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Transform each of the following functions into laplace

$L\{f(t)\} \longrightarrow F(s) = \int_0^\infty e^{-st} f(t)\,dt$

1. $3e^{-4t} - 5e^{4t}$

$L\{3e^{-4t} - 5e^{4t}\}$

$L\{3e^{-4t}\} - L\{5e^{4t}\}$

$\frac{3}{s+4} - \frac{5}{s-4}$

ii) $\sin 4t + \cos 4t$

$L\{\sin 4t + \cos 4t\}$

$L\{\sin 4t\} + L\{\cos 4t\}$

$\frac{4}{s^2+16} + \frac{s}{s^2+16}$

iii.) $t^3 + 2t^2 - t + 4$

$L\{t^3 + 2t^2 - t + 4\}$

$L\{t^3\} + L\{2t^2\} - L\{t\} + L\{4\}$

$\frac{6}{s^2} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

iv) $e^{-2t}\cos 5t$

$L\{e^{-2t}\cos 5t\}$

$= \frac{s-2}{(s-2)^2+25}$

v) $t\sin 3t$

$L\{t\sin 3t\}$

$\frac{3}{s^2+9}$

vi. $\frac{e^{-t} - e^{-2t}}{t}$

$L\{e^{-t} - e^{-2t}\}$

$L\{e^{-t}\} - L\{e^{-2t}\}$

$\frac{1}{s+1} - \frac{1}{s+2}$

$= \int_s^\infty \left(\frac{1}{\sigma+1} - \frac{1}{\sigma+2}\right) d\sigma$

$\int_s^\infty \frac{1}{\sigma+1}\,d\sigma - \int_s^\infty \frac{1}{\sigma+2}\,d\sigma$

$[\ln(\sigma+1) - \ln(\sigma+2)]_s^\infty$

$\left[\ln\frac{(\sigma+1)}{\sigma+2}\right]_s^\infty$

$\left[\frac{\ln(\infty+1)}{(\infty+2)} - \ln\frac{(s+1)}{(s+2)}\right]$

$0 - \ln\frac{(s+1)}{(s+2)}$

$\ln\frac{s+2}{s+3}$

vii. $e^{4t}\cos 2t$

$L[e^{4t}\cos 2t]$

$\frac{s-4}{(s-4)^2+4}$

viii. $t\sin 2t$

$\frac{2}{s^2+4}$

$t^3 + 4t^2 + 5$

$L\{t^3 + 4t^2 + 5\}$

$L\{t^3\} + L\{4t^2\} + L\{5\}$

$\frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$

$e^{3t}(t^2 + 4)$

$\frac{1}{s-3}\left(\frac{2}{s^3} + \frac{4}{s}\right)$

$e^t \cos t$

$\frac{s-1}{(s-1)^2 + 1}$

$\frac{\sinh 2t}{t} = L\left\{\frac{\sinh 2t}{t}\right\}$

$= \frac{\tan^{-1}(2)}{s}$

Convert the f... to ... domain

$\frac{s-5}{(s-3)(s-4)} = \frac{A}{(s-3)} + \frac{B}{(s-4)}$

$\frac{A(s-4) + B(s-3)}{(s-3)(s-4)}$

$s-5 = As - 4A + Bs - 3B$

$s-5 = As + Bs - 4A - 3B$

$A + B = 1 \times -4$

$-4A - 3B = -5 \times 1$

$-4A - 3B = -5$

$-B = 1$

$B = -1$

$A + B = 1$

$A - 1 = 1$

$A = 2$

$\frac{2}{(s-3)} - \frac{1}{(s-4)}$

$L^{-1}\left[\frac{2}{(s-3)} - \frac{1}{(s-4)}\right]$

$2e^{3t} - e^{4t}$

2.) $\frac{2s-6}{(s-2)(s-4)}$

$\frac{A}{(s-2)} + \frac{B}{(s-4)}$

$\frac{A(s-4) + B(s-2)}{(s-2)(s-4)}$

$2s - 6 = As - 4A + Bs - 2B$

$A + B = 2 \times -4$

$-4A - 2B = -6 \times 1$

$-4A + 4B = -8$

$-4A - 2B = -6$

$-2B = 2$

$B = 1$

$A + B = 2$

$A + 1 = 2$

$A = 1$

$\frac{1}{(s-2)} + \frac{1}{(s-4)}$

$L^{-1}\left(\frac{1}{(s-2)} + \frac{1}{(s-4)}\right) e^{2t} + 4^t$

3. $\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4)}$

$\frac{5s-8}{s(s-4)} = \frac{A(s-4)+Bs}{s(s-4)}$

$5s-8 = A(s-4)+Bs$

$5s-8 = As-4A+Bs$

$+4A = +8$

$A = 2$

$A+B = 5$

$2+B = 5$

$B = 3$

$L^{-1}\left(\frac{2}{s} + \frac{B}{(s-3)}\right)$

$2+e^{3t}$

5.) $\frac{s^2-3s-4}{(s-3)(s-1)^2}$

$\frac{A}{s-3)} + \frac{B}{(s-1)} + \frac{C}{(s-1)^2}$

$s^2-3s-4 = A(s-1)^2 + B(s-3)(s-1) + C(s-3)$

$(s-3) = 0$

$C = -2 \quad A = 1$

$B = -3$

$\frac{1}{s-3)} \frac{-3}{(s-1)} \frac{-2}{(s-1)^2} = L^{-1}\left[\frac{1}{s-3}\right) - L^{-1}\left(\frac{3}{s-1}\right)$

$e^{3t} - 3e^{t}$

4.

1. $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)y'' - 2xy' + 2y = 0$

$W = (1-x^2)y''$

$U^n = y^{n+2} \qquad V = (1-x^2)$

$V' = -2x$

$W_2 = 2xy' \qquad V'' = -2$

$y^{n+1} = U \qquad V = 2x \quad V' = 2$

$W_3 = 2y$

$U^n = 2y^n$

From Leibnitz

$y^n = U^nV^0 + nU^{n-1}V' + \frac{n(n-1)}{2!}U^{n-2}V^2$

from W_1

$y^{n+2}(1-x^2) + n(y^{n+2})-2x + \frac{n(n-1)}{2!}y^n$

$y^{n+2}(1-x^2) - 2xn(y^{n+2}) - n(n-1)y^n$

from W_2

$y^{n+1}2x + ny^n 2$

from W_3

$2y^n$

$y^n = (1-x^2)y^{n+2} - 2xn(y^{n+2}) - n(n-1)y^n$

$-y^{n+1}2x + 2ny^n + 2y^n$

when $x=0$

$y^n = y^{n+2} - (n^2+n)y^n + 2ny^n + 2y^n$

$y^n = y^{n+2} (n^2+n)y^n + y^n(2n-2) = 0$

$y^n = y^{n+2} - (y^n n^2 + y^n n) + (2ny^n - 2y^n)$

$y^n = y^{n+2} - y^n n^2 - y^n n + 2ny^n + 2y^n$

$y^n = y^{n+2} - y^n(n^2-n+2) = 0$

$y^{n+2} - y^n(-n^2+n-2)$

$n=0$ $(y^{(2)})_0 = y_0 \; (-2)$

$n=1$ $(y^{(3)})_0 = (y^{(1)})_0 \; (0) = 0$

$n=2$ $(y^{(4)})_0 = (y^{(2)})_0 \; (4)(-2)$

$n=3$ $(y^{(5)})_0 = (y^{(3)})_0 = 0$

$n=4$ $(y^{(6)})_0 = (y^{(4)})_0 \; (18)(4)(-2)$

$n=5$ $(y^{(7)})_0 = (y^{(5)})_0 = 0$

$n=6$ $(y^{(8)})_0 = (y^{(6)})_0 (18)(4)(-2)(40)$

$n=7$ $(y^{(9)})_0 = (y^{(7)})_0 = 0$

$y = (y^{2})_0 \frac{x^2}{2!} + (y^{3})_0 \frac{x^3}{3!} + (y^{(4)}) \frac{x^4}{4!} + (y^{5}) \frac{x^5}{5!}$

$+ (y^{6}) \frac{x^6}{6!} + (y^{(7)}) \frac{x^7}{7!} + (y^{(8)}) \frac{x^8}{8!}$

$+ (y^{9}) \frac{x^9}{9!}$

$y = (y^{0})_0 (-2) \frac{x^2}{2!} + (y^{(2)})_0 (-2) \frac{x^4}{4!} + \frac{x^6}{6!}$

$(y^{(4)})_0 \; (18)(-4)(-2) \quad y^{8}$

$(y^{8})_0 \; (18)(-4)(-2)(40) \frac{x^8}{8!}$