GET 3?? ASSIGNMENT 4 $3^4 = 64$

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15/ENG01/009

CHEMICAL ENGINEERING

1) $[1-x^2]\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$[1-x^2]y'' - 2xy' + 2y = 0$

Let $u = y''$, $u' = y'''$, $u'' = y^{iv}$

$u^n = y^{n+2}$

$V = 1-x^2$, $V' = -2x$, $V'' = -2$, $V''' = 0$

$W_1^n = u^nV + nu^{n-1}V' + \frac{n[n-1]u^{n-2}V''}{2!} +$

$= y^{n+2}[1-x^2] + ny^{n+1} - [2x] + n[n-1]$

$\frac{n[n-1]y^n \cdot -2}{2\times 1}$

$= [1-x^2]y^{n+2} - 2xny^{n+1} - n[n-1]y^n$

$W_2^n = u^nV + nu^{n-1}V' + \frac{n[n-1]u^{n-2}V''}{2!} +$

Let $u = y'$, $u' = y''$, $u^n = y^{n+1}$

$V = x$, $V' = 1$, $V'' = 0$

$W_2^n = x y^{n+1} + ny^n \cdot 1 = xy^{n+1} + ny^n$

$W_3^n = y^n$

Combining

$W_n = W_1^n + W_2^n + W_3^n$

$0 \Rightarrow [1-x^2]y^{n+2} - 2xny^{n+1} - n[n-1]y^n$

$- 2[xy^{n+1} + nyn] + 2y^n$

If $x = 0$

$0 = [1-0^2]y^{n+2} - 2[0]ny^{n+1} - [n^2-n]y^n$

$- 2[0\, y^{n+1} + ny^n] + 2y^n$

$0 = y^{n+2} + y^n[2 - 2n - n^2 + n] = y^{n+2} +$

$y^n[2 - n - n^2]$

$y^{n+2} = [n^2 + n - 2]y^n$

For $n = 1$

$y^3 = y''' = [1^2 + 1 - 2]y' = 0$

For $n = 2$

$y^{2+2} = y^{iv} = [2^2 + 2 - 2]y''$

$= 4y''$

for $n = 3$

$y^{3+2} = y^v[3^2 + 3 - 2]y''' = 10y''' = 0$

For $n = 4$

$y^{4+2} = y^{vi} = [4^2 + 4 - 2]y^{iv} = 18y^{iv} = 72y''$

$\therefore y^n = f[x] + y'f'[x] + \frac{y''f''[x]}{2!} + \frac{y'''f'''[x]}{3!} +$

$y^n = 1 + y' + \frac{y''}{2!} + 0 + \frac{4y''}{4!} + 0 + \frac{72y''}{6!}$

$y^n = 1 + y' + y''[\frac{1}{2} + \frac{1}{6} + \frac{1}{10}]$

$y^n = 1 + y' + \frac{23y''}{30}$

2 i) $L[3e^{-4t} - 5e^{4t}]$

$= L[3e^{-4t}] - L[5e^{4t}]$

$= 3L[e^{-4t}] - 5L[e^{4t}]$

$= 3\frac{1}{s+4} - 5\frac{1}{s-4}$

$L[f(t)] = [\frac{3}{s+4} - \frac{5}{s-4}]$

ii) $\sin 4t + \cos 4t$

$= \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2} = \frac{4}{s^2+16} + \frac{s}{s^2+16}$

iii) $t^3 + 2t^2 - t + 4$

$= L[t^3] + L[2t^2] - L[t] + 4$

$= \frac{3!}{s^4} + 2[\frac{2}{s^3}] - \frac{1}{s^2} + \frac{4}{s}$

$= \frac{3}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

iv) $e^{-2t}\cos 5t = [\frac{1}{s+2}] \cdot \frac{s}{s^2+5^2}$

$= \frac{s}{(s+2)\cdot(s^2+5^2)}$

$= \frac{s}{(s+2)(s^2+25)}$

$= \frac{s}{s^3 + 25s + 2s^2 + 50}$

v) $t \sin 3t = [-1]^n \cdot \frac{d^n}{dx^n}[f[t]]$

$-1 \frac{d}{dx}[L[\sin 3t]$

$= -1 \frac{d}{dx}\left[\frac{3}{s^2+3^2}\right]$

$du = 0, \quad dv = 2s$

$= -\left[-\frac{6s}{[s^2+9]^2}\right] = \frac{6s}{s^4+18s^2+81}$

vi) $\frac{e^{-t} - e^{-2t}}{t} = t^{-1}[e^{-t} - e^{-2t}]$

$= [-1]^{-1} \frac{dx}{d}\left[\frac{1}{s+1} - \frac{1}{s+2}\right]$

$= -\frac{dx}{d}\left[\frac{s+2-s-1}{[s+1][s+2]}\right]$

$= \frac{dx}{d}\left[\frac{1}{s^2+3s+2}\right]$

$du = 0, \quad dv = 2s+3$

$= -\left[\frac{2s+3}{[s^2+3s+2]^2}\right]$

vii) $e^{4t}\cos 2t = \frac{s}{[s-4]^2+2^2} = \frac{s}{s^2-8s+16+4}$

$= \frac{s}{s^2-8s+20}$

viii) $t \sin 2t = [-1]^1 \frac{d}{dx}\left[\frac{2}{s^2+2^2}\right]$

$du = 0, \quad dv = 2s$

$= -1\left[\frac{4s}{[s^2+2^2]^2}\right] = \frac{4s}{[s^4+8s^2+16]}$

) $t^3 + 4t^2 + 5 =$

$L[t^3] + L[4t^2] + L[5]$

$\frac{2}{s^4} + 4\left[\frac{1}{s^2}\right] + \frac{5}{s}$

$\frac{2}{s^4} + \frac{4}{s^2} + \frac{5}{s}$

x) $e^{3t}(t^2+4] = e^{3t}t^2 + 4e^{3t}$

$= \frac{2!}{[s-3]^{1+1}} + \frac{4}{s-3} = \frac{2}{[s-3]^2}$

$+ \frac{4}{s-3}$

xi) $t^2\cos t = [-1]^2 \frac{d^2}{dx^2}\left[\frac{s}{s^2+1}\right]$

$= du = 1$

$dv = 2s$

$= \frac{d}{dx}\left[\frac{1-s^2}{s^2+1}\right] = du = -2s$

$dv = 2s$

$= \left[\frac{-2s^3 - 2s - 2s + 2s^3}{[s^2+1]^2}\right] = \frac{-4s}{[s^2+1]^2}$

x) $\frac{\sinh 2t}{t} = \frac{1}{2}\ln[s^2-2^2] - \ln s$

$= \frac{1}{2}\ln[s^2-4] - \ln s$

3i) $\frac{s-5}{[s-3][s-4]} = \frac{A}{s-3} + \frac{B}{s-4}$

$s-5 = A[s-4] + B[s-3]$

$3-5 = A[3-4] + B[3-3]$

$A = \frac{-2}{-1} = 2$

$4-5 = A[4-4] + B(4-3]$

$B = -1$

$= \frac{2}{s-3} - \frac{1}{s-4}$

$L^{-1}\left[\frac{2}{s-3} - \frac{1}{s-4}\right]$

$= 2e^{3t} - e^{4t}$

ii) $\frac{2s-6}{[s-2][s-4]} = \frac{A}{s-2} + \frac{B}{s-4}$

$2s-6 = A[s-4] + B[s-2]$

$2[2]-6 = A[2-4]+B[2-2]$

$-2 = -2A$

$A=1$

$2(4)-6 = A(4-4)+B(4-2)$

$2 = 2B$

$B=1$

$= \frac{1}{s-2} + \frac{1}{s-4}$

$= e^{2t} + e^{4t}$

iii) $\frac{5s-8}{s[s-4]} = \frac{A}{s} + \frac{B}{s-4}$

$A[s-4]+Bs = 5s-8$

$A(0-4)+B\,0 = 5[0]-8$

$A=2$

$A[4-4]+4B = 5[4]-8$

$B = \frac{12}{4} = 3$

iv) $\frac{s^2-3s-4}{[s-3][s-1]^2} = \frac{A}{s-3} + \frac{B}{s-1} + \frac{C}{[s-1]^2}$

$s^2-3s-4 = A[s-1]^2+B[s-3][s-1]+C[s-3]$

$s^2-3s-4 = A[s^2-2s+1]+B[s^2-4s+3]+C[s-3]$

$A+B=1$

$C-2A-4B=-3$

$A+3B-3C=-4$

$1^2-3[1]-4 = C[1-3]$

$-6=-2C$

$C=3$

$A+3B-3[3]=-4$

$A+3B=-4+9=5$

$A+3B=5$

$-\quad A+B=1$

$2B=4$

$B=2$

$A+B=1$

$A=1-B=1-2=-1$

$= L^{-1}\left[\frac{-1}{s-3} + \frac{2}{s-1} + \frac{2}{[s-1]^2}\right]$

$= -e^{3t}+2e^{t}+\frac{3}{2}t^2e^{t}$

v) $\frac{s-5}{s^2+4s+20} = \frac{s-5}{s^2+4s+4+16}$

$= \frac{s-5}{[s+2]^2+16}$

$\frac{s-5}{[s+2]^2+4^2}$

$= e^{2t}\cos 4t$