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15/ENG07/008

ENGINEERING MATHEMATICS 3

CHEMICAL ENGINEERING

ASSIGNMENT FOUR

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ENGINEERING MATHEMATICS ENG 381

CHEMICAL ENGINEERING

ASSIGNMENT V

a) Transform each of the following functions with Laplace (s) domain:

$3e^{-4t} - 5e^{4t}$

$L\{3e^{-4t} - 5e^{4t}\} = L\{3e^{-4t}\} - L\{5e^{4t}\}$

$= \frac{3}{s+4} - \frac{5}{s-4}$

b) Sin4t + cos4t

$L\{\sin 4t + \cos 4t\} = L\{\sin 4t\} + L\{\cos 4t\}$

$= \frac{4}{s^2+16} + \frac{s}{s^2+16}$

c) $L\{t^3 + 2t^2 - t + 4\} = L(t^3) + L(2t^2) - L(t) + L(4)$

$= \frac{3!}{s^{3+1}} + \frac{2 \cdot 2!}{s^{2+1}} - \frac{1!}{s^{1+1}} + \frac{4}{s}$

$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

d) $L\{e^{-2t}\cos 5t\}$

$L\{\cos 5t\} = \frac{s}{s^2+5^2} = \frac{s}{s^2+25}$

$L\{e^{-2t}\cos 5t\} = \frac{(s+2)}{(s+2)^2+25} = \frac{s+2}{(s+2)^2+25}$

e) $L\{t \sin 2t\}$

$L\{\sin 2t\} = \frac{2}{s^2+4}$

$L\{t\sin 2t\} = -\frac{d}{ds}\left(\frac{2}{s^2+4}\right) = f\left(\frac{u}{v}\right)$

differentiating as quotient

let $u = 2$, let $v = s^2+4$

$\frac{du}{ds} = 0$, $\frac{dv}{ds} = 2s$

$f\left(\frac{u}{v}\right) = \frac{v\frac{du}{ds} - u\frac{dv}{ds}}{v^2} = \frac{(s^2+4)(0) - (2)(2s)}{(s^2+4)^2}$

$$F'(s) = \frac{-6s}{(s^2+9)^2}$$

Recall

$$L\{t \sin 3t\} = -F'(s)$$

$$= -\times \frac{-6s}{(s^2+9)^2}$$

$$L\{t \sin 3t\} = \frac{6s}{(s^2+9)^2}$$

vi) $L\left(\frac{e^{-t} - e^{-2t}}{t}\right)$

To the limit exists as $t \to 0$

By l'Hôpital rule

$$\lim_{x \to a}\left(\frac{f(x)}{g(x)}\right) = \lim_{x \to a}\left(\frac{f'(x)}{g'(x)}\right)$$

$$= \lim_{t \to 0}\left(\frac{-e^{-t} + 2e^{-2t}}{1}\right)$$

$$= -e^{-(0)} + 2e^{-2(0)}$$

$$= -1 + 2 = 1$$

Hence the limit exist

Recall $L\left(\frac{f(t)}{t}\right) = \int_{\sigma=s}^{\infty} F(\sigma)\, d\sigma$ $\quad L(e^{-t} - e^{-2t}) = \frac{1}{s+1} - \frac{1}{s+2}$

$$L\left(\frac{e^{-t} - e^{-2t}}{t}\right) = \int_{\sigma=s}^{\infty} \frac{1}{\sigma+1} - \frac{1}{\sigma+2}$$

$$= \left[\ln(\sigma+1) - \ln(\sigma+2)\right]_s^{\infty}$$

$$= \left[\ln\left(\frac{\sigma+1}{\sigma+2}\right)\right]_s^{\infty}$$

$$= \ln\left(\frac{\infty+1}{\infty+2}\right) - \ln\left(\frac{s+1}{s+2}\right)$$

$$= 0 - \ln\left(\frac{s+1}{s+2}\right)$$

$$= \ln\left(\frac{s+2}{s+1}\right)$$

(10) $L(e^{4t}\cos 2t)$

$L(\cos 2t) = \frac{s}{s^2+2^2} = \frac{s}{s^2+4}$

$L(e^{4t}\cos 2t) = \frac{s-4}{(s-4)^2+4}$

(11) $L(t\sin 2t)$

$L(\sin 2t) = \frac{2}{s^2+4}$

$L(t\sin 2t) = -\frac{d}{ds}\left(\frac{2}{s^2+4}\right)$

diff $L(\sin 2t)$ w.r.t s

$u = 2, \quad \frac{du}{ds} = 0 \qquad v = s^2+4 \quad \frac{dv}{ds} = 2s$

$F(s) = \frac{v\frac{du}{ds} - u\frac{dv}{ds}}{v^2} = \frac{(s^2+4)(0) - 2(2s)}{(s^2+4)^2}$

$F'(s) = \frac{-4s}{(s^2+4)^2}$

$L(t\sin 2t) = -F'(s) = \frac{-(-4s)}{(s^2+4)^2} = \frac{4s}{(s^2+4)^2}$

(12) $L(t^3+4t^2+5)$

$= L(t^3) + L\{4t^2\} + L[5]$

$= \frac{3!}{s^4} + \frac{4\cdot 2!}{s^3} + \frac{5}{s}$

$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$

(13) $L\{e^{3t}(t^2+4)\}$

$L(t^2+4) = \frac{2!}{s^3} + \frac{4}{s} = \frac{2}{s^3} + \frac{4}{s}$

$L\{e^{3t}(t^2+4)\} = \frac{2}{(s-3)^3} + \frac{4}{(s-3)}$

* $L\{t^2 \cos t\}$

$L\{t \cdot t\cos t\}$

$L\{t\cos t\} = -\frac{d}{ds} L\{\cos t\}$

$L\{\cos t\} = \frac{s}{s^2+1^2} = \frac{s}{s^2+1}$

$L\{t\cos t\} = -\frac{d}{ds}\left(\frac{s}{s^2+1}\right)$

let $u = s$ $\qquad v = s^2+1$

$\frac{du}{ds} = 1$ $\qquad \frac{dv}{ds} = 2s$

$f(s) = \frac{(s^2+1)(1) - (s)(2s)}{(s^2+1)^2}$

$f(s) = \frac{s^2+1-2s^2}{(s^2+1)^2} = \frac{1-s^2}{(s^2+1)^2}$

$L\{t^2\cos t\} = -f'(\cos t)$

$f(s) = \frac{1-s^2}{s^4+2s^2+1}$

let $u = 1-s^2$ $\qquad v = s^4+2s^2+1$

$\frac{du}{ds} = -2s$ $\qquad \frac{dv}{ds} = $ [illegible]

$L\{t^2\cos t\} = -\left[\frac{(s^4+2s^2+1)(-2s) - (1-s^2)(\text{[illegible]})}{(s^4+2s^2+1)^2}\right]$

$L\{t^2\cos t\} = \frac{\text{[illegible]}}{(s^4+2s^2+1)^2}$

$L\{t^2\cos t\} = \frac{-2s^3 \ \text{[illegible]}}{(s^4+2s^2+1)^2}$

$L\{t^2\cos t\} = \frac{-2s^3 \ \text{[illegible]}}{(s^4+2s^2+1)^2}$

1b) $L\left(\frac{\sinh 2t}{t}\right)$ [illegible]

$L\{\sinh 2t\} = \frac{2}{s^2+2^2} = \frac{2}{s^2+4}$

$L\left\{\frac{\sinh 2t}{t}\right\} = \int_{s=1}^{\infty} \frac{2}{s^2+2^2}\, ds$

let $u = s^2 + 4$

$\frac{du}{ds} = 2s + 0 = 2s$

$du = 2s\, ds$

$ds = \frac{du}{2s}$

$= \int_{s}^{\infty} \frac{2}{2s} = 2\left[\tan^{-1}\left(\frac{s}{2}\right)\right]_{s=1}^{\infty}$

$= 2\left[\tan^{-1}\left(\frac{\infty}{2}\right) - \tan^{-1}\left(\frac{1}{2}\right)\right]$

$= 2\left[90 - \tan^{-1}\left(\frac{1}{2}\right)\right]$

$= 2\left(\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{2}\right)\right)$

$L\left(\frac{\sinh 2t}{t}\right) = \pi - \tan^{-1}\left(\frac{1}{2}\right)$

2) Convert the following function in the [illegible] form

i) $\frac{s-5}{(s-3)(s-4)}$

$L^{-1}\left(\frac{s-5}{(s-3)(s-4)}\right) = L^{-1}\left(\frac{A}{s-3} + \frac{B}{s-4}\right)$

Resolving into partial fraction

$s-5 = A(s-4) + B(s-3)$

$s-5 = As - 4A + Bs - 3B$

$s-5 = (A+B)s - 4A - 3B$

$A+B = 1 \ldots (i)$, $-4A - 3B = -5 \ldots (ii)$

From Eq (i) $A = 1-B$ put in Eq (ii)

$\cancel{1-B+}\; -4(1-B) - 3B = -5$

$-4 + 4B - 3B = -5$

$B = 5 - 4 = -1$

Then $A = 1 - (-1) = 2$

$$L^{-1}\left(\frac{s-5}{(s-3)(s+1)}\right) = L^{-1}\left(\frac{2}{s-3} + \frac{1}{s+1}\right)$$

$$L^{-1}\left(\frac{s-5}{(s-3)(s+1)}\right) = 2e^{3t} + e^{4t}$$

5) $$L^{-1}\left(\frac{2s-6}{(s-2)(s-4)}\right) = L^{-1}\left(\frac{A}{s-2} + \frac{B}{(s-4)}\right)$$

$2s - 6 = A(s-4) + B(s-2)$

$2s - 6 = As - 4A + Bs - 2B$

$2s - 6 = As + Bs - 4A - 2B$

$2s - 6 = (A+B)s - 4A - 2B$

$A + B = 2$ ... (1)

$-4A - 2B = -6$ ... (2)

From (1)

$A = 2 - B$. Sub. in (2)

$-4(2-B) - 2B = -6$

$-8 + 4B - 2B = -6$

$-8 + 2B = -6$

$2B = -6 + 8$

$2B = 2$

$B = 1$

Then $A = 1$

$$L^{-1}\left(\frac{1}{s-2} + \frac{1}{s-4}\right)$$

$$L^{-1}\left(\frac{2s-6}{(s-2)(s-4)}\right) = e^{2t} + e^{4t}$$

$(s-4)(s+1)$

(a) $L^{-1}\left(\frac{5s-8}{s(s-4)}\right) = L^{-1}\left[\frac{A}{s} + \frac{B}{s-4}\right]$

$5s-8 = A(s-4) + Bs$

$5s-8 = As - 4A + Bs$

$5s-8 = (A+B)s - 4A$

$A+B = 5$ ... (i)

$-4A = -8$ ... (ii)

From Eq (ii)

$A = \frac{8}{4}$

Then $B = 5-2 = 3$

$L^{-1}\left[\frac{2}{s} + \frac{3}{s-4}\right] = 2 + 3e^{4t}$

(b) $L^{-1}\left(\frac{s^2-3s-4}{(s-3)(s-1)^2}\right) = L^{-1}\left(\frac{A}{s-3} + \frac{B}{(s-1)} + \frac{C}{(s-1)^2}\right)$

$s^2-3s-4 = A(s-1)^2 + B(s-3)(s-1) + C(s-3)$

$s^2-3s-4 = A(s^2-2s+1) + B(s^2-s-3s+3) + C(s-3)$

$s^2-3s-4 = As^2 - 2As + A + Bs^2 - Bs - 3Bs + 3B + Cs - 3C$

$s^2-3s-4 = As^2 + Bs^2 - As - 4Bs + Cs + A + 3B - 3C$

$s^2-3s-4 = s^2(A+B) + s(-A-4B+C) + A + 3B - 3C$

Comparing coefficients

$A+B = 1$ ... (i) $\quad -A - 4B + C = -3$ ... (ii)

$A + 3B - 3C = -4$ ... (iii)

Adding Eq (ii) and (iii) $\quad -B - 2C = -7$ ... (iv)

Sub $A = 1-B$ in Eq (ii)

$-1 + B - 4B + C = -3$

$-3B - 1 + C = -3$

$C - 3B = -2$ ... (v) $\times 2$

$+ \; -2C - B = -7$ ... (vi) $\times 1$

$2C - 6B = -4$ ... (vii)

$-7B = -11$

$B = \frac{11}{7}$

From eq (5)

$C - 3B = -2$

$C - 3 \times \frac{11}{7} = 2$

$C = 2 + \frac{33}{7} = \frac{47}{7}$

To get A

$A = 1 - \frac{11}{7} = \frac{-4}{7}$

$$L^{-1}\left(\frac{s^2-8s-4}{(s-3)(s-1)^2}\right) = L^{-1}\left(\frac{\frac{-4}{7}}{s-3} + \frac{\frac{11}{7}}{(s-1)} + \frac{\frac{47}{7}}{(s-1)^2}\right)$$

$$= -\frac{4}{7}e^{3t} + \frac{11}{7}e^{t} + \frac{47}{7} + L^{-1}\left(\frac{\frac{47}{7}}{(s-1)^2}\right)$$

v) $$L^{-1}\left(\frac{s-5}{s^2+4s+20}\right) = L^{-1}\left(\frac{s+2-7}{(s+2)^2+16}\right)$$

$$L^{-1}\left(\frac{s+2-7}{(s+2)^2+16}\right) = L^{-1}\left(\frac{s+2}{(s+2)^2+4^2}\right) - \frac{7}{(s+2)^2+4^2}$$

$$= L^{-1}\left(\frac{s+2}{(s+2)^2+4^2}\right) - \left(\frac{4}{(s+2)^2+4^2}\right)\frac{7}{4}$$

$$L^{-1}\left(\frac{s+2}{(s+2)^2+4^2}\right) - L^{-1}\left(\left(\frac{4}{(s+2)^2+4^2}\right)\frac{7}{4}\right)$$

$$e^{-2t}\cos 4t - \frac{7}{4}e^{-2t}\sin 4t$$

1) Use the Leibniz – Maclaurin method to determine a series solution for

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$$

$$(1-x^2)y'' - 2xy' + 2y = 0$$

Let $(1-x^2)y'' = w_1$

$V = 1-x^2 \quad V' = -2x \quad V'' = -2 \quad V''' = 0$

$U = y'' \quad U^n = y^{n+2} \quad U^{n-1} = y^{n+1} \quad U^{n-2} = y^n$

$$w_1^n = U^n V + nU^{n-1}V' + \frac{n(n-1)U^{n-2}V''}{2!} + 0$$

$$= y^{n+2}(1-x^2) + ny^{n+1}(-2x) + n(n-1)y^n$$

$$= (1-x^2)y^{n+2} - 2xny^{n+1} + n(n-1)y^n$$

Let $w_2 = -2xy'$

$V = -2x \quad V^{(1)} = -2 \quad V^{(n)} = 0$

$U = y' \quad U^n = y^{n+1}$

$$w_2^n = U^n V + nU^{n-1}V' + 0$$

$$= y^{n+1}(-2x) + ny^n(-2)$$

$$= -2xy^{n+1} - 2ny^n$$

Let $w_3 = 2y$

$V = 2 \quad V' = 0$

$U = y \quad U^n = y^n$

$$w_3^n = U^n V + 0 = 2y^n$$

$$w_T^n = (1-x^2)y^{n+2} + 2xy^{n+1}(-n-1) + y^n(n^2-3n+n)$$

$w = w_1^n + w_2^n + w_3^n$

at $x = 0$

Recurrence relation is given as:

$$(y^{(n+2)}) = (-y^n(n^2-3n+n)) = (-y^n(n^2-2n))$$

when $n=0$

$(y^{ii})_0 = 0$

$n=1$, $(y^{iii})_0 = (y^{i})_0$

$n=2$, $(y^{iv})_0 = 0$

$n=3$

$(y^{v})_0 = (-3y^{iii})_0 = -3(y^{i})_0$

$n=4$

$(y^{vi})_0 = 0$

$n=5$

$(y^{vii})_0 = -15y^{v} = -15(-3y^{i})_0$

$n=6$

$(y^{viii})_0 = 0$

Hence the series solution is given as

$$y = y_0 + x y_0' + \frac{x^2}{2!}(y'')_0 + \frac{x^3}{3!}(y''')_0 + \frac{x^4}{4!}(y^{iv})_0$$

$$+ \frac{x^5}{5!}(y^{v})_0 + \frac{x^6}{6!}(y^{vi})_0 + \frac{x^7}{7!}(y^{vii})_0 + \frac{x^8}{8!}(y^{viii})_0$$

$$y = y_0 + x(y^{i})_0 + \frac{x^3}{6}(y^{i})_0 - \frac{(y_1^{i})x^5}{40} + \frac{3y^{i}_0 x^7}{336}$$

$$y = y_0 + (y^{i})_0\left(x + \frac{x^3}{6} - \frac{x^5}{40} + \frac{x^7}{112} + \cdots\right)$$