2) Transform each of the following functions into Laplace (s) domain:

$$\mathcal{L}\{f(t)\} \longrightarrow F(s) = \int_0^\infty e^{-st} f(t)\,dt$$

i) $3e^{-4t} - 5e^{4t}$

$\mathcal{L}\{3e^{-4t} - 5e^{4t})$

$\mathcal{L}\{3e^{-4t}\} - \mathcal{L}\{5e^{4t}\}$

$\frac{3}{s+4} - \frac{5}{s-4}$

ii) $\sin 4t + \cos 4t$

$\mathcal{L}\{\sin 4t + \cos 4t\}$

$\mathcal{L}\{\sin 4t\} + \mathcal{L}\{\cos 4t\}$

$\frac{4}{s^2+16} + \frac{s}{s^2+16}$

iii) $t^3 + 2t^2 - t + 4$

$\mathcal{L}\{t^3 + 2t^2 - t + 4]$

$\mathcal{L}\{t^3\} + \mathcal{L}\{2t^2\} - \mathcal{L}\{t\} + \mathcal{L}\{4\}$

$\frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

iv) $e^{-2t}\cos 5t$

$\mathcal{L}\{e^{-2t}\cos 5t\}$

$= \frac{s-2}{(s-2)^2+25}$

v) $t\sin 3t$

$\mathcal{L}[t\sin 3t\}$

$\frac{3}{s^2+9}$

vi) $\frac{e^{-t} - e^{-2t}}{t}$

$\mathcal{L}\{e^{-t} - e^{-2t}\}$

$\mathcal{L}\{e^{-t}\} - \mathcal{L}\{e^{-2t}\}$

$\frac{1}{s+1} - \frac{1}{s+2}$

$= \int_s^\infty \left(\frac{1}{\sigma+1} - \frac{1}{\sigma+2}\right) d\sigma$

$\int_s^\infty \frac{1}{\sigma+1}\, d\sigma - \int_s^\infty \frac{1}{\sigma+2}\, d\sigma$

$\left[\ln(\sigma+1) - \ln(\sigma+2)\right]_s^\infty$

$\left[\frac{\ln(\sigma+1)}{(\sigma+2)}\right]_s^\infty$

$\left[\ln\frac{(\infty+1)}{(\infty+2)} - \ln\frac{(s+1)}{(s+2)}\right]$

$0 - \ln\frac{(s+1)}{(s+2}$

$\ln\frac{s+2}{s+3}$

vii) $e^{4t}\cos 2t$

$\mathcal{L}\{e^{4t}\cos 2t\}$

$\frac{s-4}{(s-4)^2+4}$

viii) $t\sin 2t$

$\frac{2}{s^2+4}$

ix) t^3+4t^2+5

$\mathcal{L}\{t^3+4t^2+5\}$

$\mathcal{L}\{t^3\}+\mathcal{L}\{4t^2\}+\mathcal{L}[5]$

$\frac{6}{s^4}+\frac{8}{s^3}+\frac{5}{s}$

x) $e^{3t}(t^2+4)$

$\frac{1}{s-3}\left(\frac{2}{s^3}+\frac{4}{s}\right)$

xi) $t^2\cos t$

$\frac{s-1}{(s-1)^2+1}$

xii) $\frac{\sinh 2t}{t} = \mathcal{L}\left\{\frac{\sinh 2t}{t}\right\}$

$= \frac{\tan^{-1}(2)}{s}$

3) Convert the following functions to time (t) domains

(1) $\frac{s-5}{(s-3)(s-4)} = \frac{A}{(s-3)} + \frac{B}{(s-4)}$

$\frac{A(s-4)+B(s-3)}{(s-3)(s-4)}$

$s-5 = As-4A+Bs-3B$

$s-5 = As+Bs-4A-3B$

$s-5 = s(A+B)-4A-3B$

$A+B=1 \quad \times -4$

$-4A-3B=-5 \quad \times 1$

$-4A-4B=-4$

$-4A-3B=-5$

$-B=1$

$B=-1$

$A+B=1$

$A-1=1$

$A=2$

$\frac{2}{(s-3)}-\frac{1}{(s-4)}$

$\mathcal{L}^{-1}\left\{\frac{2}{(s-3)}-\frac{1}{(s-4)}\right\}$

$2e^{3t}-e^{4t}$ //

2) $\frac{2s-6}{(s-2)(s-4)}$

$\frac{A}{(s-2)}+\frac{B}{(s-4)}$

$\frac{A(s-4)+B(s-2)}{(s-2)(s-4)}$

$2s-6 = As-4A+sB-2B$

$2s-6 = As+sB-4A-2B$

$2s-6 = s(A+B)-4A-2B$

$A+B=2 \quad \times -4$

$-4A-2B=-6 \quad \times 1$

$-4A+4B=-8$

$-4A-2B=-6$

$-2B=-2$

$B=1$

$A+B=2$

$A+1=2$

$A=1$

$\frac{1}{(s-2)}+\frac{1}{(s-4)}$

$\mathcal{L}^{-1}\left(\frac{1}{s-2}+\frac{1}{(s-4)}\right) = e^{2t}+e^{4t}$

3) $\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$\frac{5s-8}{s(s-4)} = \frac{A(s-4)+Bs}{s(s-4)}$

$5s-8 = A(s-4)+Bs$

$5s-8 = As-4A+Bs$

$5s-8 = As+Bs-4A$

$5s-8 = s(A+B)-4A$

$-4A = -8$

$A = 2$

$A+B = 5$

$2+B = 5$

$B = 3$

$\mathcal{L}^{-1}\left(\frac{2}{s} + \frac{B}{s-3}\right)$

$2 + e^{3t}$ //

iv) $\frac{s^2-3s-4}{(s-3)(s-1)^2}$

$\cancel{\frac{A}{s-A}} + \cancel{\frac{B}{s+1}}$

$\frac{A}{(s-3)} + \frac{B}{(s-1)} + \frac{C}{(s-1)^2}$

$s^2-3s-4 = A(s-1)^2 + B(s-3)(s-1) + C(s-3)$

$(s-3) = 0$

$C = -2$, $A = 1$

$B = -3$

$\frac{1}{(s-3)} - \frac{3}{(s-1)} - \frac{2}{(s-1)^2} = \mathcal{L}^{-1}\left[\frac{1}{(s-3)}\right] - \mathcal{L}^{-1}\left[\frac{3}{(s-1)}\right]$

$\frac{1}{}\, e^{3t} - 3e^{t}$ //

(i) $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)y'' - 2xy' + 2y = 0$

Let $w_1 = (1-x^2)y''$

$u = y^{(2)}$ $\quad$ $v = (1-x^2)$

$u^n = y^{n+2}$ $\quad$ $v' = -2x$

$u^{n+1} = y^{n+2}$ $\quad$ $v'' = -2$

$u^{n-1} = y^n$

Let $w_2 = 2xy'$

$u = 2y'$ $\quad$ $v = 2x$

$u^n = y^{n+1}$ $\quad$ $v' = 2$

$u^{n-1} = y^n$

Let $w_3 = 2y$

$u^n = 2y^n$

From Leibnitz

$y^n = u^n v + nu^{n-1}v' + \frac{n(n-1)}{2!}u^{n-2}v^2$

For w_1

$y^{n+2}(1-x^2) + n(y^{n+1})-2x + \frac{n(n-1)}{2!}y^n - 2$

$y^{n+2}(1-x^2) - 2xn(y^{n+1}) - n(n-1)y^n$

For w_2

$y^{n+1}2x + ny^n 2$

For w_3

$2y^n$

$y^n = (1-x^2)y^{n+2} + n(y^{n+1}) - 2x$

$y^n = (1-x^2)y^{n+2} - 2xn(y^{n+1}) - n(n-1)y^n - y^{n+1}2x$
$+ 2ny^n + 2y^n$

$y^n = y^{n+2} - x^2y^{n+2} - 2xn(y^{n+1}) - (n^2+n)y^n - y^{n+1}2x$
$+ 2ny^n + 2y^n$

when $x = 0$

$y^n = y^{n+2} - (n^2+n)y^n + 2ny^n + 2y^n$

$y^n = y^{n+2} - (n^2+n)y^n + y^n(2n-2) = 0$

$$y^n = y^{n+2} - (y^n n^2 + y^n n) + (2ny^n - 2y^n)$$

$$y^n = y^{n+2} - y^n n^2 - y^n n + 2ny^n + 2y^n$$

$$y^n = y^{n+2} - y^n(n^2 - n + 2) = 0$$

$$(y^{n+2})_0 = y^n(-n^2 + n + 2)$$

9+3 16+4
6-2 20-2 16

$n=0$, $(y^{(2)})_0 = (y^{(0)})_0(-2)$

$n=1$, $(y^{(3)})_0 = (y^{(1)})_0(0) = 0$

$n=2$, $(y^{(4)})_0 = (y^{(2)})_0(4)(-2)$

$n=3$, $(y^{(5)})_0 = (y^{(3)})_0 = 0$

$n=4$, $(y^{(6)})_0 = (y^{(4)})_0(18)(4)(-2)$

$n=5$, $(y^{(7)})_0 = (y^{(5)})_0 = 0$

$n=6$, $(y^{(8)})_0 = (y^{(6)})_0(18)(4)(-2)(40)$

$n=7$, $(y^{(9)})_0 = (y^{(7)})_0 = 0$

$$y = (y^{(2)})_0\frac{x^2}{2!} + (y^{(3)})_0\frac{x^3}{3!} + (y^{(4)})_0\frac{x^4}{4!} + (y^{(5)})_0\frac{x^5}{5!} + (y^{(6)})_0\frac{x^6}{6!} + (y^{(7)})_0\frac{x^7}{7!} + (y^{(8)})_0\frac{x^8}{8!} + (y^{(9)})_0\frac{x^9}{9!}$$

$$y = (y^{(0)})(-2)\frac{x^2}{2!} + (y^{(2)})_0(4)(-2)\frac{x^4}{4!} + \frac{x^6}{6!}(y^{(4)})_0(18)(4)(-2) + (y^{(8)})_0(18)(-4)(-2)(40)\frac{x^8}{8!}$$