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15/ENG07/010
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Transform each of the following using Laplace transform

$$L\{f(t)\} \rightarrow f(s) = \int_0^\infty e^{-st} f(t)\, dt$$

(I) $3e^{-4t} - 5e^{4t}$

$L(3e^{-4t} - 5e^{4t})$

$L(3e^{-4t}) - L(5e^{4t})$

$\frac{3}{s+4} - \frac{5}{s-4}$

(II) $\sin 4t + \cos 4t$

$L\{\sin 4t + \cos 4t\}$

$L\{\sin 4t\} + L\{\cos 4t\}$

$\frac{4}{s^2+16} + \frac{5}{s^2+16}$

(III) $t^3 + 2t^2 - t + 4$

$L\{t^3 + 2t^2 - t + 4\}$

$L\{t^3\} + L\{2t^2\} - L\{t\} + L\{4\}$

$\frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

(IX) $e^{-2t}\cos t$

$L\{e^{-2t}\cos t\}$

$= \frac{s-2}{(s-2)^2+25}$

(V) $t\sin 3t$

$L\{t\sin 3t\}$

$\frac{3}{s^2+9}$

(X) $\frac{e^{-t} - e^{-2t}}{t}$

$L\{e^{-t} - e^{-2t}\}$

$L\{e^{-t}\} - L\{e^{-2t}\}$

$\frac{1}{s+1} - \frac{1}{s+2}$

$= \int_s^\infty \left(\frac{1}{s+1} - \frac{1}{s+2}\right) ds$

$\int_s^\infty \frac{1}{s+1} ds - \int_s^\infty \frac{1}{s+2} ds$

$\left[\ln(s+1) - \ln(s+2)\right]_s^\infty$

$\left[\ln\frac{(s+1)}{(s+2)}\right]_s^\infty$

$\left[\ln\frac{(\infty+1)}{(\infty+2)} - \ln\frac{(s+1)}{(s+2)}\right]$

$0 - \ln\frac{(s+1)}{s+2}$

(VI) $e^{4t}\cos 2t$

$L\{e^{4t}\cos 2t\}$

$\frac{s-4}{(s-4)^2+4}$

(VII) $t\sin t$

$\frac{2}{s^2+4}$

$t^3 + 4t^2 + 5$

$L\{t^3 + 4t^2 + 5\}$

$L\{t^3\} + L\{4t^2\} + L\{5\}$

$\frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$

) $e^{3t}(t^2+4)$

$L\left(\frac{2}{s^3} + \frac{4}{s}\right)$
$s-3$

) $t^3 \cos t$

$\frac{s-1}{(s-1)^2+1}$

2.) $\frac{\sinh 2t}{t} = L\left\{\frac{\sinh 2t}{t}\right\}$

$= \frac{\tan^{-1}(2)}{s}$

3) Convert the forms to partial fractions

$\frac{s-5}{(s-3)(s-4)} = \frac{A}{(s-3)} + \frac{B}{(s-4)}$

$\frac{A(s-4) + B(s-3)}{(s-3)(s-4)}$

$s-5 = As - 4A + Bs - 3B$

$s-5 = As + Bs - 4A - 3B$

$A + B = 1 \times -4$

$-4A - 3B = -5 \times 1$

$-4A - 3B = -5$

$-B = 1$

$B = -1$

$A + B = 1$

$A - 1 = 1$

$A = 2$

$\frac{2}{(s-3)} - \frac{1}{(s-4)}$

$L^{-1}\left(\frac{2}{(s-3)} - \frac{1}{(s-4)}\right)$

$2e^{3t} - e^{4t}$

2.) $\frac{2s-6}{(s-2)(s-4)}$

$\frac{A}{(s-2)} + \frac{B}{(s-4)}$

$\frac{A(s-4) + B(s-2)}{(s-2)(s-4)}$

$2s-6 = As - 4A + sB - 2B$

$A + B = 2 \times -4$

$-4A - 2B = -6 \times 1$

$-4A + 4B = -8$

$-4A - 2B = -6$

$-2B = 2$

$B = 1$

$A + B = 2$

$A + 1 = 2$

$A = 1$

$\frac{1}{(s-2)} + \frac{1}{(s-4)}$

$L^{-1}\left(\frac{1}{(s-2)} + \frac{1}{(s-4)}\right) e^{2t} + 4^t$

3. $\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4)}$

$\frac{5s-8}{s(s-4)} = \frac{A(s-4)+Bs}{s(s-4)}$

$5s-8 = A(s-4)+Bs$

$5s-8 = As-4A+Bs$

$+4A=+8$

$A=2$

$A+B=5$

$2+B=5$

$B=3$

$L^{-1}\left(\frac{2}{s}+\frac{B}{(s-3)}\right)$

$2+e^{3t}$

(4.) $\frac{s^2-3s-4}{(s-3)(s-1)^2}$

$\frac{A}{(s-3)}+\frac{B}{(s-1)}+\frac{C}{(s-1)^2}$

$s^2-3s-4 = A(s-1)^2+B(s-3)(s-1)$
$+C(s-3)$

$(s-3)=0$

$C=-2$ $A=1$

$B=-3$

$\frac{1}{(s-3)} - \frac{3}{(s-1)} - \frac{2}{(s-1)^2} = L^{-1}\left[\frac{1}{s-3}\right) - L^{-1}\left(\frac{3}{s-1}\right)$

$e^{3t} - 3e^{t}$

X.

1. $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y=0$

$(1-x^2)y'' - 2xy' + 2y = 0$

$W=(1-x^2)y''$

$u^n = y^{n+2}$ $\quad v=(1-x^2)$

$v' = -2x$

$W_2 = 2xy'$ $\quad v''=-2$

$y^{n+1} = u$ $\quad v = 2x \quad v' = 2$

$W_3 = 2y$

$u^n = 2y^n$

From Leibnitz

$y^n = u^nv^0 + nu^{n-1}v' + \frac{n(n-1)}{2!}u^{n-2}v^2$

from W_1

$y^{n+2}(1-x^2) + n(y^{n+2})-2x + \frac{n(n-1)}{2!}y^n$

$y^{n+2}(1-x^2) - 2xn(y^{n+2}) - n(n-1)y^n$

From W_2

$y^{n+1}2x + nxy^n2$

From W_3

$2y^n$

$y^n = (1-x^2)y^{n+2} - 2xn(y^{n+2}) - n(n-1)y^n$
$-y^{n+1}2x + 2xy^n + 2y^n$

When $x=0$

$y^n = y^{n+2} - (n^2+n)y^n + 2xy^n + 2y^n$

$y^n = y^{n+2} (n^2+n)y^n + y^n(2n-2) = 0$

$y^n = y^{n+2} - (y^nn^2 + y^nn) + (2xy^n - 2y^n)$

$y^n = y^{n+2} - y^nn^2 - y^nn + 2xy^n + 2y^n$

$y^n = y^{n+2} - y^n(n^2-n+2) = 0$

$y^{n+2} - y^n(-n^2+n-2)$

$n=0\ (y^{(2)})_0 = y^0\ (-2)$

$n=1\ (y^{(3)})_0 = (y^{(1)})_0\ (0) = 0$

$n=2\ (y^{(4)})_0 = (y^{(2)})_0\ (4)(-2)$

$n=3\ (y^{(5)})_0 = (y^{(3)})_0 = 0$

$n=4\ (y^{(6)})_0 = (y^{(4)})_0\ (18)(4)(-2)$

$n=5\ (y^{(7)})_0 = (y^{(5)})_0 = 0$

$n=6\ (y^{(8)})_0 = (y^{(6)})_0 (18)(4)(-2)(4_0)$

$n=7\ (y^{(9)})_0 = (y^{(9)})_0 = 0$

$= (y^2)_0 \frac{x^2}{2!} + (y^{(3)})_0 \frac{x^3}{3!} + (y^{(4)}) \frac{x^4}{4!} + (y^5) \frac{x^5}{5!}$

$+ (y^6) \frac{x^6}{6!} + (y^{(7)}) \frac{x^7}{7!} + (y^{(8)}) \frac{x^8}{8!}$

$+ (y^9) \frac{x^9}{9!}$

$= (y^{0})_0 (-2) \frac{x^2}{2!} + (y^{(2)})_0 (-2) \frac{x^4}{4!} + \frac{x^6}{6!}$

$(y^4)_0\ (18)(-4)(-2) \quad y^6$

$(y^8)_0\ (18)(-4)(-2)(4_0) \frac{x^8}{8!}$