FAPOHUNDA OLUWATOMI OCHE.
15/ENG04/025.
ELECTRICAL/ELECTRONICS ENG

Assignment 4.

(1) $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)y^{(2)} - 2xy^{(1)} + 2y = 0.$

$y^{(2+n)}\cdot(1-x^2) + ny^{(1+n)}\cdot(-2x) + \frac{n(n-1)}{2!}y^{(n)}\cdot(-2) + y^{(1+n)}\cdot -2x + ny^{n} - 2$

$+ 2y^n = 0$

$\Rightarrow (1-x^2)y^{(2+n)} - 2xny^{(1+n)} - n(n-1)y^{(n)} - 2xy^{(1+n)} - 2ny^{(n)} + 2y^{(n)} = 0$

let $x = 0.$

$y^{(2+n)} - n(n-1)y^{(n)} - 2ny^{(n)} + 2y^{(n)} = 0.$

$y^{(2+n)} + y^{(n)}[-n^2 - n + 2] = 0$

$y^{(n+2)} = -(y^{(n)})_0[-n^2 - n + 2]$ → recurrence relation

$n = 0 \therefore (y^{(2)})_0 = -(y^{(0)})_0\cdot(\cdot) = -2(y^{(0)})_0$

$n = 1 : (y^{(3)})_0 = 0$

$n = 2 : (y^{(4)})_0 = (4)(-2)(y^{(0)})_0$

$n = 3 : (y^{(5)})_0 = 0$

$n = 4 : (y^{(6)})_0 = (18)(4)(-2)(y^{(0)})_0$

$n = 5 : (y^{(7)})_0 = 0$

$y = (y^{(0)})_0 + x(y^{(1)})_0 + \frac{x^2}{2!}(-2)(y^{(0)})_0 + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(4)(-2)(y^{(0)})_0 + \frac{x^5}{5!}(0) + \frac{x^6}{6!}(18)(4)(-2)(y^{(0)})_0 + \frac{x^7}{7!}(0).$

$y = (y^{(0)})_0\left[1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5}\right] + (y^{(1)})_0(x)$

(2i) $3e^{-4t} - 5e^{4t} = f(t)$.

$$L\{f(t)\} = \frac{3}{s+4} - \frac{5}{s-4}$$

(ii) $L\{\sin 4t + \cos 4t\} = \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2}$

(iii) $L\{t^3 + 2t^2 - t + 4\} = \frac{6}{s^{4}} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

(iv) $L\{t \sin 3t\}$ $= \frac{3}{s^2+9}$

$$= -\frac{d}{ds}\left(\frac{3}{s^2+9}\right).$$

$$= \frac{6s}{(s^2+9)^2}$$

(v) $L\{e^{-2t}\cos 2t\} = \frac{s}{(s+2)^2 + \cancel{2}5}$

(vi) $L\left\{\frac{e^{-t} - e^{-2t}}{t}\right\}$

check for limit.

$$\lim_{t \to 0}\left[\frac{-e^{-t} + 2e^{-2t}}{1}\right] = \frac{2}{1} = 2$$

$$L\{e^{-t} - e^{-2t}\} = \frac{1}{s+1} - \frac{1}{s+2}$$

$$L\left\{\frac{e^{-t} - e^{-2t}}{t}\right\} = \int_{s=s}^{\infty}\left(\frac{1}{s+1}\right) - \left(\frac{1}{s+2}\right) ds$$

$$= \left[\ln(s+1) - \ln(s+2)\right]_{s}^{\infty}$$

$$= -\ln\left[\frac{s+1}{s+2}\right]$$

(vii) $L\{e^{4t}\cos 2t\} = \dfrac{s}{(s-4)^2+4}$

(viii) $L\{t\sin 2t\} = \dfrac{-d}{ds}\dfrac{2}{s^2+4}$

$= -\left(\dfrac{(s^2+4)0 - 2(2s)}{(s^2+4)^2}\right)$

$= \dfrac{4s}{(s^2+4)^2}$

(ix) $L\{t^3+4t^2+5\} = \dfrac{6}{s^4} + \dfrac{8}{s^3} + \dfrac{5}{s}.$

(x) $L\{e^{3t}(t^2+4)\} = \dfrac{2}{(s-3)^3} + \dfrac{4}{s-3}$

(xi) $L\{t^2\cos t\} = (-1)^2\dfrac{d^2}{ds^2}\left[\dfrac{s}{s^2+1}\right]$

$= \dfrac{d}{ds}\dfrac{1-s^2}{(s^2+1)^2}$

$= \dfrac{(s^2+1)^2 - 2s - (1-s^2)\,4s(s^2+1)}{(s^2+1)^4}$

$= \dfrac{2s(3s^2-1)}{(s^2+1)^3}$

3) a) $L^{-1}\left\{\dfrac{s-5}{(s-3)(s-4)}\right\} = \dfrac{A}{(s-3)} + \dfrac{B}{s-4}$

$A = \left.\dfrac{s-5}{s-4}\right|_{s=3} = 2$

$B = \left.\dfrac{s-5}{s-3}\right|_{s=4} = -1$

Scanned by CamScanner

$$L^{-1}\left\{\frac{s-5}{(s-3)(s-4)}\right\} = 2e^{3t} - e^{4t}$$

(ii) $$L^{-1}\left\{\frac{2s-6}{(s-2)(s-4)}\right\} = \frac{A}{(s-2)} + \frac{B}{(s-4)}$$

$$A : \left.\frac{2s-6}{s-4}\right|_{s=2} = \frac{2(2)-6}{2-4} = 1$$

$$B : \left.\frac{2s-6}{s-2}\right|_{s=4} = \frac{2(4)-6}{4-2} = 1$$

$$= \frac{1}{s-2} + \frac{1}{s-4}$$

$$L^{-1}\{f(s)\} = \cancel{2}e^{2t} + e^{4t}$$

(iii) $$L^{-1}\left\{\frac{5s-8}{s(s-4)}\right\} = \frac{A}{s} + \frac{B}{s-4}$$

$$A : \left.\frac{5s-8}{s-4}\right|_{s=0} = \frac{8}{4} = 2$$

$$B : \left.\frac{5s-8}{s}\right|_{s=4} = \frac{12}{4} = 3$$

$$f(s) = \frac{2}{s} + \frac{3}{s-4}$$

$$L^{-1}\{f(s)\} = 2 + 3e^{4t}$$

(iv) $$L^{-1}\left\{\frac{s^2-3s-4}{(s-3)(s-1)^2}\right\} = \frac{A}{(s-3)} + \frac{B}{(s-1)^2} + \frac{C}{s-1}$$

$$A : \left.\frac{s^2-3s-4}{(s-1)^2}\right|_{s=3} = \frac{3^2-3(3)-4}{(3-1)^2} = -1$$

$$B : \left.\frac{s^2-3s-4}{s-3}\right|_{s=1} = \frac{1^2-3(1)-4}{1-3} = 3$$

$c: \frac{d}{ds}\left[\frac{s^2-3s-4}{s-3}\right]_{s=1} = \frac{(1-3)(2(1)-3)-(1^2-3(1)-4)}{(1-3)^2} = 2$

$f(s) = \frac{-1}{s-3} + \frac{3}{(s-1)^2} + \frac{2}{s-1}$

$L\{f(s)\} = -e^{3t} + 3te^{t} + 2e^{t}.$

$v)\ L^{-1}\left\{\frac{s-5}{s^2+4s+20}\right\} = \frac{s-5}{(s+2)^2+16} - \frac{5}{(s+2)^2+16}.$

$= \frac{s+2-2}{(s+2)^2+16} - \frac{5}{(s+2)^2+16}$

$= \frac{s+2}{(s+2)^2+16} - \frac{7}{(s+2)^2+16}.$

$= \frac{s+2}{(s+2)^2+4^2} - \frac{7}{4}\cdot\frac{4}{(s+2)^2+4^2}$

$L^{-1}\{f(s)\} = e^{-2t}\cos 4t - \frac{7}{4}e^{-2t}\sin 4t$