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15/ENG06/001
MECHANICAL ENGINEERING.

1. $(1-x^2)\,{}^{dy}/_{dx} - 2x\,{}^{dy}/_{dx} + 2y = 0$

$y^n = a^n v + na^{n-1}v^1 + \frac{n(n-1)}{2!}a^{n-2}v^2 + \ldots$

$[y^{(2+n)}(1-x^2) + ny^{(1+n)}(-2x) + \frac{n(n-1)}{2!}y^n(-2)] + [y^{(1+n)}\cdot -2x$

$+ ny^{n-2} + [2y^n] = 0$

$(1-x^2)y^{(2+n)} - 2xny^{(1+n)} - n(n-1)y^{(n)} - 2xy^{(1+n)} - 2ny^{(n)} + 2y^n$

when $x = 0$

$y^{(2+n)} - n(n-1)y^n - 2ny^{(n)} + 2y^n = 0$

$y^{n+2} + y^n[-n(n-1) - 2n + 2] = 0$

$y^{n+2} + y^n[-n^2 - n + 2] = 0$

$y^{n+2} = -(y^n)(-n^2 - n + 2)$

$n=0 \quad y^2 = -y^0 \cdot 2 = -2y^0$

$n=1 \quad y^3 = -y^1 \cdot [0] = 0$

$n=2 \quad y^4 = -y^2[-4] = 4y^2 = 4(-2y^0) = -8y^0$

$n=3 \quad y^5 = -y^3[-10] = +10y^3 = 0$

$n=4 \quad y^6 = -y^4 \cdot [-18] = 18y^4 = 18 \cdot y$

$n=5 \quad y^7 = -y^5 \cdot [-28] = 28(y^5) = 28 \cdot 0 = 0$

$y = y^0 + xy^1 + \frac{x^2}{2!}y^3 + \frac{x^3}{3!}y^3 + \ldots$

$y = y^0 + xy^1 + \frac{x^2}{2!}(-2)y^0 + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(4)(-2)y^0 + \frac{x^5}{5!}(0)$

$+ \frac{x^6}{6!}(18)(4)(-2)y^0 + \frac{x^7}{7}(0)$

$y = y^0 + xy^1 - x^2y^0 - \frac{x^4}{3!}y^6 - \frac{x^6}{5}y^0$

$y = y^0[1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5}] + y^1[x]$

2. $3e^{-4t} - 5e^{4t}$

$L\{3e^{-4} - 5e^{4t}\} = 3\{\frac{1}{s-(-4)}\} - 5\{\frac{1}{s-4}\}$

$$= \frac{3}{s+4} - \frac{5}{s-4}$$

ii. $\sin 4t + \cos 4t$.

$$L\{\sin 4t + \cos 4t\} = \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2}$$

$$= \frac{4}{s^2+16} + \frac{s}{s^2+16} = \frac{4+s}{s^2+16}.$$

iii. $t^3 + 2t^2 - t + 4$.

$$L\{t^3+2t^2-t+4\} = \frac{3!}{s^{3+1}} + 2\left\{\frac{2!}{s^{2+1}}\right\} - \left\{\frac{1}{s^{1+1}}\right\} + \frac{4}{s}$$

$$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}.$$

iv. $e^{-2t}\cos 5t$

$$L\{\cancel{e^{-2t}}\cos 5t\} = \frac{s}{s^2+25}$$

$$L\{e^{-2t}\cos 5t\} = \frac{s+2}{[s+2]^2+25}$$

v. $L\{t\sin 3t\} =$

$$L\{\sin 3t\} = \frac{3}{s^2+9}$$

$$L\{t\sin 3t\} = -\frac{d}{ds}\left\{\frac{3}{s^2+9}\right\}$$

$$= -\left[\frac{[s^2+9][0] - 3[2s]}{[s^2+9]^2}\right]$$

$$= -\left[\frac{-6s}{[s^2+9]^2}\right]$$

$$= \frac{6s}{[s^2+9]^2}$$

vi. $L\left\{\frac{e^{-t} - e^{-2t}}{t}\right\}$

$$\lim_{t \to 0} \left\{ \frac{e^{-t} - e^{-2t}}{t} \right\} = \frac{1-1}{0} = \frac{0}{0}$$

Applying L' Hopitals rule.

$$\lim_{t \to 0} \left\{ \frac{-1e^{-t} - (-2)e^{-2t}}{1} \right\} = \left\{ \frac{-1+2}{1} \right\} = \frac{1}{1}$$

$$L\{f(t)\} = e^{-t} - e^{-2t} = \frac{1}{s+9} - \frac{1}{s+9}$$

$$= \frac{1}{s+1} - \frac{1}{s+2}$$

$$\int_{\sigma=s}^{\infty} \frac{1}{\sigma+1} - \frac{1}{\sigma+s}$$

$$= [\ln(\sigma+1) - \ln(\sigma+2)]_0^{\infty}$$

$$= \left[\ln \frac{\sigma+1}{\sigma+2)} \right]_s^{\infty} = \ln \left[\frac{\infty+1}{\infty+2} - \frac{s+1}{s-2} \right]$$

$$= -\ln \left[\frac{s+1}{(s+2)} \right] = \ln \left[\frac{s+2}{s+1} \right]$$

vii $e^{4t} \cos 2t$

$$L\{\cos 2t\} = \frac{s}{s^2+4}$$

$$L\{e^{4t} \cos 2t\} = \frac{s-4}{(s-4)^2+4}$$

viii $t \sin 2t$

$$L\{\sin 2t\} = \frac{2}{s^2+4}$$

$$L\{t \sin 2t\} = -\left\{ \frac{[s^2+4][0] - 2[2s]}{[s^2+4]^2} \right\}$$

$$= -\left\{ \frac{-4s}{[s^2+4]^2} \right\}$$

$$= \frac{4s}{[s^2+4]^2}$$

ix. t^3+4t^2+5

$L\{t^3+4t^2+5\} = \frac{3!}{s^{3+1}} + 4\left\{\frac{2!}{s^{2+1}}\right\} + \frac{5}{s}$

$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$

x $e^{3t}(t^2+4)$

Let $x = t^2+4$

$L\{e^{3t}x\} : L\{x\}$

$L\{x\} = L\{t^2+4\} = \frac{2}{s^3} + \frac{4}{s}$

$\therefore L\{e^{3t}x\} = \frac{2}{[s-3]^3} + \frac{4}{[s-3]}$

xi $t^2 \cos t$

$L\{\cos t\} = \frac{s}{s^2+1}$

$L\{t\cos t\} = \frac{[s^2+1][1]-2s[s]}{[s^2+1]^2}$

$= \frac{s^2+1^2-2s^2}{[s^2+1^2]^2}$

$= \frac{-s^2+1}{[s^2+1^2]^2}$

$L\{t^2\cos t\} = -\frac{[s^2+1]^2 \cdot 2s - [s^2-1][4s^3+4s]}{[s^2+1]^2}$

$= -\frac{[2s^5-4s^3+2s]-[4s^5-4s]}{(s^2+1)^2}$

$= -\left[\frac{2s^5-4s^3+2s-4s^5+4s}{(s^2+1)^2}\right]$

$= -\left[\frac{-2s^5-4s^3+6s}{s^4+2s^2+1}\right]$

$= \frac{2s^5+4s^3-6s}{s^4+2s^2+1}$

3. i) $\frac{s-5}{(s-3)(s-4)} = \frac{A}{(s-3)} + \frac{B}{(s-4)}$

$\frac{s-5}{(s-3)(s-4)} = A(s-4) + B(s-3)$

when $s = 4$

$4-5 = B(4-3)$

$B = -1$

when $s = 3$

$3-5 = A(3-4)$

$-2 = -A$

$A = 2$

$\frac{s-5}{(s-3)(s-4)} = \frac{2}{s-3} - \frac{1}{s-4}$

$= 2e^{3t} - e^{4t}$

ii. L$\left\{\frac{2s-6}{(s-2)(s-4)}\right\} = \frac{A}{s-2} + \frac{B}{s-4}$

$2s-6 = A(s-4) + B(s-2)$

when $s = 4$

$8-6 = 2B$

$B = 1$

when $s = 2$

$4-6 = -2A$

$-2 = -2A$

$A = 1$

$\frac{2s-6}{(s-2)(s-4)} = \frac{1}{s-2} + \frac{1}{s-4}$

$= e^{2t} + e^{4t}$

iii. L$\left\{\frac{5s-8}{s(s-4)}\right\} = \frac{A}{s} + \frac{B}{s-4}$

$5s-8 = A(s-4) + B(s)$

when $s = 4$

$20-8 = 4B$

$12 = 4B$

$B = 3$

when $s = 0$.

$-8 = -4A$

$A = 2$

$$L^{-1}\left\{\frac{5s-8}{s(s-4)}\right\} = \frac{2}{s} + \frac{3}{s-4}$$

$$= 2 + 3e^{4t}$$

iv. $$L^{-1}\left\{\frac{s-5}{s^2+4s+20}\right\} = \frac{s-5}{s^2+4s+4+16} = \frac{s}{(s+2)^2+16} - \frac{5}{(s+2)^2+16}$$

$$= \frac{s+2-2}{(s+2)^2+16} - \frac{5}{(s+2)^2+16} = \frac{s+2}{(s+2)^2+16} - \frac{2}{(s+2)^2+16}$$

$$= \frac{s+2}{(s+2)^2+4^2} - \frac{7 \times 4/4}{(s+2)^2+4^2}$$

$$= \frac{s+2}{(s+2)^2+4^2} - \frac{7}{4} \cdot \frac{4}{(s+2)^2+4^2}$$

$$I(t) = e^{-2t}\cos 4t - \frac{7}{4} e^{-2t}\sin 4t$$

$$= e^{-2t}\left[\cos 4t - \frac{7}{4}\sin 4t\right]$$

v. $$\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{A}{s-3} + \frac{B}{(s-1)^2} + \frac{C}{s-1}$$

$$A(s-1)(s-1) + B(s-3) + C(s-3)(s-1).$$

when $s = 3$

$$A = \frac{s^2-3s-4}{(s-1)^2} = \frac{3^2-3(3)-4}{(3-1)^2} = -1$$

when $s = 1$

$$B = \frac{1^2-3(1)-4}{1-3} = 3$$

when s = 1

$$\frac{d}{ds}\left[\frac{s^2-3s-4}{s-3}\right] = \frac{(s-3)(2s-3)-[s^2-3s-4]}{(s-3)^2}$$

$$\frac{(1-3)(2(1)-3)-(1^2-3(1)-4)}{(1-3)^2} = 2$$

$$F(s) = \frac{-1}{s-3} + \frac{3}{(s-1)^2} + \frac{2}{s-1}$$

$$f(t) = -e^{-3t} + 3te^{t} + 2e^{t}$$

$$= e^{t}[3t+2] - e^{3t}$$

2. $L\left\{\frac{\sinh 2t}{t}\right\} = \ln\left\{\frac{s+2}{s+1}\right\}$