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CHEMICAL ENGINEERING.

(1) $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0.$

$(1-x^2)y^2 - 2xy^1 + 2y = 0.$

$(1-x^2)y^2 \Rightarrow$

$u = y^2 \qquad v = (1-x^2).$

$u^n = y^{n+2} \qquad v^1 = -2x$

$u^{n-1} = y^{n+1} \qquad v^2 = -2$

$u^{n-2} = y^n \qquad v^3 = 0.$

$w_1 = (1-x^2)y^{n+2} - n\cdot 2x\cdot y^{n+1} - \frac{n(n-1)}{2!}\cancel{2}\cdot y^n$

$w_1 = (1-x^2)y^{n+2} - 2xny^{n+1} - n(n-1)y^n$

$2xy^1 \Rightarrow$

$u = y^1 \qquad v = 2x$

$u^n = y^{1+n} \qquad v^1 = 2$

$u^{n-1} = y^n \qquad v^2 = 0.$

$w_2 = 2x\cdot y^{n+1} + n2y^n$

$w_2 = 2xy^{n+1} + n2y^n.$

$2y \Rightarrow$

$u = y \qquad v = 2$

$u^n = y^n \qquad v^1 = 0$

$w_3 = 2y^n.$

$w_1 - w_2 + w_3 = 0$

$(1-x^2)y^{n+2} - 2xny^{n+1} - n(n-1)y^n - 2xy^{n+1} - 2ny^n + 2y^n = 0.$

$(1-x^2)y^{n+2} - 2xny^{n+1} - 2xy^{n+1} - n(n-1)y^n - 2ny^n + 2y^n = 0.$

$(1-x^2)y^{n+2} - 2xy^{n+1}\{n-1\} - y^n\{n(n-1) + 2n - 2\} = 0.$

$(1-x^2)y^{n+2} - \cancel{2xy^{n+1}}^{\to 0}(n-1) - y^n(n^2+n-2) = 0.$

when $x = 0.$

$y^{n+2} - y^n(n^2+n-2) = 0.$

$y^{n+2} = y^n(n^2+n-2).$ recurrence relation.

when $n=0$ $\quad y^n = y^0(-2) \quad ; \quad y^2 = -2y^0.$

$n=1 \quad y^3 = y^1(0) \quad ; \quad y^3 = 0.$

$n=2 \quad y^4 = y^2(4) \quad y^4 = 4y^2 = 4\times -2y^0 = -8y^0$

$n=3 \quad y^5 = y^3(10) \quad y^5 = 10y^3 = 0.$

$n=4 \quad y^6 = y^4(18) \quad y^6 = 18y^4 = 18\times -8y^0 = -144y^0$

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$$y = y^0 + xy' + \frac{x^2}{2!}(-2y^0) + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(-8y^0) + \frac{x^5}{5!}(0) + \frac{x^6}{6!}(-144y^0)$$

$$y = y^0 + xy' - x^2y^0 - \frac{x^4}{3}y^0 - \frac{x^6}{5}y^0.$$

$$y = y^0\left\{1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5}\right\} + y'\left\{x + \frac{x^3}{2} + \frac{x^5}{4} + \frac{x^7}{6}\right\}.$$

2) i) $L\{3e^{-4t} - 5e^{4t}\}$

$$= \frac{3}{(s+4)} - \frac{5}{(s-4)}$$

ii) $\sin 4t + \cos 4t.$

$$\frac{4}{s^2+16} + \frac{s}{s^2+16}.$$

iii) $t^3 + 2t^2 - t + 4$

$$\frac{3!}{s^{3+1}} + \frac{2\cdot 2!}{s^{2+1}} - \frac{1}{s^{1+1}} + \frac{4}{s}.$$

$$\frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}.$$

iv) $e^{-2t}\cos 5t.$

$$= \frac{s}{s^2+5^2} \quad ; s = s+2$$

$$= \frac{(s+2)}{(s+2)^2 + 25}.$$

i) $t\sin 3t$

$$\frac{d}{ds}\left[\frac{3}{s^2+9}\right]$$

$u = 3$ $\quad v = s^2+9$

$\frac{du}{ds} = 0$ $\quad \frac{dv}{ds} = 2s$

$$\frac{v\frac{du}{ds} - u\frac{dv}{ds}}{v^2} = \frac{(s^2+9)(0) - 3(2s)}{(s^2+9)^2}$$

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$$= \frac{-6s}{(s^2+4)^2}.$$

vi) $\frac{e^{-t} - e^{-2t}}{t}.$

$$\lim_{t \to 0} \frac{e^{-0} - e^{-2(0)}}{0} = 0$$

L'Hopitals rule.

$$\frac{-e^{-t} + 2e^{-2t}}{1} = \frac{-1+2}{1} = 1.$$

$$\frac{1}{s+1} - \frac{1}{s+2}$$

$$\int_s^\infty \frac{1}{\sigma+1} - \int_s^\infty \frac{1}{\sigma+2}$$

$$\ln(\sigma+1) - \ln(\sigma+2) \Big|_s^\infty$$

$$\ln\frac{(\sigma+1)}{\sigma+2}\Big|_s^\infty$$

$$\ln\left(\frac{\infty+1}{\infty+2}\right) - \ln\left(\frac{s+1}{s+2}\right)$$

$$-\ln\left(\frac{s+1}{s+2}\right) = \ln\left(\frac{s+1}{s+2}\right)^{-1} = \ln\left(\frac{s+2}{s+1}\right).$$

vii) $e^{4t}\cos 2t.$

$$\frac{s}{s^2+2^2} \quad ; \quad s = s-4.$$

$$\frac{s-4}{(s-4)^2+4}$$

viii) $t\sin 2t$.

$$\frac{d}{ds}\left[\frac{2}{s^2+4}\right] \qquad u = 2 \quad v = s^2+4$$

$$\frac{du}{ds} = 0 \qquad \frac{dv}{ds} = 2s$$

$$\frac{v\frac{du}{ds} - u\frac{dv}{ds}}{v^2} = \frac{(s^2+4)(0) - 2(2s)}{(s^2+4)^2} = \frac{-4s}{(s^2+4)^2}$$

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ix) $t^3 + 4t^2 + 5.$

$$\frac{3!}{s^{3+1}} + \frac{4\times 2!}{s^{2+1}} + \frac{5}{s}.$$

$$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}.$$

x) $$\frac{2}{s^{2+1}} + \frac{4}{s} \qquad s = s-3$$

$$\frac{2}{(s-3)^3} + \frac{4}{s-3}.$$

xi) $$\frac{d^2}{ds^2}\left[\frac{s}{s^2+1}\right] \qquad u = s \quad v = s^2+1$$

$$\frac{du}{ds} = 1 \qquad \frac{dv}{ds} = 2s$$

$$\frac{v\frac{du}{ds} - u\frac{dv}{ds}}{v^2} = \frac{(s^2+1)1 - s(2s)}{(s^2+1)^2}.$$

$$= \frac{s^2+1-2s^2}{(s^2+1)^2} = \frac{1-s^2}{(s^2+1)^2}$$

$$\frac{d}{ds}\left[\frac{1-s^2}{(s^2+1)^2}\right] \qquad u = 1-s^2 \qquad v = (s^2+1)^2$$

$$\frac{du}{dx} = -2s \qquad \frac{dv}{ds} = 2(s^2+1) \times 2s.$$

$$= 4s(s^2+1).$$

$$\frac{v\frac{du}{ds} - u\frac{dv}{ds}}{v^2} = \frac{-2s(s^2+1)^2 - 4s(s^2+1)((1-s^2).}{(s^2+1)^4}$$

$$= \frac{-2s(s^2+1) - 4s(1-s^2).}{(s^2+1)^3}$$

$$= \frac{-2s^3 - 2s - 4s + 4s^3}{(s^2+1)^3}$$

$$= \frac{2s^3 - 6s}{(s^2+1)^3}$$

(xii) $$\int \frac{2}{s^2-4} = \int_s^\infty \frac{2}{\sigma^2-4.} = \frac{1}{2}\tan^{-1}\frac{\sigma}{2}\Bigg|_s^\infty$$

$$= \frac{1}{2}\tan^{-1}\frac{\infty}{2} - \frac{1}{2}\tan^{-1}\frac{s}{2}. = \frac{\pi}{2} - \frac{1}{2}\tan^{-1}\frac{s}{a}$$

$$= \tan^{-1}\frac{a}{s} + \tan^{-1}\frac{s}{a} - \tan^{-1}\frac{s}{a}$$

$$= \tan^{-1} a/s.$$

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i) $\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$

$s-5 = A(s-4) + B(s-3).$

when $s=4$.

$-1 = B.$ ; $B=-1$

when $=3$

$-2 = -A$ ; $A=2.$

$\frac{s-5}{(s-3)(s-4)} = \frac{2}{s-3} - \frac{1}{s-4}.$

$= 2e^{3t} - e^{4t}.$

ii) $\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}.$

$2s-6 = A(s-4) + B(s-2)$

when $s=4$

$2 = 2B$ ; $B=1.$

when $s=2.$

$-2 = -2A$ ; $A=1.$

$\frac{2s-6}{(s-2)(s-4)} = \frac{1}{s-2} - \frac{1}{s-4}$

$= e^{2t} - e^{4t}$

iii) $\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{(s-4)}$

$5s-8 = A(s-4) + Bs$

when $s=0$.

$-8 = -4A$

$A=2.$

when $s=4$

$12 = 4B$

$B=3.$

$\frac{5s-8}{s(s-4)} = \frac{2}{s} + \frac{3}{s-4} = 2 + 3e^{4t}$

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$$\frac{s^2-3s-4)}{(s-3)(s-1)^2} = \frac{A}{(s-3)} + \frac{B}{(s-1)} + \frac{C}{(s-1)^2}$$

$$(s^2-3s-4) = A(s-1)^2 + B(s-3)(s-1) + C(s-3).$$

when $s=1$.

$$-6 = -2C$$

$$C = 3.$$

when $s = 3$.

$$-4 = 4A.$$

$$A = -1.$$

when $s=0$.

$$-4 = -1 + 3B - 3(3)$$

$$3B = -4 + 1 + 9.$$

$$B = 2.$$

$$\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{-1}{s-3} + \frac{2}{s-1} + \frac{3}{(s-1)^2}$$

$$= -e^{3t} + 2e^{t} + e^{t}\,3t.$$

$$= -e^{3t} + 2e^{t} + 3e^{t}t.$$

v) $$\frac{s-5}{s^2+4s+20} = \frac{s-5}{(s+2)^2+16} = \frac{s-5}{(s+2)^2+4^2}$$

$$\frac{s}{(s+2)^2+4^2} - \frac{5}{(s+2)^2+4^2} \times \frac{\cancel{4}}{\cancel{4}}$$

$$\frac{s+2-2}{(s+2)^2+4^2} - \frac{5}{(s+2)^2+4^2}$$

$$\frac{s+2}{(s+2)^2+4^2} - \frac{2}{(s+2)^2-4} - \frac{5}{(s+2)^2+4^2}$$

$$\frac{s+2}{(s+2)^2+4^2} - \frac{7}{(s+2)^2-4} \times \frac{4}{4}$$

$$= e^{-2t}\cos 4t - \frac{7}{4}\sin 4t.$$