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15ENG04049
ELECTRICAL AND ELECTRONICS ENG

Assignment 4

1) $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)y^{(2)} - 2xy^{(1)} + 2y = 0$

$y^{(n)} = u^n v + nu^{(n-1)}v^{(1)} + \frac{n(n-1)}{2!}u^{(n-2)}v^{(2)} + \dots$

$[y^{(2+n)}\cdot(1-x^2) + ny^{(1+n)}\cdot(-2x) + \frac{n(n-1)}{\cancel{2}!}y^{(n)}\cdot(-\cancel{2})] + [y^{(1+n)}\cdot -2x + ny^{(n)}\cdot -2$

$+ [2y^{(n)}] = 0$

$(1-x^2)y^{(2+n)} - 2xny^{(1+n)} - n(n-1)y^{(n)} - 2xy^{(1+n)} - 2ny^{(n)} + 2y^{(n)} = 0$

let $x=0$

$y^{(2+n)} - n(n-1)y^{(n)} - 2ny^{(n)} + 2y^{(n)} = 0$

$y^{(2+n)} + y^{(n)}[-n(n-1) - 2n + 2] = 0$

$y^{(2+n)} + y^{(n)}[-n^2 + n - 2n + 2] = 0$

$y^{(2+n)} + y^{(n)}[-n^2 - n + 2] = 0$

$y^{(2+n)} = -(y^{(n)})_0[-n^2 - n + 2]$ → recurrence relation

$n=0 \quad \therefore (y^{(2)})_0 = -(y^{(0)})_0 \cdot (2) = -2(y^{(0)})_0$

$n=1 \quad \therefore (y^{(3)})_0 = -(y^{(1)})_0 \cdot [0] = 0$

$n=2 \quad \therefore (y^{(4)})_0 = -(y^{(2)})_0 \cdot [-4] = 4(y^{(2)})_0 = (4)(-2)(y^{(0)})_0$

$n=3 \quad \therefore (y^{(5)})_0 = -(y^{(3)})_0 \cdot [-10] = 10(y^{(3)})_0 = (10)(0) = 0$

$n=4 \quad \therefore (y^{(6)})_0 = -(y^{(4)})_0 \cdot [-18] = 18(y^{(4)})_0 = (18)(4)(-2)(y^{(0)})_0$

$n=5 \quad \therefore (y^{(7)})_0 = -(y^{(5)})_0 \cdot [-28] = 28(y^{(5)})_0 = (28)(0) = 0$

$y = (y^{(0)})_0 + x(y^{(1)})_0 + \frac{x^2}{2!}(y^{(2)})_0 + \frac{x^3}{3!}(y^{(3)})_0 + \dots$

$y = (y^{(0)})_0 + x(y^{(1)})_0 + \frac{x^2}{2!}(-2)(y^{(0)})_0 + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(4)(-2)(y^{(0)})_0 + \dots$

$\dots + \frac{x^5}{5!}(0) + \frac{x^6}{6!}(18)(4)(-2)(y^{(0)})_0 + \frac{x^7}{7!}(0)$

$$y = (y^{(0)})_0 + x(y^{(1)})_0 - x^2(y^{(0)})_0 - \frac{x^4}{3\times 1}(y^{(0)})_0 - \frac{x^6}{5}(y^{(0)})_0$$

$$y = (y^{(0)})_0\left[1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5}\right) + (y^{(1)})_0[x]$$

2) i) $3e^{-4t} - 5e^{4t} = f(t)$

$$L[f(t)] = \frac{3}{s+4} - \frac{5}{s-4} = \frac{3(s-4) - 5(s+4)}{(s+4)(s-4)} = \frac{3s - 12 - 5s - 20}{(s+4)(s-4)}$$

$$= \frac{-2s-32}{(s+4)(s-4)}$$

ii) $\sin 4t + \cos 4t = f(t)$

$$L\{f(t)\} = \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2} = \frac{4+s}{(s^2+4^2)}$$

iii) $t^3 + 2t^2 - t + 4$

$$\frac{n!}{s^{n+1}} = \frac{3!}{s^4} + 2\cdot\frac{2!}{s^3} - \frac{1!}{s^2} + \frac{4}{s} = \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

$$= \frac{1}{s^4}\left[6 + 4s - s^2 + 4s^3\right]$$

iv) $t\sin 3t = f(t)$

$$L\{\sin 3t\} = \frac{3}{s^2+3^2} = \frac{3}{s^2+9}$$

$$L\{t\sin 3t\} = -\frac{d}{ds}f(s) \quad \therefore u = 3 \quad v = s^2+9$$

$$du = 0 \quad dv = 2s$$

$$\frac{(s^2+9)0 - 3(2s)}{(s^2+9)^2} = \frac{-6s}{(s^2+9)^2} = \frac{6s}{(s^2+9)^2}$$

v) $e^{2t}\cos 5t = f(t)$

$$L\{f(t)\} = L[\cos 5t] = \frac{s}{s^2+5^2}$$

$$L[f(t)] = \frac{s+2}{(s+2)^2+5^2}$$

vi) $\frac{e^{-t} - e^{-2t}}{t}$

$$\lim_{t\to 0}\left[\frac{-e^{-t} + 2e^{-2t}}{1}\right] = \frac{2}{1} = 2$$

$$L[e^{-t} - e^{-2t}] = \frac{1}{s+1} - \frac{1}{s+2}$$

$$L\left[\frac{e^{-t}-e^{-2t}}{t}\right] = \int_{\sigma=s}^{\infty}\left(\frac{1}{\sigma+1}\right)-\left(\frac{1}{\sigma+2}\right)d\sigma$$

$$= \int_{\sigma=s}^{\infty}\frac{1}{\sigma+1}\,d\sigma - \int_{\sigma=s}^{\infty}\frac{1}{\sigma+2}\,d\sigma$$

$$= \left[\ln(\sigma+1) - \ln(\sigma+2)\right]_s^{\infty}$$

$$= \left[\ln(\sigma+1) - \ln(\sigma+2)\right]_s^{\infty}$$

$$= \left[\ln\frac{\sigma+1}{(\sigma+2)}\right]_s^{\infty} = \ln\left\{\frac{\infty+1}{(\infty+2)} - \frac{s+1}{(s+2)}\right\}$$

$$= -\ln\left[\frac{s+1}{(s+2)}\right] = \ln\left[\frac{(s+2)}{s+1}\right]$$

vi) $e^{4t}\cos 2t$

$$L[\cos 2t] = \frac{s}{s^2+2^2} = \frac{s}{s^2+4}$$

$$L[e^{4t}\cos 2t] = \frac{s-4}{(s-4)^2+4}$$

vii) $t\sin 2t$

$$L[\sin 2t] = \frac{2}{s^2+2^2}$$

$$L[t\sin 2t] = -\frac{d}{ds}[f(s)]$$

$u = 2 \quad du \quad v = s^2+4$

$du = 0 \quad dv = 2s$

$$\frac{(s^2+4).0 - 2(2s)}{(s^2+4)^2} = \left(\frac{-4s}{(s^2+4)^2}\right) = \frac{4s}{(s^2+4)^2}$$

ix) $t^3+4t^2+5 = f(t)$

$$= \frac{3!}{s^4} + \frac{4.2!}{s^3} + \frac{5}{s} = \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$$

$$= \frac{1}{s^4}\left[6 + 8s + 5s^3\right]$$

x) $e^{3t}(t^2+4)$

$$L[t^2+4] = \frac{2!}{s^3} + \frac{4}{s} = \frac{2}{s^3} + \frac{4}{s}$$

$$L[e^{3t}] = \frac{2}{(s-3)^3} + \frac{4}{(s-3)}$$

$$= \frac{4s^2 - 24s + 38}{(s-3)^3}$$

xi) $t^2 \cos t = f(t)$

$$L(\cos t) = \frac{s}{s^2+1} \qquad \therefore L[t^2 \cos t) = \frac{-d^2}{ds^2}\left[\frac{s}{s^2+1}\right]$$

$$\frac{d}{ds}\left[\frac{s}{s^2+1}\right] = \frac{(s^2+1)\cdot 1 - s(2s)}{(s^2+1)^2} = \frac{s^2+1-2s^2}{(s^2+1)^2}$$

$$= \frac{1-s^2}{(s^2+1)^2}$$

$$\frac{d}{ds}\left[\frac{1-s^2}{(s^2+1)^2}\right] = \quad \therefore u = 1-s^2 \qquad v = (s^2+1)^2$$

$$du = -2s \qquad dv = 4s(s^2+1)$$

$u = s^2+1 \qquad v = v^2$

$du = 2s \qquad dv = 2v$

$$\frac{du}{ds} \times \frac{dv}{du} = 2s \times 2v$$

$$= 4sv - 4s(s^2+1)$$

$$\frac{(s^2+1)^2 \cdot -2s - (1-s^2)\, 4s(s^2+1)}{(s^2+1)^4}$$

$$= \frac{-2s(s^2+1)^2 - 4s(s^2+1)(1-s^2)}{(s^2+1)^4}$$

$$\frac{\cancel{(s^2+1)}\left[-2s(s^2+1) - 4s(1-s^2)\right]}{(s^2+1)^{\cancel{4}\,3}} = \frac{-2s\left[s^2+1-2+2s^2\right)}{(s^2+1)^3}$$

$$= \frac{-2s\left[3s^2-1\right]}{(s^2+1)^3} \qquad \therefore L(t^3 \cos t)$$

$$= \frac{-d^2}{ds^2}\left[\frac{s}{s^2+1}\right]$$

$$\therefore \frac{d^2}{ds^2}\left[\frac{s}{s^2+1}\right] = \frac{2s\left[3s^2-1\right]}{(s^2+1)^3}$$

3) $\frac{s-5}{(s-3)(s-4)} = f(t) = \frac{A}{(s-3)} + \frac{B}{(s-4)}$

$$A : \cancel{(s-3)} \cdot \frac{s-5}{\cancel{(s-3)}(s-4)}\Big|_{s=3} = \frac{(3-5)}{(3-4)} = 2$$

$$B : (s-4) \cdot \frac{s-5}{\cancel{(s-3)(s-4)}} \frac{s-5}{(s-3)\cancel{(s-4)}}\Big|_{s=4} = \frac{4-5}{4-3} = -1$$

$$f(t) = \frac{2}{s-3} - \frac{1}{s-4}$$

$$f(t) = 2e^{3t} - e^{4t}.$$

ii) $\frac{2s-6}{(s-2)(s-4)} = f(t) = \frac{A}{(s-2)} + \frac{B}{(s-4)}$

$A: \left.\frac{2s-6}{s-4}\right|_{s=2} = \frac{2(2)-6}{2-4} = 1$

$B: \left.\frac{2s-6}{s-2}\right|_{s=4} = \frac{2(4)-6}{4-2} = 1$

$$f(t) = \frac{1}{s-2} + \frac{1}{s-4}$$

$$f(t) = e^{2t} + e^{4t}$$

iii) $\frac{5s-8}{s(s-4)} = f(t) = \frac{A}{s} + \frac{B}{s-4}$

$A: \left.\frac{5s-8}{s-4}\right|_{s=0} = \frac{5(0)-8}{0-4} = \frac{8}{4} = 2$

$B: \left.\frac{5s-8}{s}\right|_{s=4} = \frac{5(4)-8}{4} = \frac{\cancel{8}}{\cancel{4}} \; 3$

$$f(s) = \frac{2}{s} + \frac{3}{s-4}$$

$$f(t) = 2 + 3e^{4t}$$

iv) $\frac{s^2-3s-4}{(s-3)(s-1)^2} = f(s) = \frac{A}{(s-3)} + \frac{B}{(s-1)^2} + \frac{C}{(s-1)}$

$A: \left.\frac{s^2-3s-4}{(s-1)^2}\right|_{s=3} = \frac{3^2-3(3)-4}{(3-1)^2} = -1$

$B: \left.\frac{s^2-3s-4}{s-3}\right|_{s=1} = \frac{1^2-3(1)-4}{1-3} = 3$

$C: \frac{d}{ds}\left[\frac{s^2-3s-4}{s-3}\right]_{s=1} = \frac{(s-3)(2s-3)-[s^2-3s-4]}{(s-3)^2}$

at $s=1$

$$\frac{(1-3)(2(1)-3)-[1^2-3(1)-4]}{(1-3)^2} = 2$$

$$f(s) = \frac{-1}{s-3} + \frac{3}{(s-1)^2} + \frac{2}{s-1}$$

$$f(t) = -e^{3t} + 3te^{t} + 2e^{t}$$

$$= e^{t}[3t+2] - e^{3t}$$

v) $\frac{s-5}{s^2+4s+20} = f(s)$

$$f(s) = \frac{s-5}{s^2+4s-4+16} = \frac{s}{(s+2)^2+16} - \frac{5}{(s+2)^2+16}$$

$$= \frac{s+2-2}{(s+2)^2+16} - \frac{5}{(s+2)^2+16} = \frac{s+2}{(s+2)^2+16} - \frac{2}{(s+2)^2+16}$$

$$= \frac{s+2}{(s+2)^2+4^2} - \frac{7 \times \frac{4}{4}}{(s+2)^2+4^2}$$

$$= \frac{s+2}{(s+2)^2+4^2} - \frac{7}{4} \cdot \frac{4}{(s+2)^2+4^2}$$

$$f(t) = e^{-2t}\cos 4t - \frac{7}{4} e^{-2t}\sin 4t$$

$$= e^{-2t}\left[\cos 4t - \frac{7}{4}\sin 4t\right]$$