Duke Tonbra James

15/ENG07/012

Petroleum-Engineering

ENG381 Assignment (iii)

1. $(1-x^2)\frac{d^2y}{dx^2} - 2x\,dy/dx + 2y = 0$

Solving for $(1-x^2)y''$

$u = y, \; u^n = y^{n+2}$

$V = 1-x^2, \; v' = -2x, \; v'' = -2$

$W^n = y^{n+2} \cdot 1-x^2 + ny^{n+1} \cdot -2x + \frac{n(n-1)}{2} y^n \cdot -2$

$W^n = (1-x^2)y^{n+2} - 2nxy^{n+1} - n(n-1)y^n$ --- (1)

Solving for $2xy'$

$u = y, \; u^n = y^{n+1}$

$V = -2x, \; v' = -2$

$W^n = -2xy^{n+2} + ny^n \cdot -2$

$= -2xy^{n+1} - 2ny^n$ --- (2)

Solving for $2y$

$V = 2, \; v' = 0$

$u = y, \; u^n = 2y^n$ --- (3)

∴ Combine the three equations

$(1-x^2)y^{n+2} - 2nxy^{n+1} - n(n-1)y^n - 2xy^{n+1} - 2ny^n + 2y^n$

$(1-x^2)y^{n+2} + (-2nx - 2x)y^{n+1} + (-n^2+n-2n+2)y^n = 0$

$(1-x^2)y^{n+2} - (2nx+2x)y^{n+1} - (n^2+n-2)y^n = 0$

at $x = 0$

$y(n+2) - 0 - (n^2+n-2)y^n = 0$

$y(n+2) = (y')_0(n^2+n-2)$

$n = 0$

$(y^{(2)})_0 = (y^0)_0(-2) = -2(y^{(0)})_0$

$n = 1$

$(y^{(3)})_0 = (y')_0(0) = 0$

$n = 2$

$(y^{(4)})_0 = (y^2)_0(4) = 4 \times -2(y^0)_0 = -8(y(0))_0$

$n = 3$

$(y^{(5)})_0 = (y^3)_0(10)(y^{(3)})_0 = 10 \times 0 = 0$

$n = 4$

$(y^{(6)})_0 = (y^4)_0(18) = 18(y^{(4)})_0 = 18 \times -8(y^0)_0$

$= -144(y^0)_0$

$n = 5$

$(y^7)_0 = (y^5)_0(28) = 28 \times 0 = 0$

$n = 6$

$(y^8) = (y^6)_0(40) = 40 \times 144(y^0) = -5760(y^0)$

$y = (y)_0 + x(y')_0 + x^2/2!(y^2)_0 + x^3/3!(y^3) + x^4/4!(y^4)_0 + \cdots$

$y = (y)_0 + x(y')_0 - x^2(y^0)_0 + 0 + - x^4/3(y^0)_0 + 0 - \frac{x^6}{5}(y^0)_0 + 0 - x^8/7(y^0)_0 + \cdots$

$y = (y)_0\left[1 - x^2 - x^4/3 - x^6/5 - x^8/7\right] + (y')_0(x)$

Question (2)

(i) $3e^{-4t} - 5e^{4t}$

$L[f(t)] = F(s)$

$F(s) = L[3e^{-4t}] - L[5e^{4t}]$

$F(s) = \frac{3}{s+4} - \frac{5}{s-4}$

ii) $\sin 4t + \cos 4t = f(t)$

$F(s) = L[f(t)]$

$F(s) = L[\sin 4t] + L[\cos 4t]$

$F(s) = \frac{4}{s^2+16} + \frac{s}{s^2+16}$

(iii) $t^3 + 2t^2 - t + 4 = f(s)$

$L[f(t)] = F(s)$

$f(s) = L[t^3] + 2L[t^2] - L[t] + L[4]$

$(Fs) = \frac{2!}{s^3} + 2\left[\frac{2!}{s^3}\right] - \frac{1}{s^2} + \frac{4}{s}$

$F(s) = \frac{2}{s^3} + 4/s^3 - \frac{1}{s^2} + 4/s$

(iv) $e^{-2t}\cos 5t$

$F(s) = L[e^{-2t}\cos 5t]$

$L[\cos 5t] = \frac{s}{s^2+25}$

$\therefore f(s) = \frac{s+2}{(s+2)^2+25}$

(v) $t\sin 3t$

$L[\sin 3t] = \frac{3}{s^2+9}$

$-d/ds\ 3/s^2+9 = \frac{-1\ (s+9(0) - 3(2s))}{(s^2+9)^2}$

$= -1\left[\frac{-6s}{(s^2+9)^2}\right]$

$F(s) = \frac{6s}{(s^2+9)^2}$

(vi) $\frac{e^{-t} - e^{-2t}}{t} = \frac{1}{t}(e^{-t} - e^{-2t})$

$f(t) = \frac{e^{-t}}{t} - \frac{e^{-2t}}{t}$

$f(t) = e^{-1}e^{-t} - e^{-1}e^{-2t}$

$F(s) = \frac{-1!}{(s+1)-1+1} + \frac{1!}{(s+2)-1+2}$

$F(s) = -1 + \frac{1}{s+2}$

$F(s) = \frac{1-s-2}{(s+2)}$

(vii) $e^{4t}\cos 2t$

$L[\cos 2t] = \frac{s}{s^2+4}$

$L[e^{4t}\cos 2t] = \frac{s-4}{(s-4)^2+4}$

(viii) $t\sin 2t$

$L[\sin 2t] = \frac{2}{s^2+4}$

$-d/ds\ 2/s^2+4 = -\left[\frac{s^2+4(0) - 2(2s)}{(s^2+4)^2}\right]$

$= \frac{4s}{(s^2+4)^2}$

(ix) $t^3 + 4t^2 + 5 = f(t)$

$F(s) = L[f(t)]$

$F(s) = L[t^3] + L[4t^2] + L[5]$

$F(s) = \frac{3!}{s^4} + 4\frac{2!}{s^3} + \frac{5}{s}$

$F(s) = \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$

(x) $e^{3x}(t^2+4)$

$L[f(t)] = f(s)$

$f(s) = L[e^{3x}]\ L[t^2] + L[4]$

$= \frac{3}{s-3}\ \frac{2!}{s^3} + \frac{4}{s}$

$= \frac{3}{s-3}\left[\frac{2}{s^3} + \frac{4}{s}\right]$

(xi) $t^2\cos t$

$L[\cos t] = \frac{s}{s^2+1^2} = \frac{s}{s^2+1}$

$\therefore F'(s) = -\frac{d}{ds}\left[\frac{s}{s^2+1}\right]$

$u = s, \quad du/ds = 1$

$v = s^2+1, \quad dv/ds = 2s$

$$\frac{v \cdot du/ds - u \cdot dv/ds}{v^2}$$

$$-\left(\frac{(s^2+1)(1) - s(2s)}{(s^2+1)^2}\right) = \frac{s^2+1-2s^2}{(s^2+1)^2}$$

$$L^{-1}[t\cos t] = -\frac{s^2+1}{(s^2+1)^2} = \frac{-1(s^2+1)}{(s^2+1)^2}$$

$$= \frac{-1}{s^2+1} = \frac{1}{s^2+1}$$

$$L[t^2\cos t] = -\frac{d}{ds}\left[\frac{1}{s^2+1}\right]$$

$u = 1, \quad du/ds = 0$

$v = s^2+1, \quad dv/ds = 2s$

$$\frac{(s^2+1)(0) - (1)(2s)}{(s^2+1)^2} = \frac{-2s}{(s^2+1)^2}$$

Question (3)

(i) $\frac{s-5}{(s-3)(s-4)} = \frac{A}{(s-3)} + \frac{B}{(s-4)}$

$A(s-3)|_{s=3} = \frac{s-5}{s-4} = \frac{3-5}{3-4} = 2$

$B(s-4)|_{s=4} = \frac{s-5}{s-3} = \frac{4-5}{4-3} = -1$

$F(s) = L^{-1}[f(s)] = L^{-1}\left[\frac{2}{s-3} - \frac{1}{s-4}\right]$

$u(t) = 2e^{-3t} - e^{-4t}$

(ii) $\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$

$A(s-2)|_{s=2} = \frac{2s-6}{s-4} = \frac{2(2)-6}{2-4} = 1$

$B(s-4)|_{s=4} = \frac{2s-6}{s-2} = \frac{2(4)-6}{4-2} = 1$

$F(s) = L^{-1}[f(s)] = L^{-1}\left[\frac{1}{s-2} + \frac{1}{s-4}\right]$

$u(t) = e^{2t} + e^{4t}$

(iii) $\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$A(s)|_{s=0} = \frac{5s-8}{s-4} = \frac{5(0)-8}{0-4} = 2$

$B(s-4)|_{s=4} = \frac{5s-8}{s} = \frac{5(4)-8}{4} = 3$

$F(t) = L^{-1}[f(s)] = L^{-1}\left[\frac{2}{s} + \frac{3}{s-4}\right]$

$u(t) = 2u(t) + 3e^{4t}$

(iv) $\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{A}{s-3} + \frac{B}{(s-1)^2} + \frac{C}{(s-1)}$

$A(s-3)|_{s=3} = \frac{(3)^2-3(3)-4}{(3-1)^2} = -1$

$B(s-1)|_{s=1} \ \frac{d}{ds}\frac{s^2-3s-4}{(s-3)} = \frac{(s-3)(3s^2-3) - (s^2-3s}{}$

$$\frac{-4(1)}{(s-3)^2}$$

$$= \frac{3s^2 - \cancel{3s} - 9s^2 + 9 - s^2 + \cancel{3s} + 4}{(s-3)^2}$$

$$= \frac{2s^2 - 9s^2 + 13}{(s-3)^2} = \frac{2(1)^2 - 9(1)^2 + 13}{(1-3)^2}$$

$$= \frac{2-9+13}{(-2)^2} = \frac{6}{4} = \frac{3}{2}$$

$C(s-1)|_{s=1} \ \frac{s^2-3s-4}{(s-3)} = \frac{(1)^2-3(1)-4}{(1-3)}$

$$= \frac{1-3-4}{-2} = \frac{-6}{-2} = 3$$

$$F(s) = -e^{3t} + \tfrac{3}{2}e^{t} + 3e^{t}$$

4) $$\frac{s-5}{s^2+4s+20} = \frac{A}{(s+2+js)} + \frac{B}{(s+2+4j)}$$

$$= \frac{s-5}{(s+2-j4)(s+2+4j)}$$

$$A(s+2-j4)|_{s=-2+4j}$$

$$= \frac{-(-2+4j)-5}{-2+4j+2+4j} = \frac{-7+4j}{8j} \times \frac{j}{j}$$

$$= -\frac{7j-4}{-8} = \frac{7j+4}{8}$$

$$B(s+2+4j)|_{s=-2-4j}$$

$$= \frac{-2-4j-5}{(-2-4j+2-4j)} = \frac{-7-4j}{-8j} \times \frac{j}{j}$$

$$= -\frac{7i+4}{8}$$

$$F(s) = \frac{7i+4}{8(s+2-4j)} - \frac{7j+4}{8(s+2+4j)}$$

$$F(t) = L^{-1}[F(s)]$$

$$F(t) = \frac{7i+4}{8}\left(e^{(-2+4j)t} - e^{(-2-4j)t}\right)$$