Assignment

1) $(1-x^2)y^2 - 2xy' + 2y = 0$

w_1: $u = y^2$ $\quad v = 1-x^2$

$u^n = y^{n+2}$ $\quad v' = -2x$

$u^{n-1} = y^{n+1}$ $\quad v^2 = -2$

$u^{n-2} = y^n$

$= (1-x^2)y^{n+2} - n2xy^{n+1} - \frac{n(n-1)}{2}y^n(-2)$

$= (1-x^2)y^{n+2} - 2xny^{n+1} - n(n-1)y^n$

w_2: $u = y'$ $\quad v = 2x$

$u^n = y^{1+n}$ $\quad v' = 2$

$u^{n-1} = y^n$

$= 2xy^{n+1} + 2ny^n$

w_3 ; $u = y$ $\qquad v = 2$

$u^n = y^n$

$= 2y^n$

$(1-x^2)y^{n+2} - 2xny^{n+1} - n(n-1)y^n - 2xy^{n+1} - 2ny^n + 2y^n = 0$

~~$n(1-x^2)y^{n+2}$~~ when $x = 0$

$y^{n+2} - n(n-1)y^n - 2ny^n + 2y^n = 0$

$y^{n+2} + y^n(-n^2+n-2n+2) = 0$

$y^{n+2} + y^n(2-n^2-n) = 0$

$y^{n+2} = -y^n(2-n^2-n)$ - Recurrence Relation

$n=0$ $(y^{(2)})_0 = -2(y^{(0)})_0$

$n=1$ $(y^{(3)})_0 = 0$

$n=2$ $(y^{(4)})_0 = 4(y^{(2)})_0 = 4(-2)(y^{(0)})_0$

$n=3$ $(y^{(5)})_0 = 10(y^{(3)})_0 = 0$

$n=4$ $(y^{(6)})_0 = 18(y^{(4)})_0 = 18(4)(-2)(y^{(0)})_0$

$n=5$ $(y^{(7)})_0 = 28(y^{(5)})_0 = 0$

$y = (y^{(0)})_0 + x(y^{(1)})_0 + \frac{x^2}{2!}(y^{(2)})_0 + \frac{x^3}{3!}(y^{(3)})_0 + \frac{x^4}{4!}(y^{(4)})_0 + \cdots$

$y = (y^{(0)})_0 + x(y^{(1)})_0 - \frac{x^2}{2!}(2)(y^{(0)})_0 + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(4)(-2)(y^{(0)})_0 +$
$\frac{x^5}{5!}(0) + \frac{x^6}{6!}(18)(4)(-2)(y^{(0)})_0 + \frac{x^7}{7!}(0) + \cdots$

$y = (y^{(0)})_0 + x(y^{(1)})_0 - x^2(y^{(0)})_0 - \frac{x^4}{3}(y^{(0)})_0 - \frac{x^6}{5}(y^{(0)})_0 + \cdots$

$y = (y^{(0)})_0\left[1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5} + \cdots\right] + x(y^{(1)})_0 + \cdots$

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2) i.) $L\left[3e^{-4t} - 5e^{4t}\right]$

solution

$$\frac{3}{s+4} - \frac{5}{s-4} = \frac{3s-12-5s-20}{(s+4)(s-4)} = \frac{-2s-32}{(s+4)(s-4)}$$

ii.) $L\left[\sin 4t + \cos 4t\right]$

solution

$$\frac{4}{s^2+4^2} + \frac{s}{s^2+4^2}$$

$$= \frac{4}{s^2+16} + \frac{s}{s^2+16} = \frac{4+s}{s^2+16}$$

iii.) $L\left[t^3 + 2t^2 - t + 4\right]$

solution

$$\frac{3!}{s^4} + 2\cdot\frac{2!}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

$$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s} = \frac{6+4s-s^2+4s^3}{s^4}$$

iv.) $L\left[e^{-2t}\cos 5t\right]$

$$\frac{s+2}{(s+2)^2+5^2} = \frac{s+2}{(s+2)^2+25}$$

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v.) $L[t \sin 3t]$

$L[\sin 3t]$

$= \dfrac{3}{s^2+9}$

$L[t \sin 3t] = \dfrac{-d}{ds} f(s)$

$= \dfrac{-d}{ds}\left(\dfrac{3}{s^2+9}\right)$

$u = 3 \qquad du = 0$

$v = s^2+9 \qquad dv = 2s$

$-\left(\dfrac{-6s}{(s^2+9)^2}\right)$

$= \dfrac{6s}{(s^2+9)^2}$

vi) $L\left[\dfrac{e^{-t} - e^{-2t}}{t}\right]$

$L[e^{-t} - e^{-2t}]$

$= \dfrac{1}{s+1} - \dfrac{1}{s+2}$

$L\left[\dfrac{e^{-t} - e^{-2t}}{t}\right] = \int_{\sigma=s}^{\infty} \left(\dfrac{1}{s+1} - \dfrac{1}{s+2}\right) d\sigma$

$= \int_{\sigma=s}^{\alpha} \left(\dfrac{1}{\sigma+1} - \dfrac{1}{\sigma+2}\right) d\sigma$

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$$= \int_{\sigma=s}^{\infty} \frac{1}{\sigma+1} d\sigma - \int_{\sigma=s}^{\infty} \frac{1}{\sigma+2} d\sigma$$

$$= \left[\ln \sigma+1 - \ln \sigma+2 \right]_s^{\infty}$$

$$= \ln \left[\frac{\sigma+1}{\sigma+2} \right]_s^{\infty}$$

$$= \ln \left[\frac{\infty+1}{\infty+2} - \frac{s+1}{s+2} \right]$$

$$= \ln \left[1 - \frac{s+1}{s+2} \right]$$

$$= \ln 1 - \ln \frac{s+1}{s+2}$$

$$= - \ln \frac{s+1}{s+2}$$

$$= \ln \frac{s+2}{s+1}$$

vii.) $L[e^{4t} \cos 2t]$

solution

$$\cancel{\frac{s+4}{(s+4)}} \quad \frac{s-4}{(s-4)^2 + 2^2} = \frac{s-4}{(s-4)^2 + 4}$$

viii.) $L[t \sin 2t]$

$$L[\sin 2t] = \frac{2}{s^2+4}$$

$$L[t \sin 2t] = -\frac{d}{ds}\left(\frac{2}{s^2+4}\right)$$

$u = 2 \qquad du = 0$

$v = s^2+4 \qquad dv = 2s$

$$= (-)\frac{-4s}{(s^2+4)^2}$$

$$= \frac{4s}{(s^2+4)^2}$$

ix.) $L[t^3 + 4t^2 + 5]$

$$\frac{3!}{s^4} + 4 \cdot \frac{2!}{s^3} + \frac{5}{s}$$

$$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$$

$$= \frac{6 + 8s + 5s^3}{s^4}$$

x.) $L[e^{3t}(t^2+4)]$

$$L[t^2+4] = \frac{2}{s^3} + \frac{4}{s}$$

$$L[e^{3t}(t^2+4) = \frac{2}{(s-3)^3} + \frac{4}{(s-3)}$$

xi.) $L[t^2 \cos t]$

$L[t \cos t] = \frac{-d}{ds}\left(\frac{s}{s^2+1^2}\right)$

$u = s \quad du = 0$

$v = s^2+1 \quad dv = 2s$

$(-1)\frac{-2s^2}{(s^2+1)^2}$

$= \frac{2s^2}{(s^2+1)^2}$

$L[t^2 \cos t] = \frac{-d}{ds}\left(\frac{2s^2}{(s^2+1)^2}\right)$

$u = 2s^2 \quad du = 4s$

$v = (s^2+1)^2 \quad dv = 2s \cdot 2(s^2+1) = 4s(s^2+1)$

~~$4s(s^2+1)\ 4s$~~

$= (-)\frac{4s(s^2+1)^2 - 2s^2 \cdot 4s(s^2+1)}{(s^2+1)^4}$

$= (-)\frac{4s(s^2+1)^2 - 8s^3(s^2+1)}{(s^2+1)^4}$

$= (-)\frac{(s^2+1)[4s(s^2+1) - 8s^3]}{(s^2+1)^4}$

$= (-)\frac{[4s(s^2+1) - 8s^3]}{(s^2+1)^3}$

$$= (-)\frac{4s\left[s^2+1-2s^2\right]}{(s^2+1)^3}$$

$$= \frac{4s(-s^2-1+2s^2)}{(s^2+1)^3}$$

$$= \frac{4s(s^2-1)}{(s^2+1)^3}$$

xii) $L\left[\frac{\sinh 2t}{t}\right]$

$$L[\sinh 2t] = \frac{2}{s^2-4}$$

$$L\left[\frac{\sinh 2t}{t}\right] = \int_{\sigma=s}^{\infty} \frac{2}{s^2-4}\,d\sigma$$

$$= 2\int_{\sigma=s}^{\infty} \frac{1}{\sigma^2-4}\,d\sigma$$

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3) i.) $\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$

~~$s-5 = A(s-4) + B(s-3)$~~

$A: \left. \frac{s-5}{s-4} \right|_{s=3} = \frac{3-5}{3-4} = \frac{-2}{-1} = 2$

$B: \left. \frac{s-5}{s-3} \right|_{s=4} = \frac{4-5}{4-3} = \frac{-1}{1} = -1$

$\frac{s-5}{(s-3)(s-4)} = \frac{2}{s-3} - \frac{1}{s-4}$

$= 2e^{3t} - e^{4t}$

ii.) $\frac{2s-6}{(s-2)(s-4)} = \frac{A}{(s-2)} + \frac{B}{s-4}$

$A: \left. \frac{2s-6}{(s-4)} \right|_{s=2} = \frac{4-6}{2-4} = \frac{-2}{-2} = 1$

$B: \left. \frac{2s-6}{s-2} \right|_{s=4} = \frac{8-6}{4-2} = \frac{2}{2} = 1$

$\frac{2s-6}{(s-2)(s-4)} = \frac{1}{s-2} + \frac{1}{s-4}$

$= e^{2t} + e^{4t}$

iii.) $$\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

A: $$\left.\frac{5s-8}{s-4}\right|_{s=0} = \frac{0-8}{0-4} = \frac{-8}{-4} = 2$$

B: $$\left.\frac{5s-8}{s}\right|_{s=4} \quad \frac{20-8}{4} = \frac{12}{4} = 3$$

$$\frac{5s-8}{s(s-4)} = \frac{2}{s} + \frac{3}{s-4}$$

$$= 2 + 3e^{4t}$$

iv.) $$\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{A}{s-3} + \frac{B}{(s-1)^2} + \frac{C}{(s-1)}$$

A: $$\left.\frac{s^2-3s-4}{(s-1)^2}\right|_{s=3} = \frac{s^2-3(3)-4}{(3-1)^2} = \frac{9-9-4}{2^2} = \frac{-4}{4} = -1$$

B: $$\left.\frac{s^2-3s-4}{(s-3)}\right|_{s=1} = \frac{1-3-4}{1-3} = \frac{-6}{-2} = 3$$

C: $$\left.\frac{s^2-3s-4}{s-3}\right|_{s=1} = \frac{d}{ds}\left(\frac{s^2-3s-4}{s-3}\right)$$

$u = s^2-3s-4 \qquad du = 2s-3$

$v = s-3 \qquad dv = 1$

$$\frac{(s-3)(2s-3) - (s^2-3s-4)}{(s-3)^2}$$

$$\frac{(s-3)(2s-3) - s^2+3s+4}{(s-3)^2}$$

at $s=1$

$$= \frac{(1-3)(2-3) - 1^2+3+4}{(1-3)^2}$$

$$= \frac{2-1+7}{2^2}$$

$$= \frac{8}{4} = 2$$

$$\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{-1}{(s-3)} + \frac{3}{(s-1)^2} + \frac{2}{(s-1)}$$

$$= -e^{3t} + 3te^{t} + 2e^{t}$$

v.) $\dfrac{s-5}{s^2+4s+20} = \dfrac{s-5}{s^2+4s+4+16}$

$f(s) = \dfrac{s}{(s+2)^2+16} - \dfrac{5}{(s+2)^2+16}$

$\dfrac{s+2-2}{(s+2)^2+16} - \dfrac{5}{(s+2)^2+16} = \dfrac{s+2}{(s+2)^2+16} - \dfrac{2}{(s+2)^2+16} - \dfrac{5}{(s+2)^2+16}$

$= \dfrac{s+2}{(s+2)^2+4^2} - \dfrac{7 \cdot \frac{4}{4}}{(s+2)^2+4^2}$

$= \dfrac{s+2}{(s+2)^2+4^2} - \dfrac{7}{4} \cdot \dfrac{4}{(s+2)^2+4^2}$

$f(t) = e^{-2t}\cos 4t - \frac{7}{4} e^{-2t}\sin 4t$

$= e^{-2t}\left[\cos 4t - \frac{7}{4}\sin 4t\right]$