$$L^{-1}\left[\frac{2}{s} + \frac{3}{s-4}\right]$$
$$= 2 + 3e^{4t}$$

(iv) $\frac{s^2-3s-4}{(s-3)(s-1)^2} \Rightarrow \frac{A}{(s-3)} + \frac{B}{(s-1)} + \frac{C}{(s-1)^2}$

$$\frac{A(s-1)(s-1)^2 + B(s-3)(s-1)^2 + C(s-3)(s-1)}{(s-3)(s-1)(s-1)^2}$$

$s^2-3s-4 = A(s-1)^2 + B(s-1)^2 + B(s-3)(s-$

$3^2-3(3)-4 = A(3-1)^2 \Rightarrow A =$

$1^2-3(1)-4 = C(1-3) \Rightarrow C =$

$s^2-3s-4 = [s^2-2s+1)A + (s^2-4s$

$= -2A-4B+C = -3$

$-2(-1)-4(B)+3 = -3$

$-4B = -3-3-2$

$B = 2$

$$L^{-1}\left[\frac{-1}{s-3} + \frac{2}{s-1} + \frac{3}{(s-1)^2}\right]$$

$$= -e^{3t} + 2e^{t} + 3te^{t}$$

(v) $\frac{s-5}{s^2+4s+20} = \frac{s-5}{s^2+4s+4+16}$

$\frac{s-5}{(s+2)^2+4} \Rightarrow (e^{2t}-7)\cos 4t$

$\Rightarrow s-4=0, \quad s=4$

$4-5 = 0 + B(4-3)$

$-1 = B$

$B = -1$

$\Rightarrow \mathcal{L}^{-1}\left[\frac{2}{(s-3)} + \frac{(-1)}{(s-4)}\right]$

$= 2e^{3t} - e^{4t}$

(ii) $\frac{2s-6}{(s-2)(s-4)} \Rightarrow \frac{A}{(s-2)} + \frac{B}{(s-4)} \Rightarrow \frac{A(s-4)+B(s-2)}{(s-2)(s-4)}$

$2s-6 = A(s-4) + B(s-2)$

$\Rightarrow s-4=0, \quad s=4$

$2(4)-6 = 0 + B(4-2)$

$2 = 2B$

$B = 1$

$\Rightarrow s-2=0, \quad s=2$

$2(2)-6 = A(2-4)+0$

$-2 = -2A$

$A = 1$

$\mathcal{L}^{-1}\left[\frac{1}{(s-2)} + \frac{1}{(s-4)}\right]$

$= e^{2t} + e^{4t}$

(iii) $\frac{5s-8}{s(s-4)} \Rightarrow \frac{A}{s} + \frac{B}{s-4} \Rightarrow \frac{A(s-4)+Bs}{s(s-4)}$

$5s-8 = A(s-4) + Bs$

$\Rightarrow s=0$

$0-8 = A(-4)$

$A = 2$

$u = s \qquad du = 1$

$v = s^2+1 \qquad dv = 2s$

$\Rightarrow \dfrac{v\,du - u\,dv}{v^2} = \dfrac{(s^2+1) - 2s^2}{s^4+2s^2+1} = \dfrac{(s^2+1) - 2s^2}{(s^2+1)(s^2+1)}$

$u = (s^2+1) - 2s^2 \qquad du = 2s - 4s = -2s$

$v = (s^2+1)(s^2+1) \qquad dv = 4s^3 + 4s$

$= \dfrac{d^2}{ds^2}\left[\dfrac{s}{s^2+1}\right] = -\dfrac{(s^2+1)^2(-2s) - (1-s^2)(4s^3+4s)}{(s^4+2s^2+1)^2}$

$= -\dfrac{(s^4+2s^2+1)(-2s) - (1-s^2)(4s^3+4s)}{(s^4+2s^2+1)^2}$

$= -\dfrac{(2s^5 - 4s^4 - 6s)}{(s^4+2s^2+1)^2}$

(xii) $L\left[\dfrac{\sinh 2t}{t}\right]$

$L[\sinh 2t] = \dfrac{2}{s^2-4}$

$L\left[\dfrac{\sinh 2t}{t}\right] = \int_{s=s}^{\infty} f(s)\,ds = \int_s^{\infty} \dfrac{2}{s^2-4}\,ds$

$= 2\int_s^{\infty} \dfrac{1}{s^2-4}\,ds = 2\ln(s^2-4)$

(x) $L\left[e^{3t}(t^2+4)\right]$

$$= \int_s^\infty \left(\frac{1}{s+1} - \frac{1}{s+2}\right) ds$$

$$= \int_s^\infty \frac{1}{s+1}\, ds - \int_s^\infty \frac{1}{s+2}\, ds$$

$$= \Big[\ln(s+1) - \ln(s+2)\Big]_s^\infty$$

$$= \ln\left[\frac{s+1}{s+2}\right]\Bigg|_s^\infty$$

$$= 0 - \ln\frac{s+1}{s+2}$$

(vii) $L[e^{4t}\cos 2t]$

$$L[\cos 2t] = \frac{s}{s^2+4}$$

$$L[e^{4t}\cos 2t] = \frac{(s-4)}{(s-4)^2+4}$$

(viii) $L[t\sin 2t]$

$$L[\sin 2t] = \frac{2}{s^2+4}$$

$$L[t\sin 2t] = -\frac{d}{}\left[\frac{2}{s^2+4}\right]$$

Number 2.

2(i) $L[5e^{-4t} - 5e^{4t}] = 5L[e^{-4t}] - 5L[e^{4t}]$

$= 5\left[\frac{1}{s+4}\right] - 5\left[\frac{1}{s-4}\right]$

$= \frac{5}{s+4} - \frac{5}{s-4}$

(ii) $L[\sin 4t + \cos 4t] = L[\sin 4t] + L[\cos 4t]$

$= \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2}$

$= \frac{4}{s^2+16} + \frac{s}{s^2+16}$

(iii) $L[t^3 + 2t^2 - t + 4] = L[t^3] + 2L[t^2] - L[t] +$

$= \frac{3}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

(iv) $L[e^{-2t}\cos 5t]$

$L[\cos 5t] = \frac{s}{s^2+25}$

$L[e^{-2t}\cos 5t] = \frac{(s+2)}{(s+2)^2+25}$

(v) $L[t \sin 3t]$

$L[\sin 3t] = \frac{3}{s^2+9}$

$L[t \sin 2t] = -\frac{d}{ds}\left[\frac{3}{s^2+9}\right]$

$u = 3 \qquad V = s^2+9$

$du = 0 \qquad dv = 2s$

$-\frac{d}{ds}\left[\frac{3}{s^2+9}\right] = -\left[\frac{Vdu - Udv}{}\right] = -\left[\frac{0 - 3(2s)}{}\right] =$

$n = 1, 2, 3, 4, \ldots$

$n = 1$

$y^3 = y''' = [1^2 - 1 + 2 - 2]\, y' = 0$

$n = 2$

$y^4 = y^{iv} = [2^2 + 2 - 2]\, y'' = 4y''$

$n = 3$

$y^5 = y^{v} = [3^2 + 3 - 2]\, y''' = 10y''' = 0$

$n = 4$

$y^6 = y^{vi} = [4^2 + 4 - 2]\, y^{iv} = 18y^{iv} = 18y^{iv} = 18\,[4y$

$n = 5$

$y^7 = y^{vii} = [5^2 + 5 - 2]\, y^{v} = 28y^{v} = 0$

$n = 6$

$y^8 = y^{viii} = [6^2 + 6 - 2]\, y^{vi} = 40y^{vi} = [40 \times 18 \times 4]$

$y = 1 + y^2 + y' + \frac{y''}{2!} + \frac{4y''}{4!} + \frac{18y''(4)}{6!} + 4$

$y = 1 + y^0 + y' + y''\left[\frac{1}{2} + \frac{1}{6} + \frac{1}{10} + \frac{1}{14}\right]$

Macdonald Perez
15/Eng 02 1035
Comp. Engr
Maths Assignment (IV)

$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)y'' - 2xy' + 2y = 0$

$y = (1-x^2)y''$

$V = (1-x^2) \quad V' = -2x \quad V'' = -2 \quad V''' = 0$

$u = y'' \quad u^n = y^{n+2}$

$y^n = u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v'' + \ldots$

$= y^{n+2} \cdot (1-x^2) + (n+2) y^{(n+1)} \cdot (-2x) + \frac{(n+2)(n+1)}{2!} y^n \cdot (-2)$

$= (1-x^2) y^{n+2} - 2x(n+2) y^{(n+1)} + \frac{(-2)(n+2)(n+1)}{2!} y^n$

$y = -2xy'$

$V = -2x \quad V' = -2 \quad V'' = 0$

$u = y' \quad u^n = y^{n+1}$

$y^n = y^{(n+1)}(-2x) + (n+1)y^n \cdot (-2)$

$= -2xy^{(n+1)} - 2^{(n+1)}y^n$