Assignment 4 Awurum I. Charles
151ENG061015

(a) $(1-x^2)\,\partial^2y/\partial x - 2x\,\partial y/\partial x + 2y = 0$

$(1-x^2)y^2 - 2xy' + 2y = 0$

$y^n = U^n U + nU^{n-1} + \frac{n(n-1)}{2!} U^{n-2} U^2 + \dots$

$[y^{(2+n)} \cdot (1-x^2) + \cancel{ny(1+n)}\; ny^{(1+n)} - n(n-y)y^n \cdot (-2)] + [y^{n+n} - 2x$

$+ ny^n - 2 + [2y^n] = 0$

$(1-x^2)y^{(2+n)} - 2x + ny^{(1+n)} - n(n-1)y^{(n)} - 2xy^{(1+n)} - 2ny^{(n)} + 2y^{(n)}$.

let $x = 0$

$y^{(2+n)} - n(n-1)y^{(n)} - 2ny^{(n)} + 2y^n = 0$

$y^{n+2} + y^n[-n(n-1) - 2n + 2] = 0$

$y^{n+2} + y^n[-n^2 - n + 2] = 0$

$y^{n+2} = -(y^n) \cdot [-n^2 - n + 2]$

$n = 0 = y^2 = -y^0 \cdot 2 = -2y^0$

$n = 1 \quad y^3 = -y^1[0] = 0$

$n = 2 \quad y^4 = -y^2[-4] = 4y^2 = (4)(-2y^0) = -8y^0$

$n = 3 : \quad y^5 = -y^3[-10] = 10y^3 = 18y \; 40(0)$

$n = 4 = \quad y^6 = -y^4[-18] = 18y^4 = 18 \cdot 4 = -2y^1$

$y = y^0 + xy^1 + \frac{x^2}{2!}y^3 + \frac{x^3}{3!}y^3 + \dots$

$y = y^0 + xy^1 + \frac{x^2}{2!}(-2)y^0 + \frac{x^3}{3!}(0) + \frac{x^4}{4}(4)(-2)y^2 + \dots$

$+ \frac{x^5}{5}(0) + \frac{x^6}{6!}(18) + (-2)y^0 + \frac{x^7}{7!}(0)$

$y = y^0 + xy^1 - x^2y^0 - \frac{x^4}{3 \times 1}y^0 - \frac{x^6}{5}y^0$

$y = y^0[1 - x^2 - x^4/3 - \frac{x^6}{5}] + y_0[x]$

B) Convert the following functions

$3e^{-4t} - 5e^{4t}$

(i) $\cancel{\dfrac{S-5}{(S-3)(S-4)}}$

$L^{-1}\left[\cancel{\dfrac{S-5}{(S-3)(S-4)}}\right] = \cancel{\dfrac{A}{(S-3)} + \dfrac{B}{(S-4)}}$

$\dfrac{S-5}{(S-3)(S-4)} = \cancel{\dfrac{A(S-4) + B(S-3)}{(S-3)(S-4)}}$

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$3\left[\dfrac{1}{S+4}\right] - 5\left[\dfrac{1}{S-4}\right]$

$= \dfrac{3(S-4) - 5(S+4)}{(S+4)(S-4)} = \dfrac{3s-12-5s-20}{(S+4)(S-4)}$

$= \dfrac{-2s-32}{(S+4)(S-4)}$

(ii) $\sin 4t + \cos 4t$

$L[\sin 4t] + L[\cos 4t]$

$\dfrac{4}{S^2+16} + \dfrac{S}{S^2+16} = \dfrac{4+S}{S^2+16}$

(iii) $t^3 + 2t^2 - t + 4$

$L[t^3] + 2L[t^2] - L[t] + L[4]$

$\dfrac{3!}{S^4} + \dfrac{4}{S^3} - \dfrac{1}{S^2} + \dfrac{4}{S}$

$\dfrac{6}{S^4} + \dfrac{4}{S^3} - \dfrac{1}{S^2} + \dfrac{4}{S}$

$\dfrac{6 + 4s - 1s^2 + 4s^3}{S^4}$

(iv) $e^{-2t}\cos 5t$

$L\{e^{-2t}\cos 5t\} = \dfrac{s}{(s+2)+5}$

(v) $t\sin 3t$

$L\{t\sin 3t\}$

$L[\sin 3t] = \dfrac{3}{s^2+9}$

$L\{t\sin 3t\} = -\partial/\partial s\,(3/s^2+9)$

$-3\dfrac{\partial}{\partial s}(s^2+9)^{-1}$

$u = s^2+9 \quad \dfrac{\partial y}{\partial s} = 2s \quad y = u^{-1}$

$\dfrac{dy}{du} = -u^{-2}$

$\partial y/\partial s = \dfrac{dy}{du} \times \dfrac{dy}{\partial s} = -u^{-2} \times 2s$

$= \dfrac{2s}{u^2} = -\dfrac{2s}{(s^2+9)^2}$

$-3\left(\dfrac{-2s}{s^2+9}\right)$

$= \dfrac{6s}{s^2+9}$

(vi) $\dfrac{e^{-t}-e^{-2t}}{t}$

$L[e^{-t}] - L[e^{-2t}]$

$\dfrac{1}{s+2} - \dfrac{1}{s+2}$

$L[e^{-t} - e^{-2t}] = \left[\dfrac{1}{s+1} - \dfrac{1}{s+2}\right] = \int_s^\infty \left(1/\sigma+1 - \dfrac{1}{\sigma+2}\right) d\sigma$

$\ln(\sigma+1) - \ln(\sigma+2)]_s^\infty$

$0 - \ln s+1/s+2$

$\ln \frac{s+2}{s+3}$

(vii) $e^{4t}\cos 2t$

$L[e^{4t}\cos 2t]$

$L[\cos 2t] = \frac{s}{s^2+4}$

$L[e^{4t}\cos 2t] = \frac{s}{(s-4)^2+4}$

(viii) $t\sin 2t$

$L[t\sin 2t]$

$L[\sin 2t] = \frac{2}{s^2+4}$

$L[t\sin 2t] = -\frac{d}{ds}\left[\frac{2}{s^2+4}\right]$

$-2\frac{d}{ds}(s^2+4)^{-1}$

$u = s^2+4 \qquad y = u^{-1}$

$\frac{du}{ds} = 2s \qquad dy/du = -u^{-1}$

$\frac{dy}{ds} = -2su^{-2}$

$= \frac{2s}{u^2} = \frac{-2s}{(s^2+4)^2}$

$= \frac{4s}{(s^2+4)^2}$

(ix) t^3+4t^2+5

$L[t^3]+4L[t^2]+L[5]$

$\frac{3!}{s^4}+\frac{4x^2}{s^3}+5$

$\frac{6+8s+5s^3}{s^4}$

x) $e^{5t}(t^2+4)$

$L[t^2+4]$

$L[t^2]+L[4]$

$\frac{2}{s^3}+\frac{4}{s} = \frac{2+4s^2}{s^3}$

xi) $\frac{2+4s^2}{s^3}$

$L[e^{3t}(t^2+4)]$

$\frac{2+4s^2}{(s-3)^3}$

xii) $t^2\cos t$

$L[\cos t] = \frac{s}{s^2+1}$

$L[t^2\cos t] = -\frac{2}{ds}\int\frac{s}{s^2+1}$

$\frac{v\,du/dx - u\,dv/dx}{v^2}$

$u = s \qquad \frac{du}{ds} = 1 \qquad \frac{dv}{ds} = 2s$

$v = s^2+1$

$= \frac{(s^2+1)(1) - s(2s)}{(s^2+1)^2}$

$$= \frac{s^2+1-2s^2}{(s^2+1)^2}$$

$$= \frac{-s^2+1}{(s^2+1)^2}$$

xii) $\frac{\sinh 2t}{t}$

$$L[\sinh 2t] = \frac{2}{s^2-4}$$

$$L[\sinh 2t] = \left[\frac{2}{s^2-4}\right] d\theta$$

3)

Convert the following functions to time dominance

$$\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$$

$$s-8 = A(s-4) + B(s-3)$$

$s = 4$

$1 = B$

$s = 3$

$-2 = B - A$

$A = 2$

$$L^{-1}\left[\frac{s-8}{(s-3)(s-4)}\right] = \frac{2}{(s-3)} - \frac{1}{(s-4)}$$

$$2e^{3t} - e^{4t}$$

(ii) $\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$

$$2s-6 = A(s-4) + B(s-2)$$

$s = 4$

$2 = 2B$

$B = 1$

$s = 2$

$-2 = -2A$

$A = 2$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{1}{s-2} + \frac{1}{s-4}$$

$$= e^{2t} + e^{4t}$$

(iii) $\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$$5s-8 = A(s-4) + (B)(s)$$

when $s = 4$

$12 = 4B$

$B = 3$

$A + B = 5$

$A = 2$

$$2/s + 3/s-4 = 2 + 4e^{3t}$$

(iv) $\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{A}{s-3} + \frac{B}{s-1} + \frac{C}{(s-1)^2}$

$$s^2-3s-4 = A(s-1)^2 + B(s-1)(s-3) + C(s-3)$$

when $s = 1$

$-6 = -2C$

$C = 3$

$s = 3$

$-4 = 4A$

$A = -1$

$A + B = 1$

$B = 2$

$$-\frac{1}{s-3} + \frac{2}{s-1} + \frac{s}{(s-1)^2}$$

$$2e^{t} - e^{3t} + 3t\,e^{t}$$

$$\frac{s-5}{s^2+4s+20} = \frac{s-5}{s^2+2s+2s+4+16}$$

$$\frac{s-5}{(s+2)^2+16} = \frac{s-5}{(s+2)^2+4^2}$$

$$\frac{s-5}{(s+2)^2+4^2} = \frac{s-5+2-2}{(s+2)^2+4^2}$$

$$\frac{s+2}{(s+2)^2+4^2} = \frac{-2}{(s+2)^2+4^2}$$

$$\cancel{\frac{s+2}{(s+2)}}$$

$$\frac{s+2}{(s+2)^2+4^2} - \frac{7}{4}\left(\frac{4}{(s+2)^2+4}\right.$$

$$e^{-2t}\cos 4t - 7/4\,e^{-2t}\sin 4t$$