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15/ENG03/020
CIVIL ENGINEERING
ENG 381 Assignment IV

1. $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)\frac{d^2y}{dx^2} = (1-x^2)y''$

$u = y^2 \quad u^n = y^{(n+2)}$

$v = 1-x^2 \quad v' = -2x \quad v^2 = -2 \quad v^3 = 0$

$W^n = u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v^2 + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v^3$

$= y^{(n+2)} \cdot (1-x^2) + n y^{(n+1)} \cdot (-2x) + \frac{n(n-1)}{2} y^n (2) + 0$

$W^n = (1-x^2) y^{(n+2)} - 2xny^{(n+1)} - n(n-1)y^n$

$= (1-x^2) y^{(n+2)} - 2xny^{(n+1)} - (n^2-n)y^n$

$W_2 = -2x\frac{dy}{dx} = -2xy'$

$u = y' \quad u^n = y^{n+1}$

$v = -2x \quad v' = -2 \quad v^2 = 0$

$y^n = y^{(n+1)}(-2x) + ny^n \cdot -2 + 0$

$= -2xy^{n+1} - 2ny^n$

$W_3 = 2y$

$u = y \quad u^n = y^n$

$v = 2 \quad v' = 0$

$y^n \cdot 2 + 0$

$= 2y^n$

$= (1-x^2)y^{(n+2)} - 2xny^{(n+1)} - 2xy^{(n+1)} - (n^2-n)y^n - 2ny^n + 2y^n$

$= (1-x^2)y^{(n+2)} - 2xy^{(n+1)}(n+1) - y^n(n^2-n+2n-2)$

$= (1-x^2)y^{n+2} - (n+1)2xy^{(n+1)} - (n^2+n-2)y^n$

at $x = 0$

$= (1-0)y^{(n+2)} - (n+1)2(0)y^{n+1} - (n^2+n-2)y^n$

$[y^{n+2}]_0 = (n^2+n-2)y^n$

at $n=0$

$(y^2]_0 = -2[y^0]_0$

at $n=1$

$[y^3]_0 = 0$

at $n=2$

$[y^4]_0 = 4[y^2]_0 = 4\times -2[y^0]_0 = -8[y^0]_0$

at $n=0$

$[y^5]_0 = 10[y^3]_0 = 10\times 0 = 0$

at $n=4$

$[y^6]_0 = 18[y^4]_0 = 18\times -8[y^0]_0 = -144[y^0]_0$

$y = y_0 + x(y')_0 + \frac{x^2}{2!}[y^2]_0 + \frac{x^3}{3!}[y^3]_0 + \cdots \frac{x^r}{r!}(y^r)_0$

$= y_0 + x(y')_0 + \frac{x^2}{2!} \cdot -2[y^0]_0 + \frac{x^3}{3!}\cdot 0 + \frac{x^4}{4!} \cdot -8[y^0]_0 + \frac{x^5}{5!}\cdot 0 +$

$\frac{x^6}{6!} \cdot -144[y^0]_0$

$= [y^0]_0 + x(y')_0 - x^2[y^0]_0 - \frac{x^4}{3}[y^0]_0 - \frac{x^6}{5}(y^0]_0$

$y_0[1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5}] + x[y']_0$

$L\{3e^{-4t} - 5e^{4t}]$

$L[3e^{-4t}] - L[5e^{4t}]$

$\frac{3}{s+4} - \frac{5}{s-4}$

$L[\sin 4t + \cos 4t]$

$L[\sin 4t] + L[\cos 4t]$

$\frac{4}{s^2+16} + \frac{4}{s^2+16}$

$= \frac{4+s}{s^2+16}$

iii) $L\{t^3 + 2t^2 - t + 4\}$

$t^n = \frac{n!}{s^{n+1}}$

$= \frac{3!}{s^4} + 2\left(\frac{2!}{s^3}\right) - \frac{1}{s^2} + \frac{4}{s}$

$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

iv) $L\{e^{-2t}\cos 5t\}$

$L\{\cos 5t\} = \frac{s}{s^2+25}$

$L[e^{-2t}\cos 5t] = \frac{s+2}{(s+2)^2+25}$

$= \frac{s+2}{(s+2)^2+25}$

v) $t\sin 3t$

$L[\sin 3t] = \frac{3}{s^2+9}$

$L[t\sin 3t] = -1\frac{d}{ds}\left[\frac{3}{s^2+9}\right]$

$u = 3 \quad du = 0$

$v = s^2+9 \quad dv = 2s$

$-1\left[\frac{0-6s}{(s^2+9)^2}\right] = \frac{6s}{(s^2+9)^2}$

vi) $e^{-t} - e^{-2t}$

$L\{e^{-t} - e^{-2t}\}$

$= \frac{1}{s+1} - \frac{1}{s+2}$

$L\left\{\frac{e^{-t}-e^{-2t}}{t}\right\} = \int_{s=s}^{\infty} \frac{1}{s+1} - \frac{1}{s+2}$

$= \int_s^{\infty} \frac{1}{s+1} - \frac{1}{s+2}$

$\ln(\sigma+1) - \ln(\sigma+2)$

$= \ln\left[\frac{\sigma+1}{\sigma+2}\right]$

$= \ln\left[\frac{s+1}{s+2}\right]$

$= -\ln\left[\frac{s+1}{s+2}\right]$

$\ln\left[\frac{s+1}{s+2}\right]^{-1} = \ln\left[\frac{s+2}{s+1}\right]$

vii) $L\{e^{4t}\cos 2t\}$

$L[\cos 2t] = \frac{s}{s^2+4}$

$L\{e^{4t}\cos 2t\} = \frac{s-4}{(s-4)^2+4}$

$= \frac{s-4}{s^2-4s+20}$

viii) $t\sin 2t$

$L[\sin 2t] = \frac{2}{s^2+4}$

$L[t\sin 2t] = -1\frac{d}{ds}\left[\frac{2}{s^2+4}\right]$

$u = 2 \quad du = 0$

$v = s^2+4 \quad dv = 2s$

$-1\left[\frac{0-4s}{(s^2+4)^2}\right] = \frac{4s}{(s^2+4)^2}$

ix) $t^3 + 4t^2 + 5$

$L\{t^3\} + L[4t^2] + L(5)$

$= \frac{3!}{s^4} + 4\left[\frac{2!}{s^3}\right] + \left[\frac{5}{s}\right]$

$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$

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x) $t^2 \cos t$

$L\{\cos t\} = \frac{s}{s^2+1}$

$L\{t^2 \cos t\} = (-1)^2 \frac{d^2}{ds^2}\left(\frac{s}{s^2+1}\right)$

$\frac{d}{ds}\left(\frac{s}{s^2+1}\right)$

$u = s \quad du = 1$

$v = s^2+1 \quad dv = 2s$

x) $e^{3t}(t^2+4)$

$L[t^2 e^{3t} + 4e^{3t}]$

$L(t^2 e^{3t}) = \frac{2}{s^3}$

$= \frac{2}{(s-3)^3}$

$L[4e^{3t}]$

$\frac{4}{s-3}$

$\therefore e^{3t}[t^2+4] = \frac{2}{(s-3)^3} + \frac{4}{s-3}$

vii) $\frac{\sinh 2t}{t}$

$L\{\sinh 2t\} = \frac{2}{s^2-4}$

$L\left\{\frac{\sinh 2t}{t}\right\} = \int_{s}^{\infty} \frac{2}{s^2-4} = 2\int_s^{\infty} \frac{1}{s^2-4}$

$= 2\left[\frac{1}{2}\frac{\tan^{-1} s}{2}\right]_s^{\infty} = \left[\frac{\tan^{-1} s}{2}\right]_s^{\infty}$

$\tan^{-1}\frac{\infty}{2} - \tan^{-1}\frac{s}{2}$

$- \tan^{-1}\frac{s}{2}$

$= \left[\tan^{-1}\frac{s}{2}\right]^{-1}$

$= \tan^{-1}\frac{2}{s}$

3) Convert the following to time (t) domains

i) $\frac{s-5}{(s-3)(s-4)}$

$L^{-1}\left(\frac{A}{s-3} + \frac{B}{s-4}\right)$

$\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$

$s-3 = A(s-4) + B(s-3)$

at $s=4$

$4-5 = A(4-4) + B(4-3)$

$-1 = A(0) + B(1)$

$-1 = 0 + B$

$B = -1$

at $s=3$

$3-5 = A(3-4) + B(3-3)$

$-2 = -A + B(0)$

$A = 2$

$L^{-1}\left[\frac{2}{s-3} - \frac{1}{s-4}\right]$

$x(t) = 2e^{3t} - e^{4t}$

ii) $\frac{2s-6}{(s-2)(s-4)}$

$L^{-1}\left[\frac{A}{(s-2)} + \frac{B}{(s-4)}\right]$

$\frac{2s-6}{(s-2)(s-4)} = \frac{A}{(s-2)} + \frac{B}{s-4}$

$2s-6 = A(s-4) + B(s-2)$

at $s=4$

$2(4)-6 = A(4-4) + B(4-2)$

$2 = 2B$

$B = 1$

at $s=2$

$2(2)-6 = A(2-4) + B(2-2)$

$-2 = -2A$

$A = 1$

$x(t) = L^{-1}\left[\frac{1}{s-2} + \frac{1}{s-4}\right]$

$x(t) = e^{2t} + e^{4t}$

iii) $\frac{5s-8}{s(s-4)}$

$L^{-1}\left[\frac{A}{s} + \frac{B}{s-4}\right]$

$\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$5s-8 = A(s-4) + Bs$

at $s=4$

$5(4)-8 = A(4-4) + B(4)$

$12 = 4B$

$B = \frac{12}{4} = 3$

at $s=0$

$-8 = -4A$

$A = \frac{-8}{-4} = 2$

$L^{-1}\left[\frac{2}{s} + \frac{3}{s-4}\right]$

$x(t) = 2 + 3e^{4t}$

iv) $\frac{s^2-3s-4}{(s-3)(s-1)^2}$

$x(t) = L^{-1}\left[\frac{A}{s-3} + \frac{B}{s-1} + \frac{C}{(s-1)^2}\right]$

$s^2-3s-4 = A(s-1)^2 + B(s-3)(s-1) + C(s-3)$

$s^2-3s-4 = A(s-1)^2 + B(s-3)(s-1) + C(s-3)$

at $s=3$

$3^2 - 3(3) - 4 = A(3-1)^2 + B(3-3)(3-1)$
$C(3-3)$

$-4 = 4A$

$A = \frac{-4}{4} = -1$

at $s=1$

$1^2 - 3(1) - 4 = A(1-1)^2 + B(1-3)(1-1)$
$C(1-3)$

$-6 = -2C$

$C = \frac{6}{2}$

$C = 3$

$s^2-3s-4 = As^2 - 2As + A + Cs - C3 + Bs^2 - B4s + B3$

$A + 3B - 3C = -4$

$-1 + 3B - 3(3) = -4$

$3B = 10 - 4$

$3B = 6$

$B = \frac{6}{3}$ $B = 2$