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15/ENG06/067.
Mechanical Engineering

Engineering mathematics Assignment 4.

1) $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0.$

$(1-x^2)y'' - 2xy' + 2y = 0.$

Sub module 1.

$u = y'' \quad u^n = y^{(n+2)}$

$v = 1-x^2, \ v' = -2x, \ v'' = -2.$

$y^n = y^{(n+2)} \cdot 1-x^2 + ny^{(n+1)} \cdot -2x + \frac{n(n-1)}{2} y^n \cdot -2.$

$y^n = (1-x^2)y^{(n+2)} - 2nxy^{(n+1)} - n(n-1)y^n.$

Sub module 2.

$u = y' \quad u^n = y^{(n+1)}.$

$v = -2x \quad v' = -2.$

$y^{(n)} = -2xy^{(n+1)} + ny^{(n)} \cdot -2.$

$= -2xy^{(n+1)} - 2ny^{(n)}.$

Sub module 3.

$u = y \quad u^n = y^n.$

$v = 2.$

$y^{(n)} = 2 \cdot y^{(n)} = 2y^{(n)}$

Combination:

$(1-x^2)y^{(n+2)} - 2nxy^{(n+1)} - n(n-1)y^{(n)} - 2xy^{(n+1)} - 2ny^{(n)} + 2y^{(n)}.$

$(1-x^2)y^{(n+2)} - (2nx - 2x)y^{(n+1)} - (n^2+n-2)y^{(n)} = 0.$

at $x = 0$.

$y^{(n+2)} - 0 - (n^2+n-2)y^{(n)} = 0.$

$y^{n+2} = y^n(n^2+n-2).$

when $n = 0$.

$(y^{(2)})_0 = (y^0) \cdot (-2) = -2(y^{(0)})_0$

when $n = 1$.

$(y^{(3)})_0 = (y^{(1)}) \cdot (0) = 0.$

when $n = 2$.

$(y^{(4)})_0 = (y^{(2)}) \cdot (4) = 4 \times -2(y^{(0)})_0 = -8(y^{(0)})_0$

when $n=3$

$$\left(y^{(5)}\right)_0 = \left(y^{(3)}\right)_0 (10) = 10\left(y^{(3)}\right)_0 = 10\times 0 = 0.$$

when $n=4$.

$$\left(y^{(6)}\right)_0 = \left(y^{(4)}\right)_0 (18) = 18\left(y^{(4)}\right)_0 = 18\times -8\left(y^{(0)}\right)_0$$

$$= -144\left(y^{(0)}\right)_0$$

when $n=5$.

$$\left(y^{(7)}\right)_0 = \left(y^{(5)}\right)(28) = 28\times 0 = 0.$$

when $n=6$.

$$\left(y^{(8)}\right)_0 = \left(y^{(6)}\right)_0 (40) = 40\times -144\left(y^{(0)}\right)_0 = -5760\left(y^{(0)}\right)_0.$$

$$y = (y)_0 + x(y')_0 + \frac{x^2}{2!}\left(y^{(2)}\right)_0 + \frac{x^3}{3!}\left(y^{(3)}\right)_0 + \frac{x^4}{4!}\left(y^{(4)}\right)_0 + \cdots$$

$$y = (y)_0 + x\left(y^{(1)}\right) - x^2\left(y^{(0)}\right)_0 + 0 + -\frac{x^4}{3}\left(y^{(0)}\right)_0 + 0 - \frac{x^6}{5}\left(y^{(0)}\right)_0 + 0 - \frac{x^8}{7}\left(y^{(0)}\right)_0 + \cdots$$

$$y = (y)_0\left[1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5} - \frac{x^8}{7}\right] + (y')_0(x)$$

2) i) $L\left[3e^{-4t} - 5e^{4t}\right]$.

$$\frac{3}{s+4} - \frac{5}{s-4} = \frac{3(s-4) - 5(s+4)}{(s+4)(s-4)}.$$

$$= \frac{3s - 12 - 5s - 20}{s^2 - 16} = \frac{-2s - 32}{s^2 - 16}.$$

ii) $L\left[\sin 4t + \cos 4t\right]$.

$$\frac{4}{s^2+4^2} + \frac{s}{s^2+4^2} = \frac{4+s}{s^2+16}.$$

iii) $L\left[t^3 + 2t^2 - t + 4\right]$.

$$= \frac{3!}{s^4} + \frac{2\cdot 2!}{s^3} - \frac{1}{s^2} + \frac{4}{s}.$$

$$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s} = \frac{6 + 4s - s^2 + 4s^3}{s^4}$$

iv) $L[e^{-2t} \cos 5t]$.

$$\cos 5t = \frac{s}{s^2+5^2}.$$

Let $s = s+2$

$$L[e^{-2t} \cos 5t] = \frac{s+2}{(s+2)^2+5^2} = \frac{s+2}{s^2+4s+29}.$$

v) $L(t \sin 3t]$

$$L[\sin 3t] = \frac{3}{s^2+3^2} = \frac{3}{s^2+9}.$$

$$-F'(s) = -\frac{d}{ds}\left[\frac{3}{s^2+9}\right].$$

Let $u = 3$ $\qquad du/ds = 0$.

$v = s^2+9$ $\qquad dv/ds = 2s$.

using quotient Rule.

$$\frac{v\,du/ds - u\,dv/ds}{v^2}.$$

$$= \frac{(s^2+9)(0) - 3(2s)}{(s^2+9)^2} = \frac{0-6s}{(s^2+9)^2} = \frac{-6s}{(s^2+9)^2}.$$

vi) $\frac{e^{-t} - e^{-2t}}{t}.$

differentiating Separately

$$\frac{-e^{-t} + 2e^{-2t}}{1} = e^{-2t}$$

at $t = 0$.

$$= e^{-2(0)} = 1.$$

vii) $L[e^{4t} \cos 2t]$.

$$L[\cos 2t] = \frac{s}{s^2+2^2} = \frac{s}{s^2+4}$$

Let $s = s-4$.

$$L[e^{4t}\cos 2t] = \frac{s-4}{(s-4)^2+4} = \frac{s-4}{s^2-8s-12}.$$

viii) $L[t \sin 2t]$

$$L[\sin 2t] = \frac{2}{s^2+2^2} = \frac{2}{s^2+4}$$

$$L[t \sin 2t] = -f'(s) = \frac{-d}{ds}\left[\frac{2}{s^2+4}\right].$$

$u = 2$. $\qquad du/ds = 0$.

$v = s^2+4$. $\qquad dv/ds = 2s$.

Using quotient rule.

$$-\left[\frac{v\,du/ds - u\,dv/ds}{v^2}\right].$$

$$-\left[\frac{s^2+4(0) - 2(2s)}{(s^2+4)^2}\right] = \frac{4}{(s^2+4)^2}$$

xi) $L[t^2 \cos t]$.

$$L[\cos t] = \frac{s}{s^2+1^2} = \frac{s}{s^2+1}.$$

$$-f'(s) = \frac{-d}{ds}\left[\frac{s}{s^2+1}\right].$$

$u = s$ $\qquad du/ds = 1$.

$v = s^2+1$ $\qquad dv/ds = 2s$.

$$\frac{v\,du/ds - u\,dv/ds}{v^2}.$$

$$-\left(\frac{(s^2+1)(1) - s(2s)}{(s^2+1)^2}\right) = \left(\frac{s^2+1-2s^2}{(s^2+1)^2}\right),$$

$$L^{-1}\left[F(s)\right] = \frac{-s^2+1}{(s^2+1)^2} = \frac{-1\,\cancel{(s^2+1)}}{(s^2+1)^{\cancel{2}}} = \frac{-1}{s^2+1} = \frac{1}{s^2+1}$$

$$L\left[t^2\cos t\right] = \frac{-d}{ds}\left[\frac{1}{s^2+1}\right]$$

$u = 1 \qquad du/ds = 0$

$v = s^2+1 \qquad dv/ds = 2s$

$$\frac{(s^2+1)(0) - (1)(2s)}{(s^2+1)^2}$$

$$= \frac{-2s}{(s^2+1)^2}$$

Question 3.

i) $$\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$$

$s-5 = A(s-4) + B(s-3)$

at $s = 4$

$-1 = B(1)$

at $s = 3$

$-2 = A(-1) + 0$

$-A = -2$

$A = 2$

$$= \frac{2}{s-3} - \frac{1}{s-4}$$

$= 2e^{3t} - e^{4t}$

ii) $$\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$$

$A(s-4) + B(s-2) = 2s-6$

at $s = 4$

$0 + B(2) = 2(4) - 6$

$B = 2/2 = 1.$

at $s = 2.$

$A(2-4) + 0 = 2(2) - 6.$

$-2A = -2.$

$A = 1.$

$= \frac{1}{s-2} + \frac{1}{s-4}$

$= e^{2t} + e^{4t}$

vii) $\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$A(s-4) + Bs = 5s - 8.$

at $s = 0.$

$A(-4) + 0 = 5(0) - 8$

$-4A = -8$

$A = 2.$

at $s = 4.$

$0 + 4B = 5(4) - 8.$

$4B = 12.$

$B = 3.$

$= \frac{2}{s} + \frac{3}{s-4}$

$= 2 + 3e^{4t}$

w) $\frac{s^2 - 3s - 4}{(s-3)(s-1)^2} = \frac{A}{s-3} + \frac{B}{s-1} + \frac{C}{(s-1)}.$

$A(s-1)^2 + B(s-3)(s-1) + C(s-3) = s^2 - 3s - 4$

at $s = 1.$

$0 + 0 + c(-2) = 1^2 - 3(-1) - 4$

$-2c = -6.$

$c = 3.$

at $s = 3.$

$A(2)^2 + 0 + 0 = 3^2 - 3(3) - 4$

$4A = -4.$

$A = -1.$

$A + B = 1.$

$B = 1 - A = 1 - (-1) = 1 + 1 = 2$

$$= \frac{-1}{s-3} + \frac{2}{s-1} + \frac{3}{(s-1)^2}$$

$= -e^{3t} + 2e^{t} + 3te^{t}$

v) $$\frac{s-5}{s^2+4s+20} = \frac{s-5}{(s+2)^2+4^2}.$$

$$= \frac{A}{(s+2)} + \frac{B}{(s+2)^2} + \frac{C}{16}.$$

$= A(s+2)(16) + B(16)\cancel{s} + C(s+2)^2 = s-5.$

$= (16s+32)A + 16B + (s^2+4s+4)C = s-5.$

$\cancel{s^2}\ C = 0\cancel{s^2}.$

$\cancel{s}(16A + 4C) = 1\cancel{s}$

$32A + 16B + 4C = -5.$

$\therefore \quad C = 0.$

$16A + 4C = 1.$

$16A + 4(0) = 1.$

$16A = 1.$

$A = 1/16.$

$32A + 16B + 4C = -5.$

$32\left(\frac{1}{16}\right) + 16B + 4(0) = -5$

$2 + 16B = -5.$

$16B = -7.$

$B = -7/16.$

$$= \frac{1}{16(s+2)} - \frac{7}{16}\frac{1}{(s+2)^2} + 0.$$

$$= \frac{1}{16}e^{-2t} - \frac{7}{16}te^{2t}$$