AMOO, EMMANUEL
15/ENG06/011
ENG 381. ASSIGNMENT 4.

1. $(1-x^2)\,dy/dx - 2x\,dy/dx + 2y = 0$

$(1-x^2)y'' - 2xy' + 2y = 0.$

$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \ldots$

$[y^{(2+n)} \cdot (1-x^2) + n y^{(1+n)} \cdot (-2x) + \frac{n(n-1)y^n \cdot (-2)}{2!}] + [y^{(1+n)} \cdot -2x + ny^n \cdot -2$

$+ [2y^n] = 0.$

$(1-x^2)y^{(2+n)} - 2xny^{(1+n)} - n(n-1)y^{(n)} - 2xy^{(1+n)} - 2ny^{(n)} + 2n^{(n)} = 0.$

let $x = 0$.

$y^{(2+n)} - n(n-1)y^{(n)} - 2ny^{(n)} + 2y^n = 0.$

$y^{n+2} + y^n[-n(n-1) - 2n + 2] = 0.$

$y^{n+2} + y^n(-n^2 - n + 2) = 0.$

$y^{n+2} = -(y^n)_0 (-n^2 - n + 2).$

when $n=0$; $y^2 = -y^0 \cdot 2 = -2y^0.$

$n=1$; $y^3 = -y^1 \cdot [0] = 0.$

$n=2$; $y^4 = -y^2[-4] = 4y^2 = 4(-2y^0) = -8y^0.$

$n=3$; $y^5 = -y^3[-10] = 10y^3 = 10 \cdot 0 = 0.$

$n=4$; $y^6 = -y^4 \cdot [-18] = 18y^4 = 18 \cdot y^4 = -2y^0.$

$n=5$; $y^7 = -y^5 \cdot [-28] = 28(y^5)_0 = 28 \cdot 0 = 0.$

$y = y^0 + xy^1 + \frac{x^2}{2!}y^3 + \frac{x^3}{3!}y^3 + \ldots$

$y = y^0 + xy^1 + \frac{x^2}{2!}(-2)y^0 + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(4)(-2)y^0 + \frac{x^5}{5!}(0)$

$+ \frac{x^6}{6!}(4)4(-2)y^0 + \frac{x^7}{7!}(0).$

$y = y^0 + xy^1 - x^2y^0 - \frac{x^4}{3\times1}y^0 - \frac{x^6}{5}y^0.$

$y = y^0[1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5}] + y'_0(x).$

2i. $3e^{-4t} - 5e^{4t}.$

Recall $L\{f(t) \pm g(t)\} = L\{f(t)\} \pm L\{g(t)\}.$

$= L\{3e^{-4t} - 5e^{4t}\} \Rightarrow L\{3e^{-4t}\} - L\{5e^{4t}\} \Rightarrow 3L\{e^{-4t}\} - 5L\{$

$= 3.\left\{\frac{1}{s+4}\right\} - 5.\left\{\frac{1}{s-4}\right\}.$

$$= 3\left\{\frac{1}{s-(-4)}\right\} - 5\left\{\frac{1}{s-4}\right\}.$$

$$= \frac{3}{s+4} - \frac{5}{s-4}.$$

ii. $\sin 4t + \cos 4t$.

$\Rightarrow L\{\sin 4t + \cos 4t\} = L\{\sin 4t\} + L\{\cos 4t\}$.

$$= \frac{s}{s^2+a^2} + \frac{s}{s^2+a^2}$$

$$= \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2}$$

$$= \frac{4}{s^2+16} + \frac{s}{s^2+16} = \frac{4+3}{(s^2+16)}$$

iii. $t^3+2t^2-\cancel{t}+4$.

$\Rightarrow L\{t^3+2t^2-t+4\} = L\{t^3\} + L\{2t^2\} - L\{t\} + L\{4\}$.

$$t^n = \frac{n!}{s^{n+1}}.$$

$$= \frac{3!}{s^{3+1}} + 2\left\{\frac{2!}{s^{2+1}}\right\} - \left\{\frac{1!}{s^{1+1}}\right\} + \frac{4}{s}.$$

$$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}.$$

iv. $e^{-2t}\cos 5t$.

Recall the first shift theorem.

$$L\{\cos 5t\} = \frac{s}{s^2+a^2}$$

$$= \frac{s}{s^2+5^2}$$

$$= \frac{s}{s^2+25}.$$

replacing s by a shift of e^{-2t}; $s+2$.

$$\therefore L\{e^{-2t}\cos 5t\} = \frac{s+2}{(s+2)^2}$$

s^2+3^2

$$F(s) = \frac{3}{s^2+9}$$

$$L\{t \sin 3t\} = -F'(s)$$

$F'(s)$ = using quotient rule.

$u = 3 \quad du/ds = 0$.

$v = s^2+9 \quad du/ds = 2s$.

$$\therefore \frac{v\,du/ds - u\,dv/ds}{v^2}$$

$$= \frac{[s^2+9].0 - 3(2s)}{(s^2+9)^2}$$

$$= \frac{-6s}{(s^2+9)^2}$$

$$-F'(s) = -1 \cdot \left\{\frac{-6s}{(s^2+9)^2}\right\}$$

$$= \frac{6s}{(s^2+9)^2}$$

vi. $\dfrac{e^{-t} - e^{-2t}}{t}$

$$L\left\{\frac{e^{-t} - e^{-2t}}{t}\right\}$$

$$\lim_{t\to 0}\left\{\frac{e^{-t} - e^{-2t}}{t}\right\} = \frac{1-1}{0} = \frac{0}{0} = \text{Indeterminate}.$$

Applying L'Hospital's rule.

$$\lim_{t\to 0}\left\{\frac{-1e^{-t} - (-2)e^{-2t}}{1}\right\} = \left\{\frac{-1+2}{1}\right\} = \frac{1}{1} = 1 \text{ (determinate)}.$$

$$L\left\{\frac{f(t)}{t}\right\} = \int_{\sigma=s}^{\infty} F\sigma \, d\sigma$$

$$s+5 \qquad s+5$$

$$= \frac{1}{s+1} - \frac{1}{s+2}.$$

$$F\sigma = \frac{1}{\sigma+1} - \frac{1}{\sigma+2}$$

$$\int F\sigma \quad L\left\{\frac{f(t)}{t}\right\} = \int_{\sigma=s}^{\infty} \frac{1}{\sigma+1} - \frac{1}{\sigma+2} \, d\sigma$$

$$= \int_{\sigma=s}^{\infty} \frac{1}{\sigma+1} \, d\sigma - \int_{\sigma=s}^{\infty} \frac{1}{\sigma+2} \, d\sigma.$$

$$= [\ln(\sigma+1) - \ln(\sigma+2)]_s^{\infty}$$

$$= [\ln(\sigma+1) - \ln(\sigma+2)]_s^{\infty}$$

$$= \left[\ln \frac{\sigma+1}{(\sigma+2)}\right]_s^{\infty} = \ln\left[\frac{\infty+1}{(\infty+2)} - \frac{s+1}{(s+2)}\right].$$

$$= -\ln\left[\frac{s+1}{(s+2)}\right] = \ln\left[\frac{(s+2)}{s+1}\right].$$

$e^{4t}\cos 2t$

$$L\{e^{4t}\cos 2t\} = e^{4t} L\{\cos 2t\}.$$

$$L\{\cos 2t\} = \frac{s}{s^2+2^2}$$

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$F'(s)$ = using quotient rule.

$u = 2 \quad du/ds = 0$.

$v = s^2+4 \quad dv/ds = 2s$.

$$\frac{v\,du/ds - u\,dv/ds}{v^2}$$

$$= \frac{(s^2+4)\cdot 0 - 2\cdot 2s)}{(s^2+4)^2}$$

$$= \frac{-4s}{(s^2+4)^2}$$

$\therefore L\{t \sin 2t\} = -f'(s)$

$$= -1 \cdot \left\{\frac{-4s}{(s^2+4)^2}\right\}$$

$$= \frac{4s}{(s^2+4)^2}$$

Ex. t^3+4t^2+5.

$L\{t^3+4t^2+5\} = L\{t^3\} + 4L\{t^2\} + L(5)$.

$$= \frac{3!}{s^{3+1}} + 4\left\{\frac{2!}{s^{2+1}}\right\} + \frac{5}{s}$$

$$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$$

x. $e^{3t}(t^2+4)$

let $x = t^2+4$.

$L\{e^{3t}x\}$.

$\therefore L\{x\} = L\{t^2+4\}$.

$= L\{t^2\} + L\{4\}$

$$= \frac{2!}{s^{2+1}} + \frac{4}{s}$$

$$= \frac{2}{s^3} + \frac{4}{s}$$

replacing s by a shift of s-3.

$$L\{e^{3t}x\} = \frac{2}{(s-3)^3} + \frac{4}{(s-3)}$$

xi) $t^2 \cos t$.

$L\{t^2 \cos t\} = t^2 \cdot L\{\cos t\}$

$\mathcal{L}(s) = L\{\cos t\} = \frac{s}{s^2+1^2}$

$f(s) = \frac{s}{s^2+1^2}$

$f'(s)$ = using quotient rule.

$u = s \quad \frac{du}{ds} = 1$

$v = s^2+1^2 \quad \frac{dv}{ds} = 2s$.

$\frac{v\,du/ds - u\,dv/ds}{v^2}$

$\Rightarrow \frac{\{s^2+1^2\} \cdot 1 - 2s(s)}{(s^2+1^2)^2}$.

$= \frac{s^2+1^2 - 2s^2}{(s^2+1^2)^2}$.

$= \frac{-s^2+1}{(s^2+1^2)^2}$

Recall

$-f''(s) = -\frac{d}{ds}\left\{\frac{s^2-1}{(s^2+1^2)^2}\right.$

using quotient rule;

$\frac{v\,du/ds - u\,dv/ds}{v^2}$

$u = s^2-1 \quad \frac{du}{ds} = 2s$

$v = (s^2+1)^2 \quad \frac{dv}{ds} = 4s(s^2+1)$.

$\frac{(s^2+1)^2 \cdot 2s - (s^2+1)(4s^3+4s)}{(s^2+1)^2}$

$= \frac{(2s^5 - 4s^3 + 2s - 4s^5 + 4s)}{(s^2+1)^2}$

$= \frac{-2s^5 - 4s^3 + 6s}{s^4+2s^2+1}$

$f''(s) = -\frac{d}{ds} = \left\{\frac{-2s^5 - 4s^3 + 6s}{s^4+2s^2+1}\right\}$.

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$$f''(s) = \frac{2s^5 + 4s^3 \cdot 6 - 6s}{s^4 + 2s^2 + 1}$$ //

xvi) $\frac{\sinh 2t}{t}$

$= L\left\{\frac{\sinh 2t}{t}\right\}$.

3i. $\frac{s-5}{(s-3)(s-4)}$

$$L^{-1}\left\{\frac{s-5}{(s-3)(s-4)}\right\} = \frac{A}{(s-3)} + \frac{B}{(s-4)}$$

$$\frac{s-5}{(s-3)(s-4)} = \frac{A(s-4) + B(s-3)}{(s-3)(s-4)}$$

if $s = 4$.

$4 - 5 = A(4-4) + B(4-3)$

$-1 = B(1)$

$B = -1$

Assuming $s = 3$.

$3 - 5 = A(3-4) + B(3-3)$

$-2 = A(-1)$

$A = 2$.

$$\therefore \frac{s-5}{(s-3)(s-4)} = \frac{2}{(s-3)} - \frac{1}{s-4}$$

$$= 2\left\{\frac{1}{s-3}\right\} - \left\{\frac{1}{s-4}\right\}$$

$$= 2\left\{\frac{1}{s-3}\right\} - \left\{\frac{1}{s-4}\right\}$$

$= 2e^{3t} - e^{4t}$ //

ii. $\frac{2s-6}{(s-2)(s-4)}$

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$$L^{-1}\left\{\frac{2s-6}{(s-2)(s-4)}\right\}$$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{A}{(s-2)} + \frac{B}{(s+4)}$$

$$2s-6 = A(s-4)+B(s-2)$$

Assuming $s=4$.

$$[2(4)-6] = A(4-4) + B(4-2)$$

$$8-6 = A(0) + B(2)$$

$$2 = 2B$$

$$B = 1.$$

Assuming $s=2$.

$$2(2)-6 = A(2-4) + B(2-2)$$

$$4-6 = -2A + 0.$$

$$-2 = -2A$$

$$A = 1.$$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{1}{s-2} + \frac{1}{s-4}$$

$$L^{-1}\left\{\frac{2s-6}{(s-2)(s-4)}\right\} = e^{2t} + e^{4t}$$ //

Assuming $s = 0$.

$$5(0) - 8 = A(0-4) + B(0)$$

$$-8 = -4A$$

$$A = 2.$$

$$L^{-1}\left\{\frac{5s-8}{s(s-4)}\right\} = \frac{2}{s} + \frac{3}{s-4}$$

$$= \frac{2}{s} + 3\left\{\frac{1}{s-4}\right\}.$$

$$= 2t \quad 3e^{4t}$$

iv. $\dfrac{s-5}{s^2+4s+20}$

$$L^{-1}\left\{\frac{s-5}{s^2+4s+20}\right\}$$

$$\Rightarrow F(s) = \frac{s-5}{s^2+4s+4+16} \Rightarrow \frac{s}{(s+2)^2+16} - \frac{5}{(s+2)^2+16}$$

$$= \frac{s+2-2}{(s+2)^2+16} - \frac{5}{(s+2)^2+16} = \frac{s+2}{(s+2)^2+16} - \frac{2}{(s+2)^2+16}$$

$$= \frac{s+2}{(s+2)^2+4^2} - \frac{7 \times 4/4}{(s+2)^2+4^2}$$

$$= \frac{s+2}{(s+2)^2+4^2} - \frac{7}{4} \cdot \frac{4}{(s+2)^2+4^2}.$$

$$f(t) = e^{-2t}\cos 4t - 7/4\, e^{-2t}\sin 4t.$$

$$= e^{-2t}[\cos 4t - 7/4 \sin 4t].$$

v. $\dfrac{s^2-3s-4}{(s-3)(s-1)^2}$

$$f(s) = \frac{A}{s-3} + \frac{B}{(s-1)^2} + \frac{C}{s-1}$$

$$A = \left.\frac{s^2-3s-4}{(s-1)^2}\right]_{s=3} = \frac{3^2-3(3)-4}{(3-1)^2} = -1$$

$$B = \left.\frac{s^2 - 3s - 4}{s - 3}\right|_{s=1} = \frac{1^2 - 3(1) - 4}{1 - 3} = 3.$$

$$C = \frac{d}{ds}\left[\frac{s^2 - 3s - 4}{s - 3}\right]_{s=1} = \frac{(s-3)(2s-3) - (s^2 - 3s - 4)}{(s-3)^2}.$$

at $s = 1$.

$$\frac{(1-3)(2(1)-3) - [1^2 - 3(1) - 4]}{(1-3)^2} = 2.$$

$$f(s) = \frac{-1}{s-3} + \frac{3}{(s-1)^2} + \frac{2}{s-1}.$$

$$f(t) = -e^{-3t} + 3t\,e^{t} + 2e^{t}$$

$$= e^{t}(3t+2) - e^{3t} \,//.$$