{2}{(s-3)^3} + \frac{4}{s-3}$$

xvi) $\frac{\sinh 2t}{t}$

$L[\sinh 2t] = \frac{2}{s^2-4}$

$x(s) = L\left[\frac{\sinh 2t}{t}\right] = \int_s^{\infty} \frac{2}{s^2-4} = 2\int_s^{\infty} \frac{1}{\sigma^2-4}$

$= 2\left[\frac{1}{2}\tan^{-1}\frac{\sigma}{2}\right]_s^{\infty} = \left[\tan^{-1}\frac{\sigma}{2}\right]_s^{\infty}$

$= \tan^{-1}\frac{\infty}{2} - \tan^{-1}\frac{s}{2}$

$= \tan^{-1}\frac{2}{s}$

3] Convert the following to time (t) domains:

i) $\frac{s-5}{(s-3)(s-4)}$

$= L^{-1}\left[\frac{A}{(s-3)} + \frac{B}{(s-4)}\right]$

$\frac{s-5}{(s-3)(s-4)} = \frac{A}{(s-3)} + \frac{B}{(s-4)}$

$s-5 = A(s-4) + B(s-3)$

at $s=4$

$4-5 = 0 + B(4-3)$

$-1 = B$; $B = -1$

at $s=3$

$3-5 = A(3-4) + 0$

$-2 = -A$

$\therefore A = 2$

$x(t) = L^{-1}\left[\frac{2}{(s-3)} + \frac{-1}{(s-4)}\right]$

$\therefore x(t) = 2e^{3t} - e^{4t}$

ii) $\frac{2s-6}{(s-2)(s-4)}$ $\therefore x(t) = L^{-1}\left[\frac{A}{(s-2)} + \frac{B}{(s-4)}\right]$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{A}{(s-2)} + \frac{B}{(s-4)}$$

$\therefore 2s-6 = \cancel{\#} A(s-4) + B(s-2)$

at $s=2$.

$2(2)-6 = A(2-4) + 0$

$4-6 = -2A$

$-2 = -2A$

$\therefore A = 1$

at $s=4$

$2(4)-6 = \cancel{A(4} 0 + B(4-2)$

$8-6 = 2B$

$2 = 2B$

$\therefore B = 1$

$$\therefore x(t) = L^{-1}\left[\frac{1}{(s-2)} + \frac{1}{(s-4)}\right]$$

$x(t) = e^{2t} + e^{4t}$

iii) $\frac{5s-8}{s(s-4)}$

$$x(t) = L^{-1}\left[\frac{A}{s} + \frac{B}{s-4}\right]$$

$\therefore 5s-8 = A(s-4) + Bs$

at $s=0$

$0-8 = A(0-4) + 0$

$\frac{-8}{-4} = \frac{-4}{-4}A$

$A = 2$

at $s=4$

$5(4)-8 = A + 4B$

$20-8 = 4B$

$12 = 4B$

$B = 3$

$$\therefore L^{-1}\left[\frac{2}{s} + \frac{3}{s-4}\right]$$

$x(t) = 2 + 3e^{4t}$

iv) $\dfrac{s^2-3s-4}{(s-3)(s-1)^2}$

$x(t) = L^{-1}\left[\dfrac{A}{(s-3)} + \dfrac{B}{(s-1)} + \dfrac{C}{(s-1)^2}\right]$

~~$x(t)$~~ $s^2-3s-4 = A(s-1)^2 + B(s-3)(s-1) + C(s-3)$

at $s=3$

$3^2 - 3(3) - 4 = A(3-1)^2 + 0 + 0$

$9-9-4 = 4A$

$-4 = 4A$

$\therefore A = -1$

$1-3-4$ at $s=1$

$1^2 - 3(1) - 4 = C(1-3)$

$-6 = -2C$

$C = 3$

$s^2-3s-4 = As^2 - 2As + A + Cs - C3 + Bs^2 - B4s + B3$

Comparing Coefficients:

$A + 3B + 3C = -4$

$-1 + 3B - 3(3) = -4$

$3B = 10-4$; $3B = 6$ $\therefore B = 2$

$\therefore x(t) = L^{-1}\left[\dfrac{-1}{(s-3)} + \dfrac{2}{(s-1)} + \dfrac{3}{(s-1)^2}\right]$

$x(t) = -e^{3t} + 2e^{t} + 3te^{t}$