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1) $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$ — (1)

Equation can also be written as

$(1-x^2)y'' - 2xy' + 2y = 0$ —(2)

Taking the first term $(1-x^2)y''$

$U = y''$ $V = 1-x^2$

$U^n = y^{(n+2)}$ $V' = -2x$

$V''' = 0$

$y^n = y^{(n+2)}(1-x^2) + ny^{(n+2-1)}\,2x + n(n-1)y^{(n+2-1)}$

$-2 + n(n-1)(n-2)\,y^{(n+2-3)}\cdot 0$

$y^n = (1-x^2)y(n+2) - 2xny^{(n+1)} - 2n(n-1)y^{(n)}$ —(3)

Taking the second term $-2xy'$

$U = y'$ $V = -2x$

$U^n = y^{(n+1)}$ $V' = -2$

$V'' = 0$

$y^n = y^{(n+1)} - 2x + ny(n+1-1)\cdot -2 + n(n-1)y^{(n+1-2)}\cdot 0$

$y^n = -2xy^{(n+1)} - 2ny^{(n)} + 0$ —(4)

Taking the third term $2y$

$U = y$ $V = 2$

$U^n = y^n$ $V' = 0$

$y^n = y^n 2 + ny^{(n-1)} = 0$

$$(1-x^2)y^{(n+2)} - 2xny^{(n+1)} - 2n(n-1)y^{(n)} - 2xy^{(n+1)}$$
$$-2ny^{(n)} + 2y^{(n)} = 0$$

$$(1-x^2)y^{(n+2)} - 2x(ny^{(n+1)} + y^{(n+1)}) + y^{(n)}(-2n + 2) = 0 \quad \text{———} \; (6)$$

now at $x = 0$, eqn 6 becomes

$$y^{(n+2)} + 2y^{(n)}(-n+1) = 0$$

$$y^{(n+2)} = -(-n+1)2y^{(n)}$$

$$y^{(n+2)} = (n-1)2y^{(n)} \quad \text{———} \; (7)$$

eqn(7) gives the recurrence equation.

now at $n = 0$

$$y^2 = (0-1)2y^{(0)}$$

$$(y^2)_0 = -2y_0 \quad \text{———} \; (7)$$

now at $n = 1$

$$(y^3)_0 = 0 \quad \text{———} \; (8)$$

now at $n = 2$

$$(y^4)_0 = 2(y^2)_0 = 2(-2y_0) = -4y_0 \quad \text{———} \; (9)$$

now at $n = 3$

$$(y^5)_0 = 4(y^{(3)})_0 = 4(0) = 0 \quad \text{———} \; (10)$$

now at $n = 4$

$$(y^6)_0 = 6y^{(4)} = 6(-4y_0) = -24y_0 \quad \text{———} \; (11)$$

now at $n = 5$

$$(y^7)_0 = 0 \quad \text{———} \; (12)$$

$$y''(t) - 2y'(t) + 5y(t) = e^{2t}$$

$$s^2 y(s) - sy(0) - y'(0) - 2(sy(s) - y(0)) + 5y(s) = \frac{1}{s-2}$$

$$s^2 y(s) - 2s - 1 - 2(sy(s) - 2) + 5y(s) = \frac{1}{s-2}$$

$$s^2 y(s) - 2s - 1 - 2sy(s) + 4 + 5y(s) = \frac{1}{s-2}$$

$$y(s)(s^2 - 2s + 5) - 2s + 3 = \frac{1}{s-2}$$

$$y(s)(s^2 - 2s + 5) = \frac{1}{s-2} + 2s - 3$$

$$y(s)(s^2 - 2s + 5) = \frac{1 + 2s^2 - 4s - 3s + 6}{s-2}$$

$$y(s)(s^2 - 2s + 5) = \frac{7 + 2s^2 - 7s}{s-2}$$

$$y(s) = \frac{7 + 2s^2 - 7s}{(s-2)(s^2 - 2s + 5)}$$

$$\frac{A}{s-2} + \frac{Bs + C}{s^2 - 2s + 5} = \frac{2s^2 + 7s + 7}{(s-2)(s^2 - 2s + 5)}$$

$$A(s^2 - 2s + 5) + B(s^2 - 2s) + C(s-2) = 2s^2 - 7s + 7$$

$s^2 \Rightarrow A + B = 2$

$A = 2 - B$ —— (i)

$s \Rightarrow -2A - 2B + C = -7$

$-2(2-B) - 2B + C = -7$

$-4 + \cancel{2B} - \cancel{2B} + C = -7$

$-4 + C = -7$

$C = -7 + 4$

$C = -3$ —— (ii)

(2) (i) $3e^{-4t} - 5e^{4t} = f(t)$

- this is a linear equation so the laplace transform

$$L[ft] = f(s)$$

$$f(s) = L[3e^{-4t}] - L[5e^{4t}]$$

$$f(s) = 3\frac{1}{s+4} - 5\frac{1}{s-4} \quad //$$

(ii) $\sin 4t + \cos 4t = f(t)$

$$f(s) = L[f(t)]$$

$$f(s) = L[\sin 4t] + L[\cos 4t]$$

$$f(s) = \frac{4}{s^2+16} + \frac{s}{s^2+16} \quad //$$

(iii) $t^2 + 2t^2 - t + 4 = f(t)$

$$L\{f(t)\} = f(s)$$

$$f(s) = L(t^2] + 2L[t^2] - L[t] + L[4]$$

$$f(s) = \frac{2!}{s^3} + 2\left[\frac{2!}{s^3}\right] - \frac{1}{s^2} + \frac{4}{s}$$

$$f(s) = \frac{2}{s^3} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

(iv) $e^{-2t} \cos 5t$

$$f(s) = L(e^{-2t} \cos 5t)$$

first taking the laplace transform of $\cos 5t$

$y''(t) - 2y'(t) + 3y...$

v $\frac{s-5}{s^2+4s+20} = F(s)$

$\frac{A}{(s+2-j4)} + \frac{B}{(s+2+4j)}$

$= \frac{5-s}{(s+2-j4)(s+2+4j)}$

$A|_{s=-2+4j} = \frac{(-2+4j)-5}{(\cancel{-2}+4j+\cancel{2}+4j)}$

$A|_{s=-2+4j} = \frac{-7+4j}{8j} \times \frac{j}{j}$

$A = \frac{-7j-4}{-8} = \frac{7j+4}{8}$

$B|_{s=-2-4j} = \frac{-2-4j-5}{(\cancel{-2}-4j+\cancel{2}-4j)}$

$= \frac{-7-4j}{-8j} \times \frac{j}{j}$

$B|_{s=-2-4j} = \frac{-7j+4}{8}$

$F(s) = \frac{7j+4}{8(s+2-4j)} - \frac{7j+4}{8(s+2+4j)}$

$f(t) = L^{-1}[F(s)]$

$f(t) = \frac{7j+4}{8}\left(e^{(-2+4j)t} - e^{(-2-4j)t}\right)$ //

$f(t) = \frac{7j+4}{8}\left[e^{(-2+4j)t} - e^{(-2-4j)t}\right]$ //

vii) $e^{4t}\cos 2t = f(t)$

$L[\cos 2t] = \frac{s}{s^2+4}$

$f(s) = \frac{s-4}{(s-4)^2+4}$ //

viii) $t \sin 2t$

$L[\sin 2t] = \frac{2}{s^2+4}$

Recall

$f(s) = -\frac{d^n}{ds^n}\frac{2}{s^2+4}$

$f(s) = -\left[\frac{s^2+4(0)-2(2s)}{(s^2+4)^2}\right]$

$f(s) = \frac{4s}{(s^2+4)^2}$ //

ix $t^3+4t^2+5 = f(t)$

$f(s) = L(f(t))$

$f(s) = \frac{3!}{s^4} + 4\frac{2!}{s^3}$

$+ 5/s$

$f(s) = 6/s^4 + 8/s^3 + 5/s$ //

x) $e^{3t}(t^2+4) = f(t)$

$\Rightarrow t^2e^{3t} + 4e^{3t}$

$f(s) = L(f(t))$

$f(s) = \frac{2}{(s-3)^3} + \frac{4}{s-3}$ //

(xi) $t^2 \cos t$

$f(s) = (-1)^n \frac{d^n}{ds^n}$

$L[\cos t] = \frac{s}{s^2+1}$

$f(s) = (-1)^2 \frac{d^2 s}{ds^2(s^2+1)}$

taking the first derivative using quotient rule

$-1\left[\frac{s^2+1(1) - s(2s)}{(s^2+1)^2}\right]$

$\Rightarrow \left[\frac{s^2+1-2s^2}{(s^2+1)^2}\right] = \frac{-1+s^2}{(s^2+1)^2}$

taking the second derivative

$\Rightarrow \frac{(s^2+1)^2(-2s) - (1-s^2)(4s^3+4s)}{(s^2+1)^4}$

$\Rightarrow$ ~~$(s^2+1)^2(-2s) - (4s^3+4s-4s^5$~~

$\frac{(s^2+1)^2(+2s) + (s^2+1)(4s^3+4s)}{(s^2+1)^4}$

$\Rightarrow \frac{(s^2+1)[2s - 4s^5+4s]}{(s^2+1)^4}$

$f(s) = \dfrac{6s - 4s^3}{(s^2+1)^3}$ //

(xii) $\dfrac{\sinh 2t}{t} = f(t)$

$f(t) = t^{-1} \sinh 2t$

$L[\sinh 2t] = \dfrac{a}{s^2 - a^2}$

$f(s) =$

$\dfrac{2}{s^2+4} + 3$

$\dfrac{2 + 3s^2 + 12}{(s^2+4)(3s-6)}$

$14 + 3s^2$

(3)

i $\dfrac{s-5}{(s-3)(s-4)} = f(s)$

$\dfrac{A}{(s-4)} + \dfrac{B}{(s-3)} = \dfrac{s-5}{(s-4)(s-3)}$

$A|_{s=4} = \dfrac{4-5}{(4-3)} = \dfrac{-1}{1} = -1$

$B|_{s=3} = \dfrac{3-5}{(3-4)} = \dfrac{-2}{-1} = 2$

$A = -1$ and $B = 2$

$f(s) = \dfrac{-1}{(s-4)} + \dfrac{2}{(s-3)}$

$f(t) = L^{-1}[f(s)]$

$f(t) = -e^{4t} + 2e^{3t}$

$f(t) = 2e^{3t} - e^{4t}$ //

ii $\dfrac{2s-6}{(s-2)(s-4)}$

$\dfrac{A}{(s-2)} + \dfrac{B}{(s-4)} = \dfrac{2s-6}{(s-2)(s-4)}$

$A|_{s=2} = \dfrac{2(2)-6}{(2-4)} = 1$

$$B\big|_{s=-2-4j} = \frac{-2-4j-5}{(\cancel{-2}-4j+\cancel{2}-4j)} = \frac{-7-4j}{-8j} \times \frac{j}{j}$$

$$B\big|_{s=-2-4j} = \frac{-7j+4}{8}$$

$$f(s) = \frac{7j+4}{8(s+2-4j)} - \frac{7j+4}{8(s+2+4j)}$$

$$f(t) = L^{-1}[f(s)]$$

$$f(t) = \frac{7j+4}{8} e^{(-2+4j)t} - \frac{7j+4}{8} e^{(-2-4j)t}$$

$$f(t) = \frac{7j+4}{8}\left(e^{(-2+4j)t} - e^{(-2-4j)t}\right)$$