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 16/ENG07/024
 Petroleum Engr

Transform each of the functions into Laplace transforms

$$L\{f(t)\} \rightarrow F(s) = \int_0^{\infty} e^{-st} f(t) dt$$

i) $3e^{-4t} - 5e^{+4t}$

$$L\{3e^{-4t} - 5e^{+4t}\}$$

$$L\{3e^{-4t}\} - L\{5e^{+4t}\}$$

$$\frac{3}{s+4} - \frac{5}{s-4}$$

ii) $\sin 4t + \cos 4t$

$$L\{\sin 4t + \cos 4t\}$$

$$L\{\sin 4t\} + L\{\cos 4t\}$$

$$\frac{4}{s^2+16} + \frac{s}{s^2+16}$$

iii) $t^3 + 2t^2 - t + 4$

$$L\{t^3 + 2t^2 - t + 4\}$$

$$L\{t^3\} + L\{2t^2\} - L\{t\} + L\{4\}$$

$$\frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

iv) $e^{-2t} \cos t$

$$L\{e^{-2t} \cos t\}$$

$$= \frac{s-2}{(s-2)^2 + 25}$$

v) $t \sin 3t$

$$L\{t \sin 3t\}$$

?

vi) $\frac{e^{-t} - e^{-2t}}{t}$

$$L\left\{\frac{e^{-t} - e^{-2t}}{t}\right\}$$

$$L\{e^{-t}\} - L\{e^{-2t}\}$$

$$\frac{1}{s+1} - \frac{1}{s+2}$$

$$= \int_s^{\infty} \left(\frac{1}{\delta+1} - \frac{1}{\delta+2}\right) d\delta$$

$$\int_s^{\infty} \frac{1}{\delta+1} d\delta - \int_s^{\infty} \frac{1}{\delta+2} d\delta$$

$$[\ln(\delta+1) - \ln(\delta+2)]_s^{\infty}$$

$$\left[\frac{\ln(\delta+1)}{\delta+2} \right]_s^{\infty}$$

$$\left[\ln \frac{(\infty+1)}{(\infty+2)} - \ln \frac{(s+1)}{(s+2)} \right]$$

$$0 - \frac{\ln(s+1)}{s+2}$$

$$\frac{\ln(s+2)}{s+3}$$

vii) $e^{4t} \cos 2t$

$$L\{e^{4t} \cos 2t\}$$

$$\frac{s-4}{(s-4)^2 + 4}$$

viii) $t \sin t$

$$\frac{2}{s^2+4}$$

$$\begin{aligned}
 n=1 & \binom{1}{0} y^0 (-2) \\
 n=1 & \binom{1}{1} y^1 (-2)^0 = 0 \\
 n=2 & \binom{2}{0} y^0 (-2)^2 + \binom{2}{1} y^1 (-2)^1 = 0 \\
 n=2 & \binom{2}{2} y^2 (-2)^0 = (4) - 2 \\
 n=3 & \binom{3}{0} y^0 (-2)^3 + \binom{3}{1} y^1 (-2)^2 + \binom{3}{2} y^2 (-2)^1 = 0 \\
 n=3 & \binom{3}{3} y^3 (-2)^0 = (18)(4) - 2 \\
 n=4 & \binom{4}{0} y^0 (-2)^4 + \binom{4}{1} y^1 (-2)^3 + \binom{4}{2} y^2 (-2)^2 + \binom{4}{3} y^3 (-2)^1 = 0 \\
 n=4 & \binom{4}{4} y^4 (-2)^0 = (24)(8) - 2(4) - 2(4) \\
 n=5 & \binom{5}{0} y^0 (-2)^5 + \binom{5}{1} y^1 (-2)^4 + \binom{5}{2} y^2 (-2)^3 + \binom{5}{3} y^3 (-2)^2 + \binom{5}{4} y^4 (-2)^1 = 0 \\
 n=5 & \binom{5}{5} y^5 (-2)^0 = (120)(16) - 2(40) - 2(40) - 2(40) \\
 n=6 & \binom{6}{0} y^0 (-2)^6 + \binom{6}{1} y^1 (-2)^5 + \binom{6}{2} y^2 (-2)^4 + \binom{6}{3} y^3 (-2)^3 + \binom{6}{4} y^4 (-2)^2 + \binom{6}{5} y^5 (-2)^1 = 0 \\
 n=6 & \binom{6}{6} y^6 (-2)^0 = (720)(64) - 2(120) - 2(120) - 2(120) - 2(120) - 2(120) \\
 n=7 & \binom{7}{0} y^0 (-2)^7 + \binom{7}{1} y^1 (-2)^6 + \binom{7}{2} y^2 (-2)^5 + \binom{7}{3} y^3 (-2)^4 + \binom{7}{4} y^4 (-2)^3 + \binom{7}{5} y^5 (-2)^2 + \binom{7}{6} y^6 (-2)^1 = 0 \\
 n=7 & \binom{7}{7} y^7 (-2)^0 = 0
 \end{aligned}$$

$$\begin{aligned}
 & y^2 \binom{2}{0} \frac{x^2}{2!} + \binom{2}{1} \frac{x^3}{3!} + \binom{2}{2} \frac{x^4}{4!} + \binom{2}{3} \frac{x^5}{5!} \\
 & + \binom{2}{4} \frac{x^6}{6!} + \binom{2}{5} \frac{x^7}{7!} + \binom{2}{6} \frac{x^8}{8!} \\
 & + \binom{2}{7} \frac{x^9}{9!}
 \end{aligned}$$

$$\begin{aligned}
 & y^2 \binom{2}{0} (-2) \frac{x^2}{2!} + \binom{2}{1} (-2) \frac{x^3}{3!} + \binom{2}{2} (-2) \frac{x^4}{4!} + \binom{2}{3} (-2) \frac{x^5}{5!} \\
 & + \binom{2}{4} (-2) \frac{x^6}{6!} + \binom{2}{5} (-2) \frac{x^7}{7!} + \binom{2}{6} (-2) \frac{x^8}{8!} \\
 & + \binom{2}{7} (-2) \frac{x^9}{9!}
 \end{aligned}$$

$$3. \frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$\frac{5s-8}{s(s-4)} = \frac{A(s-4) + Bs}{s(s-4)}$$

$$5s-8 = A(s-4) + Bs$$

$$5s-8 = As - 4A + Bs$$

$$+4A = +8$$

$$A = 2$$

$$A+B=5$$

$$2+B=5$$

$$B=3$$

$$L^{-1} \left(\frac{2}{s} + \frac{3}{s-4} \right)$$

$$2 + e^{4t}$$

$$1.) \frac{s^2 - 3s - 4}{(s-3)(s-1)^2}$$

$$A + \frac{B}{s-1} + \frac{C}{(s-1)^2}$$

$$\frac{s^2 - 3s - 4}{(s-3)(s-1)(s-1)^2} = \frac{A(s-1)^2 + B(s-3)(s-1) + C(s-3)}{(s-3)(s-1)^2}$$

$$(s-3) = 0$$

$$(-2)A = 1$$

$$A = -\frac{1}{2}$$

$$B = -3$$

$$\frac{1}{s-3} - \frac{3}{s-1} - \frac{2}{(s-1)^2} = L^{-1} \left(\frac{1}{s-3} \right) - L^{-1} \left(\frac{3}{s-1} \right) - L^{-1} \left(\frac{2}{(s-1)^2} \right)$$

$$e^{3t} - 3e^t - 2te^t$$

$$1. (1-x^2) \frac{d^2 y}{dx^2} - 2xy' + 2y = 0$$

$$(1-x^2)y'' - 2xy' + 2y = 0$$

$$W = (1-x^2)y''$$

$$U^n = y^{n+2}$$

$$v = (1-x^2)$$

$$v' = -2x$$

$$v'' = -2$$

$$W_2 = 2xy'$$

$$y^{n+1} = U$$

$$v = 2x \quad v' = 2$$

$$W_3 = 2y$$

$$U^n = 2y^n$$

From Leibnitz

$$y^n = U^n v^n + n U^{n-1} v^{n-1} v' + \frac{n(n-1)}{2!} U^{n-2} v^{n-2} v'^2$$

$$y^{n+2} (1-x^2) + n(y^{n+2}) (-2x) + \frac{n(n-1)}{2!} y^{n-2}$$

$$y^{n+2} (1-x^2) - 2x n y^{n+2} - n(n-1) y^n$$

From W_2

$$y^{n+1} 2x + n y^n 2$$

From W_3

$$2y^n$$

$$y^n (1-x^2) y^{n+2} - 2x n (y^{n+2}) - n(n-1) y^n$$

$$- y^{n+1} 2x + 2n y^n + 2y^n$$

When $n=0$

$$y^n = y^{n+2} - (n^2 + n) y^n + 2n y^n + 2y^n$$

$$y^n = y^{n+2} - (n^2 + n) y^n + 2n y^n + 2y^n$$

$$y^n = y^{n+2} - (n^2 + n) y^n + 2n y^n + 2y^n$$

$$y^n = y^{n+2} - (n^2 + n) y^n + 2n y^n + 2y^n$$

$$y^{n+2} - y^n (-n^2 + n - 2) = 0$$

$$\begin{aligned}
 & t^3 + 4t^2 + c \\
 & \left. \begin{aligned}
 & 8t^2 + 4c + 5 \\
 & 4t + 8 + \frac{5}{s} \\
 & 4t + \frac{8}{s} + \frac{5}{s}
 \end{aligned} \right\} \text{Laplace}
 \end{aligned}$$

$$\begin{aligned}
 & 2^{8t} (t^2 + 4) \\
 & \frac{1}{s} \left(\frac{2}{s} + \frac{4}{s} \right) \\
 & \frac{2}{s^3} + \frac{4}{s^2}
 \end{aligned}$$

$$\begin{aligned}
 & t^3 \cos t \\
 & \frac{s}{(s-1)^2 + 1}
 \end{aligned}$$

$$\begin{aligned}
 & \int_0^{\infty} \sin t \, dt = \int_0^{\infty} \frac{\sin t}{t} \, dt \\
 & = -\int_0^{\infty} \frac{\cos t}{s} \, dt
 \end{aligned}$$

Convert the fms to
 into common

$$\begin{aligned}
 s-5 &= A + B \\
 -8(s-4) &= A(s-3) + B(s-4) \\
 &= A(s-3) + B(s-4) \\
 -5 &= A s - 4A + B s - 2B \\
 s-5 &= A s + B s - 4A - 2B \\
 A+B &= 1 \quad -4A - 2B = -5 \\
 -4A - 2B &= -5 \\
 -4A + 2B &= -5
 \end{aligned}$$

$$\begin{aligned}
 B &= -1 \\
 A+B &= 1 \\
 A &= 2 \\
 A &= 2 \\
 2 &= \frac{1}{(s-3)} - \frac{1}{(s-4)} \\
 &= \frac{2}{(s-3)} - \frac{1}{(s-4)} \\
 &= 2e^{3t} - e^{4t}
 \end{aligned}$$

$$\begin{aligned}
 & 2s-6 \\
 & \frac{2s-6}{(s-2)(s-4)} \\
 & \frac{A}{s-2} + \frac{B}{s-4} \\
 & A(s-4) + B(s-2) \\
 & (s-2)(s-4) \\
 & 2s-6 = As - 4A + Bs - 2B \\
 & A+B = 2 \quad -4A - 2B = -6 \\
 & -4A + 4B = -8 \\
 & -4A - 2B = -6 \\
 & -2B = 2 \\
 & B = -1 \\
 & A+B = 2 \\
 & A+1 = 2 \\
 & A = 1
 \end{aligned}$$

$$\begin{aligned}
 & -4A + 4B = -8 \\
 & -4A - 2B = -6 \\
 & -2B = 2 \\
 & B = -1 \\
 & A+B = 2 \\
 & A+1 = 2 \\
 & A = 1
 \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{(s-2)} - \frac{1}{(s-4)} \\
 & = \frac{1}{(s-2)} + \frac{1}{(s-4)} \\
 & = e^{2t} + e^{4t}
 \end{aligned}$$