EBIS BLESSING OLOKPA 15/ENG07/013

(1) $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

using Leibnitz-maclaurin series

Soln

$(1-x^2)y'' - 2xy' + 2y = 0$

Sub 1

$(1-x^2)y''$

$u = y''$ $\quad u^n = y^{n+2}$

$v = 1-x^2$

$v' = -2x$

$v'' = -2$

$\therefore u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + \dots$

$y^n = y^{n+2}(1-x^2) + ny^{n+1} \cdot -2x + \frac{n(n-1)}{2!} y^n \cdot -2$

$y^n = (1-x^2)y^{n+2} - 2nxy^{n+1} - n(n-1)y^n$

Sub 2

$-2xy'$

$u = y'$ $\quad u^n = y^{n+1}$

$v = -2x$ $\quad v' = -2$

$y^{n+1} \cdot -2x + ny^n \cdot -1$

$= -2xy^{n+1} - 2ny^n$

Sub 3

$2y$

$u = y$ $\quad u^n = y^n$

$v = 2$ $\quad v' = 0$

$y^n \cdot 2 = 2y^n$

combining sub (1), (2) and (3)

$(1-x^2)y^{n+2} - 2nxy^{n+1} - n(n-1)y^n - 2xy^{n-1}$

$-2ny^n + 2y^n$

$(1-x^2)y^{n+2} + (-2nx - 2x)y^{n+1} + (-n^2 + n - 2n + 2)y^n = 0$

$(1-x^2)y^{n+2} - 2nx + 2xy^{n+1} - (n^2 - n + 2)y^n$

at $x = 0$

$y^{(n+2)} - 0(-n^2 - n + 2)y^n = 0$

$y^{n+2} = (y^n)_0 (n^2 + n - 2)$

when $n = 0$

$(y^2)_0 - (y^0)_0(-2) = -2(y^0)_0$

when $n = 1$

$(y^3)_0 = (y')_0(0) = 0$

$n = 2$

$(y^4)_0 = (y^2)_0(4)$

$= 4(-2)(y^0)_0 = -8(y^0)_0$

$n = 3$

$(y^5)_0 = (y^3)_0(10) = 10(y^3)_0 = 0$

$n = 4$

$(y^6)_0 = (y^4)_0(18) = 18(y^4)_0$

$= 18 \times -8(y^0)_0 = -144(y^0)_0$

$n = 5$

$(y^7)_0 = (y^5)_0(28) = 0$

$n = 6$

$(y^8)_0 = (y^6)_0(40) = 40 \times -144(y^{(0)})_0$

$= -5760(y^0)_0$

$\therefore y = y_0 + x(y')_0 + \frac{x^2}{2!}(y^2)_0 +$

$\frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^4)_0 + \dots$

$y = y_0 + x(y')_0 + (-2y^0)_0 \cdot \frac{x^2}{2!} + 0 +$

$(-8y^0)\frac{x^4}{4!} + 0 + \frac{x^6}{6!}(-144)(y^0)_0$

$+ 0 + (-5760)(y^0)_0 \frac{x^8}{8!}$

$y = y_0\left(1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5} - \frac{x^8}{7}\right)$

$+ (y')_0(x)$

(i) $L\{3e^{-4t}+5e^{4t}\}$

$= \frac{3}{s+4} - \frac{5}{s-4} = \frac{3(s-4)-5(s+4)}{(s+4)(s-4)}$

$= \frac{3s-12-5s-20}{s^2-16} = \frac{-2s-32}{s^2-16}$

ii) $L\{\sin 4t + \cos 4t\}$

$\frac{4}{s^2+4^2} + \frac{s}{s^2+4^2} = \frac{4+s}{s^2+16}$

iii) $L\{t^3+2t^2-t+4\}$

$= \frac{3!}{s^4} + 2\left(\frac{2!}{s^3}\right) - \frac{1}{s^2} + \frac{4}{s}$

$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

$= \frac{6+4s-s^2+4s^3}{s^4}$

iv) $L\{e^{-2t}\cos 5t\}$

$L\{\cos 5t\} = \frac{s}{s^2+25}$

let $s = s+2$

$L\{e^{-2t}\cos 5t\} = \frac{s+2}{(s+2)^2+25}$

$= \frac{s+2}{s^2+4s+29}$

(v) $L\{t\sin 3t\}$

$L\{\sin 3t\} = \frac{3}{s^2+9}$

$-f'(s) = -\frac{d}{ds}\left[\frac{3}{s^2+9}\right]$

$\frac{s^2+9(0)-3(2s)}{(s^2+9)^2} = \frac{-6s}{(s^2+9)^2}$

(vi) $L\left\{\frac{e^{-t}-e^{-2t}}{t}\right\}$

$\lim_{t\to 0} \frac{e^{-t}-e^{-2t}}{t} = \frac{e^{-0}-e^{-2(0)}}{0} \quad \frac{1-1}{0} = \frac{0}{0}$

$L\left\{\frac{-e^{-t}+2e^{-2t}}{1}\right\} = \frac{-1+2}{1} = \frac{1}{1}$

(vii) $L\{e^{4t}\cos 2t\}$

$L\{\cos 2t\} = \frac{s}{s^2+4}$

$s = s-4$

$= \frac{s-4}{(s-4)^2+4} = \frac{s-4}{s^2-8s-12}$

viii) $L\{t\sin 2t\}$

$L\{\sin 2t\} = \frac{2}{s^2+4}$

$= -f'(s) = -\frac{d}{ds}\left(\frac{2}{s^2+4}\right)$

$\frac{s^2+4(0)-2(2s)}{(s^2+4)^2} = \frac{-4s}{(s^2+4)^2}$

ix) $L\{t^3+4t^2+5\}$

$\frac{3!}{s^4} + \frac{4\cdot 2!}{s^3} + \frac{5}{s}$

$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$

$= \frac{6+8s+5s^3}{s^4}$

(x) $L\{e^{3t}(t^2+4)\}$

$L\{t^2e^{3t}\} + L\{4e^{3t}\}$

$L\{t^2\} = \frac{2!}{s^3}$

$s = s-3$

$\frac{2}{(s-3)^3}$

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① $L\{e^{3t}(t^2+4)\} = \frac{2}{(s-3)^2} + \frac{4}{(s-3)}$

$= \frac{2+4s^2-24s+3}{(s-3)^3} = \frac{4s^2-24s+38}{(s-3)^3}$

(ii) $L\{t^2\cos t\}$

$L\{\cos t\} = \frac{s}{s^2+1^2} = \frac{s}{s^2+1}$

$-f'(s) = \frac{-d}{ds}\left[\frac{s}{s^2+1}\right]$

$\frac{(s^2+1)(1)-s(2s)}{(s^2+1)^2}$

$= \frac{s^2+1-2s^2}{(s^2+1)^2}$

$= \frac{-s^2+1}{(s^2+1)^2} = \frac{-1}{s^2+1}$

$L\{t^2\cos t\} = \frac{-d}{ds}\left[\frac{-1}{s^2+1}\right]$

$\frac{(s^2+1)(0)-1(2s)}{(s^2+1)^2} = \frac{-2s}{(s^2+1)^2}$

③ $\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$

$A(s-4)+B(s-3) = s-5$

if $s=4$

$B(1) = -1$

$B = -1$

if $s=3$

$-A = -2$

$A = 2$

$\therefore \frac{A}{s-4} + \frac{B}{s-3} = \frac{2}{s-3} - \frac{1}{s-4}$

$= 2e^{3t} - e^{4t}$

ii) $\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$

$A(s-4)+B(s-2) = 2s-6$

if $s=4$

$B(2) = 2(4)-6$

$B = 1$

if $s=2$

$A(2-4) = 2(2)-6$

$-2A = -2$

$A = 1$

$\Rightarrow \frac{1}{s-2} + \frac{1}{s-4}$

$= e^{2t} + e^{4t}$

iii) $\frac{5s-8}{s(s-4)} \quad \frac{A}{s} + \frac{B}{s-4}$

$A(s-4)+B(s) = 5s-8$

at $s=0$

$-4A = -8$

$A = 2$

$s=4$

$4B = 12$

$B = 3$

$\Rightarrow \frac{2}{s} + \frac{3}{s-4} = 2 + 3e^{4t}$

iv) $\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{A}{(s-3)} + \frac{B}{(s-1)} + \frac{C}{(s-1)^2}$

$A(s-1)^2 + B(s-3)(s-1) + C(s-3) = s^2-3s-4$

at $s=1$

$0+0+C(-2) = 1^2-3(1)-4$

$-2C = -6$

$C = 3$

at $s=3$

$A(2)^2 = 3^2-3(3)-4$

$4A = -4$

$A = -1$

$A+B = 1$

$B = 1-A$

$= 1-(-1)$

$B = 1+1 = 2$

$\Rightarrow \frac{-1}{s-3} + \frac{2}{s-1} + \frac{3}{(s-1)^2}$

① $\frac{s-5}{s^2+4s+20} = \frac{s-5}{(s+2)^2+4^2}$

$= \frac{A}{(s+2)} + \frac{B}{(s+2)^2} + \frac{C}{16}$

$A(s+2)(16) + B(16) + C(s+2)^2 = s-5$

$(16s+32)A + 16B + C(s^2+4s+4) = s-5$

comparing coefficients

$C = 0$

$16A + 4C = 1$

$32A + 16B + 4C = -5$

$\therefore C = 0$

$16A + 4(0) = 1$

$A = 1/16$

$32A + 16B + 4C = -5$

$32\left(\frac{1}{16}\right) + 16B + 4(0) = -5$

$2 + 16B = -5$

$B = -7/16$

$\Rightarrow \frac{1}{16(s+2)} - \frac{7}{16(s+2)^2} + 0$

$= \frac{1}{16}e^{-2t} - \frac{7}{16}te^{2t}$