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15/ENG06/023.

1. $(1-x^2)\, d^2y/dx^2 - 2x\, dy/dx + 2y = 0$

$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \ldots$

$\left(y^{(n+2)} \cdot (1-x^2) + n y^{(1+n)} \cdot (-2x) \frac{2!}{} + \frac{n(n-1) y^n \cdot (-2)}{2!} \right] + \left[y^{(1+n)} - 2x + n y^n - 2 + [2y^n] \right) = 0$

Let $x = 0$

$y^{(2+n)} - n(n-1) y^n - 2n y^{(n)} + 2y^n = 0$

$y^{n+2} + y^n(-n^2 - n + 2) = 0$

$y^{n+2} = -(y^n)_0 \ (-n^2 - n + 2)$

when $n = 0$, $y^2 = y^0 \cdot 2 = -2y^0$

$n = 1$; $y^3 = -y^1 \cdot 0 = 0$

$n = 2$; $y^4 = -y^2(-4) = 4y^2 = -8y^0$

$n = 3$; $y^5 = -y^3(-10) = 10y^3 = 10 \times 0 = 0.$

$n = 4$; $y^6 = -y^4(-18) = 18y^4 = -2y^0$

$n = 5$; $y^7 = -y^5 - (-28) = 28(y^5)_0 = 28 \times 0 = 0$

$y = y^0 + xy' + \frac{x^2}{2!} y^2 + \frac{x^3}{3!} y^3 + \ldots$

$y = y^0 + xy' + \frac{x^2}{2!}(-2) y^0 + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(4)(-2) y^0 + \frac{x^5}{5!}(0) + \frac{x^6}{6!}(18)\, 4(-2) y^0$

$y = y^0 \left(1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5} \right] + y'_0 \ (x)$

2. $3e^{-4t} - 5e^{4t}$

Recall that $L\{f(t) + G(t)\} = L\{f(t)\} \pm L\{G(t)\}$

$= L\{3e^{-4t} - 5e^{4t}\} \Rightarrow L\{3e^{-4t}\} - L\{5e^{4t}\} \rightarrow 3L\{e^{-4t}\} - 5L\{e^{4t}\}$

$= \frac{3}{s+4} - \frac{5}{s-4}$

3. $\sin 4t + \cos 4t$

$L\{\sin 4t + \cos 4t\}$

$\frac{4}{s^2+16} + \frac{s}{s^2+16} = \frac{4+s}{s^2+16}.$

iii t^3+2t^2-t+4

$L\{t^3+2t^2-t+4\}$

$$\frac{3!}{s^4}+\frac{4}{s^3}-\frac{1}{s^2}+\frac{4}{s}$$

$$\frac{6}{s^4}+\frac{4}{s^3}-\frac{1}{s^2}+\frac{4}{s}$$

iv $e^{-2t}\cos 5t$

Recall Shift theorem

$$L\{\cos 5t\} = \frac{s}{s^2+25} \quad ; \quad (s = s+2)$$

$$\frac{s+2}{(s+2)^2+25}$$

5.

v $t\sin 3t$

$$L\{\sin 3t\} = \frac{3}{s^2+9}$$

Using quotient rule to differentiate.

$u = 3, \quad v = s^2+9$

$du = 0 \quad dv = 2s$

$$= \frac{v\,du/ds - u\,dv/ds}{v^2}$$

$$= \frac{(s^2+9)\cdot 0 - 3(2s)}{(s^2+9)^2}$$

$$= \frac{-6s}{(s^2+9)^2}$$

$$-F'(s) = -1 \cdot \left\{\frac{-6s}{(s^2+9)^2}\right\} = \frac{6s}{(s^2+9)^2}$$

vi $\frac{e^{-t}-e^{-2t}}{t}$

$$L\left\{\frac{e^{-t}-e^{-2t}}{t}\right\}$$

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$$\lim_{t \to 0} \left\{ \frac{e^{-t} - e^{-2t}}{t} \right\} = \frac{1-1}{0} = \frac{0}{0} = \text{Indeterminate.}$$

. Apply l'hopital's rule.

$$\lim_{t \to 0} \left\{ \frac{-1 e^{-t} - (-2) e^{-2t}}{1} \right\} = \left(\frac{-1 + 2}{1} \right) = +1 \text{ (determinate.}$$

$$L \left\{ \frac{f(t)}{t} \right\} = \int_{\sigma = s}^{\infty} F \sigma \, d\sigma$$

$$F \sigma = L \{ F (t) \}$$

$$L \{ f(t) \} = e^{-t} - e^{-2t} = \frac{1}{s+a} - \frac{1}{s+a}$$

$$= \frac{1}{s+1} - \frac{1}{s+2}$$

$$F \sigma = \frac{1}{\sigma + 1} - \frac{1}{\sigma + 2}$$

$$\int F \sigma \quad L \left\{ \frac{f(t)}{t} \right\} = \int_{\sigma}^{\infty} \frac{1}{\sigma + 1} - \frac{1}{\sigma + 2} \, d\sigma$$

$$= \int_{\sigma = s}^{\infty} \frac{1}{\sigma + 1} \, d\sigma - \int_{\sigma}^{\infty} \frac{1}{\sigma + 2} \, d\sigma$$

$$= \Big[\ln (\sigma + 1) - \ln (\sigma + 2) \Big]_{s}^{\infty}$$

$$= -\ln \left(\frac{s+1}{s+2} \right) - \ln \left(\frac{s+2}{s+1} \right)$$

vii $\quad e^{4t} \cos 2t$

$$L \{ e^{4t} \cos 2t \} = e^{4t} \, L \{ \cos 2t \}$$

$$L \{ \cos 2t \} = s / s^2 + 2^2$$

$$= s / s^2 + 4$$

replacing s by a shift of e^{4t} ; $(s-4)$

$$L \{ e^{4t} \cos 2t \} = \frac{s-4}{(s-4)^2 + 4}$$

viii $\quad 1 \; \sin 2t$

$$L \{ \sin 2t \} = -d/dt \; \{ t \, (s) \}$$

$$F(s) = L\{ \sin 2t \} = 2/s^2 + 4$$

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$F'(s)$ = using quotient rule

$u = 2 \qquad du/ds = 0$

$v = s^2+4 \qquad dv/ds = 2s$

$= \frac{v\, du/ds - u\, dv/ds}{v^2}$

$= \frac{(s^2+4)\cdot 0 - 2(2s)}{(s^2+4)^2}$

$= \frac{-4s}{(s^2+4)^2}$

$L\{t(-\sin 2t)\} = -f(s)$

$= \frac{4s}{(s^2+4)^2}$

ix $\quad t^3+4t^2+5$

$L\{t^3+4t^2+5\} = L\{t^3\} + 4L\{t\} + L\{5\}$

$= \frac{3!}{s^4} + \frac{4\cdot 2!}{s^3} + \frac{5}{s}$

$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$

x $\quad e^{3t}(t^2+4)$

let $x = t^2+4$

$L\{e^{3t}x\}$

$L\{x\} = L\{t^2+4\}$

$= \frac{2}{s^3} + \frac{4}{s}$

replacing $\to$ by a shift of $s-3$

$L\{e^{3t}x\} = \frac{2}{(s-3)^2} + \frac{4}{s-3}$

x. $\quad t^2\cos t$

$L\{t^2\cos t\} = t^2 L\{\cos t\}$

$f(s) = \frac{s}{s^2+1}$

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$f'(s)$ = using quotient rule.

$u = s \quad du/ds = 1$

$v = s^2 + 1^2 \quad dv/ds = 2s$

$= \dfrac{v\,du/ds - u\,dv/ds}{v^2}$

$= \dfrac{(s^2+1^2)\cdot 1 - 2s(s)}{(s^2+1^2)^2}$.

$= \dfrac{s^2+1^2 - 2s^2}{(s^2+1^2)^2} = -\dfrac{s^2+1}{(s^2+1)^2}$

Recall

$-f''(s) = -d/ds \left\{ \dfrac{s^2-1}{(s^2+1)^2} \right\}$

using quotient rule

$\dfrac{v\,du/ds - u\,dv/ds}{v^2}$

$u = s^2 - 1 \quad du = 2s$

$v = (s^2+1)^2 \quad dv = 4s(s^2+1)$

$= \dfrac{(s^2+1)^2 \cdot 2s - (s^2+1)(4s^3+4s)}{(s^2+1)^2}$

$= \dfrac{-2s^5 - 4s^3 + 6s}{s^4 + 2s^2 + 1}$

$f''(s) = -d/ds = -\left\{ \dfrac{2s^5 - 4s^3 + 6s}{s^4 + 2s^2 + 1} \right\}$

$= \dfrac{2s^5 + 4s^3 - 6s}{s^4 + 2s^2 + 1}$

x ii $\quad \dfrac{\sinh 2t}{t}$

$= L \left\{ \dfrac{\sinh 2t}{t} \right\}$

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3;

$$\frac{s-5}{(s-5)(s-4)}$$

$$L^{-1}\left\{\frac{s-5}{(s-3)(s-4)}\right\} = \frac{A}{s-3} + \frac{B}{s-4}$$

$$s-5 = A(s-4) + B(s-3)$$

if $s = 4$

$$B = -1$$

if $s = 3$

$$A = 2$$

$$\therefore \frac{s-5}{(s-3)(s-4)} = \frac{2}{s-3} - \frac{1}{s-4}$$

$$= 2e^{3t} - e^{4t}$$

ii; $$\frac{2s-6}{(s-2)(s-4)}$$

$$L^{-1}\left\{\frac{2s-6}{(s-2)(s-4)}\right\}$$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{(s-4)}$$

$$2s-6 = A(s-4) + B(s-2)$$

Assuming $s = 4$

$$B = 1$$

Assuming $s = 2$

$\cancel{2(2)}$ $A = 1$

$$\therefore \frac{2s-6}{(s-2)(s-4)} = \frac{1}{s-2} + \frac{1}{s-4}$$

$$L^{-1}\left\{\frac{2s-6}{(s-2)(s-4)}\right\} = e^{2t} + e^{4t}$$

iii; $$\frac{5s-8}{s(s-4)}$$

$$L^{-1}\left\{\frac{5s-8}{s(s-4)}\right\} = \frac{A}{s} + \frac{B}{s-4}$$

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$5s - 8 = A(s-4) + B(s)$

Assuming $s = 4$

$5(4) - 8 = A(4-4) + B(4)$

$B = 3.$

Assuming $s = 0.$

$A = 2.$

$\therefore \quad L^{-1}\left\{\frac{5s-8}{s(s-4)}\right\} = \frac{2}{s} + \frac{3}{s-4}$

$= 2 + 3e^{4t}$

ix $\left\{\frac{s-5}{s^2+4s+20}\right\}$

$L^{-1}\left\{\frac{s-5}{s^2+4s+20}\right\}$

$\Longrightarrow F(s) = \frac{s-5}{s^2+4s+20} = \frac{s}{(s+2)^2+16} - \frac{5}{(s+2)^2+16}.$

$= \frac{s+2}{(s+2)^2+4^2} - \frac{7 \times 4/4}{(s+2)^2+4^2}$

$f(t) = e^{-2t}\cos 4t - 7/4\, e^{-2t}\sin 4t$

$= e^{-2t}\{\cos 4t - 7/4 \sin 4t\}$

iv $\frac{s^2-3s-4}{(s-3)(s-1)^2}$

$F_s = \frac{A}{s-3} + \frac{B}{(s-1)^2} + \frac{C}{s-1}$

$A = \frac{s^2-3s-4}{(s-1)^2}\Big|_{s=3} = \frac{3^2-3(3)-4}{2^2} = -1$

$B = \frac{s^2-3s-4}{s-3}\Big|_{s=1} = \frac{1^2-3(1)-4}{-2} = 3$

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$$C = \frac{d}{ds}\left\{ \frac{s^2-3s-4}{s-3} \right\}_{s=1} = \frac{(s-3)(2s-3)-(s^2-3s-4)}{(s-3)^2}$$

$$= \frac{(1-3)(2-3)-(1-3-4)}{(1-3)^2}$$

$$= 2$$

$$f(s) = \frac{-1}{s-3} + \frac{3}{(s-1)^2} + \frac{2}{s-1}$$

$$L^{-1}\{f\} = -e^{3t} + 3te^{t} + 2e^{t}$$

$$= e^{t}(3t+2) - e^{3t}$$

5.

2

3

3

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