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ENG 381 ENGINEERING MATHEMATICS ASSIGNMENT

Answers

1) $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)y^2 - 2xy' + 2y = 0$

$y^n = u^n v + nu^{n-1}v' + \frac{n(n-1)u^{n-2}v^2}{2!}$

$\left[y^{(2+n)}.(1-x^2) + ny^{(1+n)}.(-2x) + \frac{n(n-1)y^n(-2)}{2!}\right] + [y^{1+n}.2x + ny^n$

$-2 + [2y^n] = 0$

$(1-x^2)y^{n+2} - 2xny^{n+1} - n(n-1)y^n - 2xy^{n+1} - 2ny^n + 2y^n = 0$

Let $x = 0$

$y^{n+2} - (n-1)y^n - 2ny^n + 2y^n = 0$

$y^{n+2} + y^n[-n(n-1) - 2n + 2] = 0$

$y^{n+2} + y^n[-n^2 + n - 2n + 2] = 0$

$y^{n+2} + y^n[-n^2 - n + 2] = 0$

$y^{n+2} = -(y^n).[-n^2 - n + 2]$

$n = 0 : y^2 = -y^0 . 2 = -2y^0$

$n = 1 : y^3 = -y^1 . [0] = 0$

$n = 2 : y^4 = -y^2[-4] = 4y^2 = 4(-2y^0] = -8y^0$

$n = 3 . y^5 = -y^3 . [-10] = 10y^3 = 10.0 = 0$

$n = 4 = y^6 = -y^4 . [-18] = 18y^4 = 18.4 - 2y^0$

$n = 5 : y^7 = -y^5[-28] = 28(y^5) = 28.0 = 0$

$y = y^0 + xy^1 + \frac{x^2y^3}{2!} + \frac{x^3y^3}{3!}$

$y = y^0 + xy^1 + \frac{x^2}{2!}(-2)y^0 + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(4)(-2)(y^0) + \frac{x^5}{5!}(0) + \frac{x^6}{6!}(18)4 - 2y^0 + \frac{x^7}{7!}(0)$

$y = y^0 + xy^1 - x^2y^0 - \frac{x^4}{3}y^0 - \frac{x^6}{5}y^0$

$y = y^0\left[1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5}\right] + y^1{}_0[x]$

2i) $3e^{-4t} - 5e^{4t}$

$= L[3e^{-4t} - 5e^{4t}] \Rightarrow L[3e^{-4t}] - L[5e^{4t}]$

$= 3\left[\frac{1}{s-a}\right] - 5\left[\frac{1}{s-a}\right]$

$= 3\left[\frac{1}{s+4}\right] - 5\left[\frac{1}{s-4}\right]$

$= \frac{3}{s+4} - \frac{5}{s-4}$

2(ii) $\sin 4t + \cos 4t$

$L[\sin 4t] + L[\cos 4t]$

$= \frac{4}{s^2+16} + \frac{s}{s^2+16}$

$= \frac{4+s}{(s^2+16)}$

2(iii) $t^3 + 2t^2 - t + 4$

$t^n = \frac{n!}{s^{n+1}}$

$= \frac{3!}{s^{3+1}} + 2\left[\frac{2!}{s^{2+1}}\right] - \left[\frac{1!}{s^{1+1}}\right] + \frac{4}{s}$

$= \frac{6}{s^4} + \frac{4}{s^4} - \frac{1}{s^2} + \frac{4}{s}$

(iv) $e^{-2t}\cos 5t$

$L[\cos 5t] = \frac{s}{s^2+a^2}$

$= \frac{s}{s^2+25}$

$L[e^{-2t}\cos 5t] = \frac{s+2}{[s+2]^2+25}$

(v) $t(\sin 3t]$

$L[\sin 3t] = \frac{a}{s^2+a^2} = \frac{3}{s^2+3^2} = \frac{3}{s^2+9}$

$L[t\sin 3t] = -F'(s) ; \frac{v\,du/ds - u\,dv/ds}{v^2}$

$u = 3 \quad du/ds = 0$

$v = s^2+9 . \; dv/ds = 2s \quad = \frac{[s^2+9]\,0 - 3[2s]}{[s^2+9]^2} = -1\left[\frac{-6s}{(s^2+9)^2}\right] = \frac{6s}{[s^2+9]^2}$

$$\frac{e^{-t} - e^{-2t}}{t}$$

$$L\left[\frac{e^{-t} - e^{-2t}}{t}\right]$$

$$L[F(t)] = e^{-t} - e^{-2t} = \frac{1}{s+a} - \frac{1}{s+a} = \frac{1}{s+1} - \frac{1}{s+2}$$

$$F\sigma = \frac{1}{\sigma+1} - \frac{1}{\sigma+2}$$

$$\int F\sigma \cdot L\left[\frac{f(t)}{t}\right] = \int_{\sigma=s}^{\infty} \frac{1}{\sigma+1} - \frac{1}{\sigma+2}\, d\sigma$$

$$= \int_{\sigma}^{\infty} \frac{-1}{\sigma+1}\, d\sigma = \int_{\sigma=s}^{\infty} \frac{1}{\sigma+2}\, d\sigma$$

$$\ln[\sigma+1] - \ln[\sigma+2]_s^{\infty}$$

$$= [\ln[\sigma+1] - \ln[\sigma+2]]_s^{\infty}$$

$$= \left[\ln\frac{\sigma+1}{\sigma+2}\right]_s^{\infty} = \ln\left[\frac{\infty+1}{[\infty+2]} - \frac{s+1}{(s+2)}\right]$$

$$= -\ln\left[\frac{s+1}{(s+2)}\right] = \ln\left[\frac{(s+2)}{(s+1)}\right]$$

$e^{4t}\cos 2t$

$$L[\cos 2t] = \frac{s}{s^2+2^2} = \frac{s}{s^2+4}$$

$$L[e^{4t}\cos 2t] = \frac{s-4}{[s-4]^2+4}$$

$t\sin 2t$

$$L[t\sin 2t] = -\frac{d}{ds}[F(s)]$$

$$F(s) = L[\sin 2t] = \frac{2}{s^2+2^2}, \quad F(s) = \frac{2}{s^2+4}$$

$u = 2 \quad du/ds = 0$

$v = s^2+4 \quad dv/ds = 2s$

$\cdot\ p^2 4\ f^\circ$ let $y = [u]^2$ $\quad dy/du = 2u$, $\frac{dy}{ds} = \frac{dy}{du}$

$u = s^2+1$ $\quad du/ds = \cdot\ 2s$

2i $t^2 \cos t$

$L[t^2 \cos t] = t^2 L[\cos t]$

$F(s) = L[\cos t] = \frac{s}{s^2+1^2}$, $F(s) = \frac{s}{s^2+1^2}$

$f'(s) = u = s \quad du/ds = 1$

$v = s^2+1^2 \quad dv/ds = 2s$

$= \frac{[s^2+1^2] \cdot 1 - 2s[s]}{[s^2+1^2]^2}$

$= \frac{s^2+1^2-2s^2}{[s^2+1^2]^2} = \frac{-s^2+1}{[s^2+1^2]^2}$

$f''(s) = -\frac{d}{ds}\left[\frac{s^2-1}{[s^2+1^2]^2}\right]$

$u = s^2-1 \quad du/ds = 2s$

$v = [s^2+1]^2 \quad dv/ds = 4s[s^2+1]$

$\frac{[s^2+1]^2 \cdot 2s - [s^2-1][4s^3+4s]}{[s^2+1]^2}$

$= \frac{[2s^5-4s^3+2s]-[4s^5-4s]}{[(s^2+1)^2]}$

$= \frac{-2s^5-4s^3+6s}{[(s^2+1)^2]}$

$= \frac{-2s^5-4s^3+6s}{s^4+4s^2+1}$

$f''(s) = -\frac{d}{ds}\left[\frac{-2s^5-4s^3+6s}{s^4+2s^2+1}\right]$

$f''(s) = \frac{2s^5+4s^3-6s}{s^4+2s^2+1}$

$$\frac{v\,{du}/{ds} - u\,{dv}/{ds}}{v^2}$$

$$= \frac{(s^2+4)\cdot 0 - 2-(2s)}{(s^2+4)^2}$$

$$= \frac{-4s}{(s^2+4)^2}$$

$$L[t\sin 2t] = F'(s)$$

$$= -1\cdot\left[\frac{-4s}{(s^2+4)^2}\right]$$

$$= \frac{4s}{(s^2+4)^2}$$

) t^3+4t^2+5

$$L[t^3+4t^2+5]$$

$$= \frac{3!}{s^{3+1}} + 4\left[\frac{2!}{s^{2+1}}\right] + \frac{5}{s}$$

$$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$$

) $e^{3t}(t^2+4)$

let $x = t^2+4$

$$L[e^{3t}x]$$

$$L[x] = L[t^2+4] - L[t^2] + L[4]$$

$$= \frac{2!}{s^{2+1}} + \frac{4}{s}$$

$$= \frac{2!}{s^3} + \frac{4}{s}$$

$$L[e^{3t}] = \frac{2}{[s-3]^3} + \frac{4}{[s-3]}$$

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$S = 2$

$2(2) - 6 = A(2-4) + B(2-2)$

$4 - 6 = -2A + 0$

$A = 1$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{1}{s-2} + \frac{1}{s+4}$$

$$L^{-1}\left[\frac{2s-6}{(s-2)(s-4)}\right] = e^{2t} + e^{4t}$$

(iii) $\dfrac{5s-8}{s(s-4)}$

$$L^{-1}\left[\frac{5s-8}{s(s-4)}\right] = \frac{A}{s} + \frac{B}{s-4}$$

$5s - 8 = A(s-4) + B(s)$

Assuming $s = 4$

$5(4) - 8 = A(4-4) + B(4)$

$20 - 8 = 4B$

$B = 3$

Assuming $s = 0$

$5(0) - 8 = A(0-4) + B0$

$-8 = -4A$

$A = 2$

$$L^{-1}\left[\frac{5s-8}{s(s-4)}\right] = \frac{2}{s} + \frac{3}{s-4}$$

$$= \frac{2}{s} + 3\left[\frac{1}{s-4}\right]$$

$$= 2 + 3e^{4t}$$

(iv) $\dfrac{s-5}{s^2+4s+20}$

$$L^{-1}\left[\frac{s-5}{s^2+4s+20}\right]$$

$$F(s) = \frac{s-5}{s^2+4s+20} = \frac{s}{(s+2)^2+16} - \frac{5}{(s+2)^2+16}$$

3i $\dfrac{s-5}{(s-3)(s-4)}$

$$L^{-}\left[\frac{s-5}{(s-3)(s-4)}\right] = \frac{A}{(s-3)} + \frac{B}{(s-4)}$$

$$\frac{s-5}{(s-3)(s-4)} = \frac{A(s-4)+B(s-3)}{(s-3)(s-4)}$$

Assuming $s=4$

$4-5 = A(4-4) + B(4-3)$

$-1 = B(1)$

$B = -1$

Assuming $s=3$

$3-5 = A(3-4) + B(3-3)$

$-2 = A(-1)$

$A = 2$

$$\frac{s-5}{(s-3)(s-4)} = \frac{2}{s-3} - \frac{1}{s-4}$$

$$= 2\left[\frac{1}{s-3}\right] - \left[\frac{1}{s-4}\right]$$

$$= 2e^{3t} - e^{4t}$$

(ii) $\dfrac{2s-6}{(s-2)(s-4)}$

$$L^{-}\left[\frac{2s-6}{(s-2)(s-4)}\right]$$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{A}{(s-2)} + \frac{B}{(s-4)}$$

$2s-6 = A(s-4) + B(s-2)$

Assuming $s=4$

$2(4) - 6 = A(4-4) + B(4-2)$

$8-6 = A(0) + B(2) \Rightarrow 2 = 2B$

$B = 1$

$$= \frac{s+2-2}{(s+2)^2+16} - \frac{5}{(s+2)^2+16} = \frac{s+2}{(s+2)^2+16} - \frac{2}{(s+2)^2+6}$$

$$= \frac{s+2}{(s+2)^2+4^2} - \frac{7}{4} \cdot \frac{4}{(s+2)^2+4^2}$$

$$F(t) = e^{-2t}\cos 4t - \tfrac{7}{4} e^{-2t} \sin 4t$$
$$= e^{-2t}\left[\cos 4t - \tfrac{7}{4}\sin 4t\right]$$

(iv) $$\frac{s^2-3s-4}{(s-3)(s-1)^2}$$

$$F(s) = \frac{A}{s-3} + \frac{B}{(s-1)^2} + \frac{C}{(s-1)}$$

$$A: \left.\frac{s^2-3s-4}{(s-1)^2}\right|_{s=3} = \frac{3^2-3(3)-4}{(3-1)^2} = -1$$

$$B: \left.\frac{s^2-3s-4}{(s-1)^2}\right|_{s=1} = \frac{1^2-3(1)-4}{1-3} = 3$$

$$C: \frac{d}{ds}\left[\frac{s^2-3s-4}{s-3}\right]_{s=1} = \frac{(s-3)(2s-3)-[s^2-3s-4)}{(s-3)^2}$$

$$= \frac{(1-3)(2(1)-3)-[1^2-3(1)-4]}{(1-3)^2} = C$$

$$F(s) = -\frac{1}{s-3} + \frac{3}{(s-1)^2} + \frac{2}{s-1}$$

$$F(t) = -e^{-3t} + 3te^{t} + 2e^{t}$$
$$= e^{t}[3t+2] - e^{3t}$$