BIANWE MALCOLM EBANGHAYOR ENG 381
15/ENG04/037
Elec/elect Engineering

1 $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)y^{(2)} - 2xy^{(1)} + 2y = 0$

$y^{(n)} = u^n y + n u^{(n-1)} v^{(1)} + \frac{n(n-1)}{2!} u^{(n-2)} y^{(2)} + \dots$

$[y^{(2+n)} \cdot (1-x^2) + ny^{(1+n)} \cdot (-2x) + \frac{n(n-1)}{2!} y^{(n)} \cdot (-2)] +$
$[y^{(1+n)} \cdot -2x + ny^{(n)} + (2y^{(n)}] = 0$
$y^{(2+n)}$ $+ y^{(n)}$

$(1-x^2)y^{(2+n)} - 2xny^{(1+n)} - n(n-1)y^{(n)} - 2xy^{(1+n)} - 2ny^{(n)} + 2y^{(n)}$
$= 0$

at $x = 0$

$y^{(2+n)} - n(n-1)y(n) - 2ny(n) + 2y^{(n)} = 0$
$y^{(2+n)} + y^{(n)}[-n(n-1) - 2n + 2] = 0$
$y(2+n) + y(n)[-n^2 + n - 2n + 2] = 0$
$y(2+n) + y(n)[-n^2 - n + 2] = 0$
$[(y^{(2+n)}] = -(y^{(n)}]_0 [-n^2 + n + 2]$

$n = 0$ $(y^{(2)}]_0 = -(y^{(0)})_0 \cdot (2) = -2(y^{(0)})_0$
$n = 1$ $(y^{(3)})_0 = -(y^{(1)})_0 \cdot (0) = 0$
$n = 2$ $(y^{(4)})_0 = -(y^{(2)})_0 (-4) = 4(y^{(2)})_0$
$n = 3$ $(y^{(5)})_0 = -(y^{(3)})_0 \cdot (-10) = 10(y^{(3)})_0$
$n = 4$ $(y^{(6)})_0 = -(y^{(4)})_0 \cdot (-18) = 18(y^{(4)})_0$
$n = 5$ $(y^{(7)})_0 = -y(y^{(5)})_0 \cdot (-28) = 28(y^{(5)})_0$

$= (4)(-2)(y^{(0)})_0$
$= 10(0) = 0$
$= (18)(4)(-2)(y^{(0)})_0$
$= (28)(0) = 0$

$y = (y^0)_0 + x(y^{(n)})_0 + \frac{x^2}{2!}(-2)(y^{(0)})_0 + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(4)(-2)(y^{(0)})_0$

$+ \dots + \frac{x^5}{5!}(0) + \frac{x^6}{6!}(18)(4)(-2)(y^{(0)})_0$

$+ \frac{x^7}{7!}(0) -$

$y = (y^0)_0 + x(y^{(1)})_0 - x^2(y^{(0)})_0 - \frac{x^4}{3 \times 1}(y^{(0)})_0 - \frac{x^6}{5}(y^{(0)})_0$

$y = (y^0)_0 \left[1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5} \ldots\right] + (y^{(1)})_0 \left[x_7 \ldots\right]$

QUESTION 2

$3e^{-4t} - 5e^{4t} = f(t)$

$L(f(t)) = \frac{3}{s+4} - \frac{5}{s-4} = \frac{3(s-4) - 5(s+4)}{(s+4)(s-4)}$

$= \frac{3s - 12 - 5s - 20}{(s+4)(s-4)} = \frac{-2s - 32}{(s+4)(s-4)}$

ii) $\sin 4t + \cos 4t = f(t)$

$L(f(t)) = \frac{4}{s^2 + 4^2} + \frac{s}{s^2 + 4^2} = \frac{4 + s}{(s^2 + 4^2)}$

iii $t^3 + 2t^2 - t + 4$

$\frac{n!}{s^{n+1}} = \frac{3!}{s^4} + \frac{2 \cdot 2!}{s^3} - \frac{1!}{s^2} + \frac{4}{s}$

$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

$= \frac{1}{s^4}\left[6 + 4s - s^2 + 4s^3\right]$

(iv) $t \sin 3t = f(t)$

$L(\sin 3t) = \frac{3}{s^2 + 3^2} = \frac{3}{s^2 + 9}$

$L(t \sin 3t) = -\frac{\delta}{\delta s} F(s)$

Scanned by CamScanner

$u = 3 \quad v = s^2+9 \quad = \dfrac{(s^2+9)0 - 3(2s)}{(s^2+9)^2}$

$du = 0 \quad dv = 2s$

$= + \left[\dfrac{+6s}{(s^2+9)^2} \right] = \dfrac{6s}{(s^2+9)^2}$

c) $e^{-2t}\cos 5t = f(t)$

$L\{f(t)\} = L(\cos 5t) = \dfrac{s}{s^2+5^2}$

$L\{f(t)\} = \dfrac{s+2}{(s+2)^2+5^2}$

4i) $\dfrac{e^{-t} - e^{-2t}}{t}$

$\lim_{t \to 0} \left[\dfrac{-e^{-t} + 2e^{-2t}}{1} \right] = \dfrac{2}{1} = 2$

$L\left[e^{-t} - e^{-2t}\right] = \dfrac{1}{s+1} - \dfrac{1}{s+2}$

$L\left[\dfrac{e^{-t} - e^{-2t}}{t}\right] = \int_{s=s}^{\infty} \left(\dfrac{1}{s+1}\right) - \left(\dfrac{1}{s+2}\right) ds$

$= \int_{s=s}^{\infty} \dfrac{1}{s+1}\, ds - \int_{s=s}^{\infty} \dfrac{1}{s+2}\, ds$

$= \left[\ln(s+1) - \ln(s+2)\right]_s^{\infty}$

$= \left[\ln(s+1) - \ln(s+2)\right]_s^{\infty}$

$= \left[\ln \dfrac{s+1}{(s+2)}\right]_s^{\infty} = \ln \left[\dfrac{\infty+1}{(\infty+2)} - \dfrac{s+1}{(s+2)}\right]$

$$= -\ln\left[\frac{s+1}{(s+2)}\right] = \ln\left[\frac{(s+2)}{s+1}\right]$$ //

v(ii) $e^{4t}\cos 2t$

$$L(\cos 2t] = \frac{s}{s^2+2^2} = \frac{s}{s^2+4}$$

$$L(e^{4t}\cos 2t) = \frac{s-4}{(s-4)^2+4}$$

v(iii) $t\sin 2t$

$$L(\sin 2t) = \frac{2}{s^2+2^2}$$

$$L(t\sin 2t] = -\frac{d}{ds}[F(s)]$$

$u = 2$ $du = 0$, $v = s^2+4$ $du = 2s$

$$\frac{(s^2+4)\cdot 0 - 2(2s)}{(s^2+4)^2} = -\left(\frac{-4s}{(s^2+4)^2}\right) = \frac{4s}{(s^2+4)^2}$$

(x) $t^3+4t^2+5 = f(t)$

$$\frac{3!}{s^4} + 4\cdot\frac{2!}{s^3} + \frac{5}{s} = \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$$

$$= \frac{1}{s^4}\left[6+8s+5s^3\right]$$

(xi) $e^{3t}(t^2+4)$

$$L(t^2+4] = \frac{2!}{s^3} + \frac{4}{s} = \frac{2}{s^3} + \frac{4}{s}$$

$$L(e^{3t}) = \frac{2}{(s-3)^3} + \frac{4}{(s-3)} = \frac{4s^2-24s+38}{(s-3)^3}$$

X_1

$t^2 \cos t = f(t)$

$L(\cos t) = \frac{s}{s^2+1}$ $\therefore L(t^2 \cos t) = \frac{-\partial^2}{\partial s^2}\left(\frac{s}{s^2+1}\right)$

$\frac{\partial}{\partial s}\left[\frac{s}{s^2+1}\right) = \frac{(s^2+1)\cdot 1 - s(2s)}{(s^2+1)^2} = \frac{s^2+1-2s^2}{(s^2+1)^2} = \frac{1-s^2}{(s^2+1)^2}$.

$\frac{d}{ds}\left[\frac{1-s^2}{(s^2+1)^2}\right]$ $\therefore$ $u = 1-s^2$ $\quad \partial u = -2s$

$v = (s^2+1)^2$ $\quad \partial v = 4s(s^2+1)$

$y = s^2+1$ $\quad \partial y = 2s$

$w = y^2$ $\quad \partial w = 2y$

$\frac{\partial y}{\partial s} \times \frac{\partial w}{\partial y} = 2s \cdot 2y$

$= 4sy = 4s(s^2+1)$

$$\frac{(s^2+1)^2 \cdot -2s - (1-s^2)\,4s\,(s^2+1)}{(s^2+1)^4}$$

$$= \frac{-2s(s^2+1)^2 - 4s(s^2+1)(1-s^2)}{(s^2+1)^4}$$

$$= \frac{\cancel{(s^2+1)}\left[-2s(s^2+1) - 4s(1-s^2)\right]}{(s^2+1)^{\cancel{4}\,3}} = \frac{-2s\left[s^2+1-2+2s^2\right]}{(s^2+1)^3}$$

$$= \frac{-2s\left[3s^2-1\right)}{(s^2+1)^3} \quad \therefore L(t^2\cos t] = \frac{-\partial^2}{\partial s^2}\left[\frac{s}{s^2+1}\right]$$

$$\therefore \frac{\partial^2}{\partial s^2}\left[\frac{s}{s^2+1}\right] = \frac{2s(3s^2-1)}{(s^2+1)^3}$$

xii) $\frac{\sinh 3t}{t} \cdot ((t)$

$\lim_{t \to 0}\left[\frac{\sinh 2t}{t}\right] = \frac{2\cosh 2t}{1} = \frac{2}{1} = 2$

$L\left[\frac{\sinh 2t}{t}\right] = \int_{s=s}^{\infty} \frac{2}{s^2-4}\, ds = 2\int_{s=s}^{t} \frac{1}{s^2-4}\, ds$

QUESTION 3

$$\frac{s-5}{(s-3)(s-4)}$$

$$L^{-1}\left[\frac{s-5}{(s-3)(s-4)}\right] = \frac{A}{(s-3)} + \frac{B}{(s-4)}$$

$$\frac{s-5}{(s-3)(s-4)} = \frac{A(s-4)+B(s-3)}{(s-3)(s-4)}$$

Assuming $s=4$

$4-5 = A(4-4)+B(4-3)$

$-1 = B(1)$

$B = -1$

Assuming $s=3$

$3-3 = A(3-4)+B(3-3)$

$-2 = A(-1), \quad A = 2$

$$\frac{s-5}{(s-3)(s-4)} = \frac{2}{s-3} + \frac{-1}{s-4}$$

$$= \frac{2}{s-3} - \frac{1}{s-4}$$

$$= 2\left[\frac{1}{s-3}\right] - \left[\frac{1}{s-4}\right] = 2e^{3t} - e^{4t}.$$

$$\frac{2s-6}{(s-2)(s-4)}$$

ii. $L^{-1}\left[\frac{2s-6}{(s-2)(s-4)}\right]$, $\frac{2s-6}{(s-2)(s-4)} = \frac{A}{(s-2)} + \frac{B}{(s-4)}$

$2s-6 = A(s-4) + B(s-2)$

Assuming $s=4$

$2(4)-6 = A(4-4) + B(4-2)$

$8-6 = A(0) + B(2)$, $2 = 2B$, $B = 1$

$s = 2$

$2(2)-6 = A(2-4) + B(2-2)$

$4-6 = -2A + 0$

$-2 = -2A$

$A = 1$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{1}{s-2} + \frac{1}{s-4}$$

$$L^{-1}\left[\frac{2s-6}{(s-2)(s-4)}\right] = e^{2t} + e^{4t}$$

iii) $\frac{5s-8}{s(s-4)}$

$L^{-1}\left[\frac{5s-8}{s(s-4)}\right] = \frac{A}{s} + \frac{B}{s-4}$

$5s-8 = A(s-4) + B(s)$

Assuming $s=4$

$5(4)-8 = A(4-4) + B(4)$

$20-8 = 4B$

$12 = 4B$, $B = 3$

Assuming $s=0$

$5(0)-8 = A(0-4) + B(0)$

$-8 = -4A$

$A = 2$

$$L^{-1}\left[\frac{5s-8}{s(s-4)}\right] = \frac{2}{s} + \frac{3}{s-4}$$

$$= \frac{2}{s} + 3\left[\frac{1}{s-4}\right]$$

Scanned by CamScanner

$= 2 + 3e^{4t}$

v) $\dfrac{s-5}{s^2+4s+20}$

$L^{-1}\left[\dfrac{s-5}{s^2+4s+20}\right] =$

$F(s) = \dfrac{s-5}{s^2+4s+4+16} = \dfrac{s}{(s+2)^2+16} - \dfrac{5}{(s+2)^2+16}$

$= \dfrac{s+2}{(s+2)^2+4^2} - \dfrac{7}{4}\,\dfrac{4}{(s+2)^2+4^2}$

$F(t) = e^{-2t}\cos 4t - \dfrac{7}{4}e^{-2t}\sin 4t$

$= e^{-2t}\left[\cos 4t - 7/4 \sin 4t\right]$

vi) $\dfrac{s^2-3s-4}{(s-3)(s-1)^2}$

$= F(s) = \dfrac{A}{s-3} + \dfrac{B}{(s-1)^2} + \dfrac{C}{s-1}$

$A: \left.\dfrac{s^2-3s-4}{(s-1)^2}\right|_{s=3} = \dfrac{3^2-3(3)-4}{(3-1)^2} = -1$

$B: \left.\dfrac{s^2-3s-4}{s-3}\right|_{s=1} = \dfrac{1^2-3(1)-4}{1-3} = 3$

$C: \dfrac{\partial}{\partial s}\left[\dfrac{s^2-3s-4}{s-3}\right]_{s=1} = \dfrac{(s-3)(2s-3)-(s^2-3s-4)}{(s-3)^2}$

at $s=1$

$\dfrac{(1-3)(2(1)-3)-(1^2-3(1)-4)}{(1-3)^2} = 2$

$F(s) = \dfrac{-1}{s-3} + \dfrac{3}{(s-1)^2} + \dfrac{2}{s-1}$

$F(t) = -e^{3t} + 3te^{t} + 2e^{t} = e^{t}\left[3t+2\right] - e^{3t}$