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Electrical-Electronics Engineering

1] $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$$\underbrace{[1-x^2]y''}_{w_1} - \underbrace{2x\,y'}_{w_2} + \underbrace{2y}_{w_3} = 0$$

For $w_1$

$u = y''$ $\qquad v = 1-x^2$

$u^n = y^{n+2}$ $\qquad v' = -2x$

$u^{n-1} = y^{n+1}$ $\qquad v'' = -2$

$u^{n-2} = y^n$ $\qquad v''' = 0$

$y^{n+2}[1-x^2] + n y^{n+1} x - 2x + \frac{n(n-1)}{2!} y^n x - 2$

$= y^{n+2} - \cancel{x^2 y^{n+2}} + \cancel{n y^{n+1} x} - 2x + n(n-1) y^n x - 1$

$= y^{n+2} - n(n-1) y^n$

For $w_2$

$-2x y'$

$u = y'$ $\qquad v = -2x$

$u^n = y^{n+1}$ $\qquad v' = -2$

$u^{n-1} = y^n$ $\qquad v'' = 0$

$w_2 = \cancel{y^{n+1} x - 2x} + n y^n x - 2$

$= -2n y^n$

For $w_3$

$2y$

$u^n = y^n$ $\qquad v = 2$

$v' = 0$

$w_3 = 2y^n$

$W = w_1 + w_2 + w_3$

$= y^{n+2} - n(n-1)y^n - 2n y^n + 2y^n$

$= y^{n+2} + 2y^n - n(n-1)y^n - 2n y^n$

$= y^{n+2} + y^n(2 - n(n-1) - 2n)$

$= y^{n+2} + y^n(2 - n^2 + n - 2n)$

$= y^{n+2} = -y^n[2 - n^2 + n - 2n]$

$= y^{n+2} = y^n[-2 + n^2 + n]$

when $n = 0, 1, 2, 3, 4$

$n=0$ $[y^{(2)}]_0 = [y^{(0)}]_0[-2+0+0]$

$[y^{(2)}]_0 = [y^{(0)}]_0[-2]$

$n=1$ $[y^{(3)}]_0 = [y^{(1)}]_0[-2+1+1]$

$= [y^{(1)}]_0[0]$

$n=2$ $[y^{(4)}]_0 = [y^{(2)}]_0[-2+2^2+2]$

$[y^{(4)}]_0 = [y^{(2)}]_0[4] = -8[y^{(0)}]_0$

$n=3$ $[y^{(5)}]_0 = [y^{(3)}]_0[-2+3^2+3]$

$[y^{(5)}]_0 = [y^{(3)}]_0[10] = 10(0)[y']_0$

$= 0[y']_0$

$n=4$ $[y^{(6)}]_0 = [y^{(4)}]_0[-2+4^2+4]$

$= [y^{(4)}]_0[18] = -8 \times 18[y^{(0)}]_0$

$= -144[y^{(0)}]_0$

Using Marclauring Series

$y = [y]_0 + x[y']_0 + \frac{x^2}{2!}[y'']_0 + \frac{x^3}{3!}[y''']_0 + \frac{x^4}{4!}[y^{iv}]_0 + \frac{x^5}{5!}[y^{v}]_0 + \frac{x^6}{6!}[y^{vi}]_0$

$= [y]_0 + x[y']_0 + \frac{x^2}{2!}[-y]_0 - 3 + \frac{x^3}{3!}\cancel{[y']} + \frac{x^4}{4!}[-8[y]_0] + \frac{x^5}{5!}\cancel{[-y']} + \frac{x^6}{6!}[-144[y]_0]$

$= [y]_0 + x[y']_0 + \frac{x^2}{2\times1}[-y]_0 - 3\times\frac{x^4}{4\times3\times2\times1} \cdot 8[-y]_0$

$-\frac{x^6}{6\times5\times4\times3\times2\times1} \cdot 144[y]_0$

$= [y]_0 + x[y']_0 - x^2[-y]_0 - \frac{x^4}{3}[y]_0 - \frac{x^6}{5}[y]_0$

$= [y]_0\left[1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5}\right] + [y']_0[x]$

ii) $L[3e^{-4t}] - L[5e^{4t}]$

$= \frac{3}{s+4} - \frac{5}{s-4}$

iii) $L[\sin 4t] + L[\cos 4t]$

$\frac{4}{s^2+4^2} + \frac{s}{s^2+4^2}$

iii) $L[t^3] + L[2t^2] - L[t] + L[4]$

$\frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} - \frac{4}{s}$

iv) $L[e^{-2t}\cos 5t]$

$L[\cos 5t] = \frac{s}{s^2+5^2}$

$L[e^{-2t}] = \frac{1}{s+2}$

$= L[e^{-2t}\cos 5t]$

$\frac{s+2}{(s+2)^2+5^2}$

v) $L[t\sin 3t]$

$L[\sin 3t] = -\frac{d}{ds}\frac{3}{s^2+3^2}$

$L[t\sin 3t] = \frac{(s^2+3^2)0 - 6s}{(s^2+3^2)^2}$

$-\frac{d}{ds}\frac{3}{s^2+3^2} = -\left[\frac{-6s}{(s^2+3^2)^2}\right]$

$= \frac{6s}{(s^2+9)^2}$

vi) $L\left[\frac{e^{-t} - e^{-2t}}{t}\right]$

$\lim_{t\to 0}\left[\frac{-e^{-t} + e^{-2t}}{t}\right]$

$-e^{-0} + e^{-2(0)}$

$= 1 + 1$

$= 2$

$L[e^{-t} - e^{-2t}] = \frac{1}{s+1} - \frac{1}{s+2}$

$= \int_{\theta=s}^{\infty} \frac{1}{\theta+1} - \frac{1}{\theta+2}$

$= [\ln \theta+1 - \ln \theta+2]_{\theta=s}^{\infty}$

$= \ln[\infty+1] - \ln[s+1] - [\ln[\infty+2] - \ln(s+2)]$

$= \ln[\infty+1] - \ln(s+1) - \ln(\infty+2) - \ln(s+2)$

$= \ln[\infty+1 - (s+1)] - \ln[\infty+2] - s - 2]$

$\ln[\infty-5] - \ln(\infty-3]$

$= \ln\left[\frac{\infty-5}{\infty-3}\right]$

$= \ln 1$

$= 0$

vii] $e^{4t}\cos 2t$

$L[\cos 2t] = \frac{2}{s^2+2^2}$

$L[e^{4t}] = \frac{1}{s-4}$

$L[e^{4t}\cos 2t] = \frac{2}{(s^2+4)-4}$

viii] $L[t\sin 2t]$

$L[\sin 2t] = \frac{2}{s^2+4}$

$-\frac{d}{ds}\frac{2}{s^2+4} = \frac{[s^2+4]\times 0 - 4s}{(s^2+4)^2}$

$= -\left[\frac{-4s}{[s^2+4]^2}\right]$

$= \frac{4s}{(s^2+4)^2}$

ix] $t^3 + 4t^2 + 5$

$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$

x] $e^{3t}\cdot[t^2+4t]$

$L[t^2+4t] = \frac{2}{s^3} + \frac{4}{s^2}$

$L[e^{3t}] = \frac{1}{s-3}$

$L[e^{3t}[t^2+4t]] = \frac{2}{(s-3)^3} + \frac{4}{(s-3)}$

xi] $t^2\cos t$

$\cos t = \frac{s}{s^2+1^2}$

$= \frac{s^2+1^2[1] - s(2s)}{(s^2+1^2]^2}$

$= \frac{s^2+1^2-2s^2}{[s^2+1^2]^2}$

$= -\left[\frac{3s^2+1}{(s^2+1)^2}\right]$

$= -\frac{[s^2+1]\times 6s + [3s^2+1](4s)}{(s^2+1)^4}$

$= -\left[\frac{6s^3-6s+12s^3-4s}{(s^2+1)^4}\right]$

$= -\frac{(18s^3-2s)}{[s^2+1)^4}$

$= \frac{-18s^3+2s}{[s^2+1)^4}$

xii] $\frac{\sinh 2t}{t}$

$\left[\frac{2\cosh 2t}{1}\right]$

lim $t=0$

$\frac{2\cosh 2(0)}{1} = 2$

$L[\sinh 2t] = \frac{2}{s^2 - 2^2}$

$\int_{\sigma = s}^{\infty} \frac{2}{\sigma^2 - 2^2}$

$v = \sigma^2 - 2^2$

$\frac{dv}{d\sigma} = 2\sigma$

$d\sigma = \frac{dv}{2\sigma}$

$\int_{\sigma}^{\infty} \frac{2}{v} \frac{dv}{2\sigma}$

$\frac{1}{\sigma} \ln v \Big|_{\sigma}^{\infty}$

$\frac{1}{\sigma} \ln(\sigma^2 - 4]$

$\frac{1}{\infty} \ln[\infty^2 - 4] - \ln \frac{1}{s}(s^2 - 4]$

$= -\frac{1}{s} \ln(s^2 - 4]$

5.] $\frac{s-5}{(s-3)(s-4)}$

$\frac{A}{s-3} + \frac{B}{s-4}$

for A $s=3$   for B $s=4$

$A[s-4] + B[s-3] = s-5$

for $s=3$

$A[3-4] + B[3-3] = 3-5$

$-A = -2$

$A = 2$

for $s=4$

$A[s-4] + B(s-3] = s-5$

$A[4-4] + B[4-3] = 4-5$

$B = -1$

$\therefore \frac{2}{s-3} + \frac{-1}{s-4}$

$2e^{3t} + -e^{4t}$

11] $\frac{2s-6}{[s-2][s-4]}$

$= \frac{A}{[s-2]} + \frac{B}{s-4}$

for $s=2$   $s=4$

$A[s-4] + B[s-2] = 2s-6$

for $s=2$

$A[2-4] + B[2-2] = 2(2) - 6$

$-2A = -2$

$A = 1$

for $s=4$

$B[4-2] = 2(4] - 6$

$2B = 2$

$B = 1$

$\frac{1}{s-2} + \frac{1}{s-4}$

$= e^{2t} + e^{4t}$

$\boxed{iii}$ $\frac{5s-8}{s(s-4)}$

$= \frac{A}{s} + \frac{B}{s-4}$

$s=0 \qquad s=4$

$A(s-4) + Bs = 5s-8$

$A(0-4) + B(0) = 5(0) - 8$

$-4A = -8$

$A = \frac{-8}{-4} = 2$

$A(4-4) + B(4) = 5(4) - 8$

$4B = 20 - 8$

$4B = 12$

$B = \frac{12}{4}$

$B = 3$

$\frac{2}{s} + \frac{3}{s-4}$

$2 + 3e^{4t}$

$\boxed{iv}$ $s^2 - 3s - 4 = A + B + C$

$= \frac{-4}{4}$

$= -1$

for B

$\frac{s^2-3s-\ldots}{(s-3)(s-\ldots}$

when $s = \ldots$

for C

$\frac{s^2-3s-\ldots}{(s-3)}$

when $s = -1$

$= \frac{-1}{(s-3)}$

$= -e^{3t}$

$s-5$

$-5$

$-5$

[c] $\dfrac{s^2-3s-4}{(s-3)(s-1)^2}$

$= \dfrac{(s-4)(s+1)}{(s-3)(s-1)^2} = \dfrac{A}{s-3} + \dfrac{B}{s-1} + \dfrac{C}{(s-1)^2}$

$(s-4)(s+1) = A(s-1)^2 + B(s-3)(s-1) + C(s-3)$

when $s=1$

$(1-4)(1+1) = C(1-3)$

$-6 = -2C$

$C = 3$

when $s=3$

$(3-4)(3+1) = A(2)^2$

$-4 = 4A$

$A = -1$

coefficient of $s^2$

$1 = A + B$

$B = 1+1$

$= 2$

$= \dfrac{-1}{s-3} + \dfrac{2}{s-1} + \dfrac{3}{(s-1)^2}$

$= -e^{3t} + 2e^{t} + 3te^{3t}$

[d] $\dfrac{s-5}{s^2+4s+20} = \dfrac{A+Bs}{s^2+4s+20}$

$s-5 = A+Bs$

$s=0$

$-5 = A$

$B = 1$

$f(s) = \dfrac{-5+s}{s^2+4s+20}$

$= \dfrac{s-5}{s^2+4s+20}$

$= \dfrac{s-5}{s^2+4s+4+16}$

$= \dfrac{s-5}{(s+2)^2+16}$

$= \dfrac{s}{(s+2)^2+16} - \dfrac{5}{(s+2)^2+16}$

$= \dfrac{s+2-2}{(s+2)^2+16} - \dfrac{5}{(s+2)^2+16}$

$= \dfrac{s+2}{(s+2)^2+16} - \dfrac{2}{(s+2)^2+16} - \dfrac{5}{(s+2)^2+16}$

$= \dfrac{s+2}{(s+2)^2+16} - \dfrac{2}{(s+2)^2+16} - \dfrac{5}{(s+2)^2+16}$

$= \dfrac{s+2}{(s+2)^2+16} - \dfrac{7}{(s+2)^2+16}$

$= e^{-2t}\cos 4t - \dfrac{7}{4}e^{-2t}\sin 4t$