$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$$

$$w_1 - w_2 + w_3 = 0$$

for w_1

$(1-x^2) = v$ $\qquad y'' = u$

$-2x = v'$ $\qquad y^{n+2} = u^n$

$-2 = v''$ $\qquad y^{n+1} = u^{n-1}$

$0 = v'''$ $\qquad y^n = u^{n-2}$

$$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + 0$$

$$= (y^{n+2})(1-x^2) + (-2xy^{n+1}) + \left(\frac{-\cancel{2}y^n}{\cancel{2}}\right)(n^2-n)$$

$$= y^{n+2}(1-x^2) - 2xy^{n+1} - (n^2-n)y^n$$

for w_2

$2x = v$ $\qquad y' = u$

$2 = v'$ $\qquad y^{n+1} = u^n$

$0 = v''$ $\qquad y^n = u^{n-1}$

$$y^n = u^n v + n u^{n-1} v' + 0$$

$$y^n = 2xy^{n+1} + 2ny^n$$

for w_3

$2 = v$ $\qquad y = u$

$0 = v'$ $\qquad y^n = u^n$

$$y^n = u^n v$$

$$= 2y^n$$

$$w_1 - w_2 + w_3$$

$$y^{n+2}(1-x^2) - 2xy^{n+1} - (n^2-n)y^n - [2xy^{n+1} + 2ny^n] + 2y^n$$

$$= y^{n+2}(1-x^2) - 2xy^{n+1} - 2xy^{n+1} - n^2y^n + ny^n - 2ny^n + 2y^n$$

$$= y^{n+2}(1-x^2) - 4xy^{n+1} - n^2y^n - ny^n + 2y^n$$

$$= y^{n+2}(1-x^2) - 4xy^{n+1} - y^n(n^2+n-2)$$

$$y^{n+2}(1-x^2) - 4xy^{n+1} - y^n(n^2+n-2) = 0$$

when $x = 0$

$$y^{n+2} \quad - y^n(n^2+n-2) = 0$$

$$(y^{n+2})_0 = y^n(n^2+n-2)_0$$

$n = \cancel{0}\ 1$

$\cancel{(y^2)_0 =}$

$$(y^3)_0 = y(1+1-2)$$
$$= 0$$

$n = 2$

$$(y^4)_0 = y^2(4+2-2)$$
$$= (4y^2)_0$$

$n = 3$

$$(y^5)_0 = y^3(9+3-2)$$
$$= 10(y^3)_0 = 0$$

$\cancel{(y^6)}$

$n = 4$

$$(y^6)_0 = (y^4)_0(16+4-2)$$
$$= \cancel{10}\ 18(y^4)_0$$

$n = 5$

$$(y^7)_0 = (y^5)(25+5-2)$$
$$= 28(y^5)_0 = 0$$

$n = 6$

$$(y^8)_0 = (y^6)_0(36+6-2)$$
$$= 40(y^6)_0$$

$n = 7$

$$(y^9)_0 = (y^7)_0(49+7-2)$$
$$= 54(y^7)_0 = 0$$

$$y = y_0 + x(y')_0 + \frac{x^2}{2!}(y^3$$

$$y^n = y^n v + n\, y^{(n-1)} v' + \frac{n(n-1)(n-2)\, y^{n-2} v''}{2!} \ldots$$

$y = ($

all odd y raise to the power of odd no is equals to zero

$$y = (y)_0 + x(y')_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^4}{4!}4(y^2)_0 + \frac{x^6}{6!}(18\times4)(y^2)_0 +$$

$$\frac{x^8}{8!}40\times18\times4(y^2)_0$$

$$y = (y)_0 + x(y')_0 + \frac{x^2(y^2)_0}{2} + \frac{\cancel{4}x^4(y^2)_0}{6} + \frac{18\times4\; x^6(y^2)_0}{\cancel{6}\times5\times\cancel{4}\times\cancel{3}\times2} + \cancel{x^8(y^2)_0}$$

$$+ \frac{x^8\;\cancel{40}\times\cancel{18}\times\cancel{4}(y^2)_0}{\cancel{8}\times7\times\cancel{6}\times\cancel{5}\times\cancel{4}\times\cancel{3}\times2}$$

$$y = (y)_0 + x(y')_0 + \frac{x^2(y^2)_0}{2} + \frac{x^4(y^2)_0}{6} + \frac{x^6(y^2)_0}{10} + \frac{x^8}{14}$$

2

i) $3e^{-4t} - 5e^{4t}$

$3L[e^{-4t}] - 5L[e^{4t}]$

$$3\left[\frac{1}{s+4}\right] - 5\left[\frac{1}{s-4}\right]$$

$$\frac{3(s-4) - 5(s+4)}{(s+4)(s-4)} = \frac{3s-12-5s-20}{(s+4)(s-4)}$$

$$= \frac{-2s-32}{(s+4)(s-4)}$$

ii) $\sin 4t + \cos 4t$

$L[\sin 4t] + L[\cos 4t]$

$$\frac{4}{s^2+16} + \frac{s}{s^2+16} = \frac{4+s}{s^2+16}$$

iii) $t^3 + 2t^2 - t + 4$

$L[t^3] + 2L[t^2] - L[t] + L\{4\}$

$$\frac{3!}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

$$\frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

$$\frac{6+4s-1s^2+4s^3}{s^4}$$

w) $e^{-2t}\cos 5t$

$L\{e^{-2t}\cos 5t\} = \frac{\cancel{s}}{s} \quad \frac{s}{(s+2)^2+25}$

x) $t\sin 3t$

$L\{t\sin 3t\} = B$

$L\{\sin 3t\} = \frac{3}{s^2+9}$

$L\{t\sin 3t\} = -\frac{d}{ds}\left(\frac{3}{s^2+9}\right)$

$= \cancel{4} - \frac{9s}{s^2+9}$

$-3\frac{d}{ds}(s^2+9)^{-1}$

$u = s^2+9 \qquad \frac{du}{ds} = 2s \qquad y = u^{-1}$

$dy/du = -u^{-2}$

$= -u^{-2} \times 2s$

$\frac{dy}{ds} = \frac{dy}{du} \times \frac{du}{ds} \qquad = -\frac{2s}{u^2} = -\frac{2s}{(s^2+9)^2}$

$= -3\left(\frac{-2s}{s^2+9}\right)$

$= \frac{6s}{s^2+9}$

w) $\frac{e^{-t} - e^{-2t}}{t}$

$L[e^{-t}] - L[e^{-2t}]$

$\frac{1}{s+1} - \frac{1}{s+2}$

$L\left[e^{-t} - e^{-2t}\right] = \left[\frac{1}{s+1} - \frac{1}{s+2}\right] = \int_s^\infty \left(\frac{1}{\sigma+1} - \frac{1}{\sigma+2}\right)d\sigma$

$\ln(\sigma+1) - \ln(\sigma+2)\Big]_s^\infty$

$0 - \ln s+1/s+2$

$\ln \frac{s+2}{s+3}$

vii) $e^{4t}\cos 2t$

$L[e^{4t}\cos 2t]$

$L[\cos 2t] = \frac{s}{s^2+4}$

$L[e^{4t}\cos 2t] = \frac{s}{(s-4)^2+4}$

viii) $t\sin 2t$

$L[t\sin 2t]$

$L[\sin 2t] = \frac{2}{s^2+4}$

$L[t\sin 2t] = -\frac{d}{ds}\int\frac{2}{s^2+4}$

$-2\frac{d}{ds}(s^2+4)^{-1}$

$u = s^2+4$ $\quad y = u^{-1}$

$du/ds = 2s$ $\quad dy/du = -u^{-}$

$\frac{dy}{ds} = -2su^{-2}$

$= -\frac{2s}{u^2} = \frac{-2s}{(s^2+4)^2}$

$= \frac{4s}{(s^2+4)^2}$

ix t^3+4t^2+5

$L[t^3] + 4L[t^2] + L[5]$

$\frac{3!}{s^4} + \frac{4\times 2}{s^3} + \frac{5}{s}$

$\frac{6+8s+5s^3}{s^4}$

x $e^{3t}(t^2+4)$

$L[t^2+4]$

$L[t^2] + L[4]$

$\frac{2}{s^3} + \frac{4}{s} = \frac{2 + 4s^2}{s^3}$

$\frac{2+4s^2}{s^3}$

$L[e^{3t}(t^2+4)]$

$= \frac{2+4s^2}{(s-3)^3}$

vi) $t^2\cos t$

$L[\cos t] = \frac{s}{s^2+1}$

$L[t^2\cos t] =$

$= -\frac{d}{ds}\left[\frac{s}{s^2+1}\right]$

$\frac{v\,du/ds - u\,dv/ds}{v^2}$

$u = s$

$v = s^2+1$

$\frac{du}{ds} = 1 \quad \frac{dv}{ds} = 2s$

$\frac{(s^2+1)1 - s(2s)}{(s^2+1)^2} = \frac{s^2+1-2s^2}{(s^2+1)^2}$

$= \frac{-s^2+1}{(s^2+1)^2}$

ii) $\frac{\sinh 2t}{t}$

$L[\sinh 2t] = \frac{2}{s^2-4}$

$L[\sinh 2t] = \left[\frac{2}{s^2-4}\right] d\sigma$

$= \frac{\tan^{-1}(2)}{s}$

3) Convert the following functions to time (t) domains.

i) $\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$

$s-5 = A(s-4) + B(s-3)$

$s = 4$

$-1 = B$

$s = 3$

$-2 = -A$

$A = 2$

$L^{-1}\left[\frac{s-5}{(s-3)(s-4)}\right] = \frac{2}{s-3} - \frac{1}{s-4}$

$2e^{3t} - e^{4t}$

ii) $\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$

$2s-6 = A(s-4) + B(s-2)$

$s=4$

$2 = 2B$

$B = 1$

$s = 2$

$-2 = -2A$

$A = 1$

$\frac{2s-6}{(s-2)(s-4)} = \frac{1}{s-2} + \frac{1}{s-4}$ $\quad e^{2t} + e^{4t}$

iii) $\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$5s-8 = A(s-4) + B(s)$

$s = 4$

$12 = 4B$

$B = 3$

$A + B = 5$

$A = 2$

$= 2/s + 3/s-4 = 2 + e^{3t}$

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