NAME: OPARA NDAMENIUM
MATRIC: 15/ENG06/057
DEPT: MECHANICAL

1) $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)y'' - 2xy' + 2y = 0$

Sub 1

$u = y''$; $u^n = y^{(n+2)}$

$v = 1-x^2$; $v' = -2x$; $v'' = -2$

$y^{(n)} = y^{(n+2)}\cdot(1-x^2 + ny^{(n+1)})\cdot -2x + \frac{n(n-1)}{2}y^n \cdot -2$

$y^n = (1-x^2)y^{n+2} - 2nxy^{(n+1)} - n(n-1)y^n$

Sub 2

$u' = y'$ $\quad u^n = y^{n+1}$

$v = -2x$ $\quad v' = -2$

$y^{(n)} = -2xy^{(n+1)} + ny^{(n)}\cdot -2$

$y^{(n)} = -2xy^{n+1} - 2ny^n$

Sub 3

$u = y$; $u^n = y^n$

$v = 2$

$y^n = 2\cdot y^{(n)} = 2y^n$

Combine sub 1, 2 and 3

$(1-x^2)y^{(n+2)} - 2nxy^{(n+1)} - n(n-1)y^n - 2xy^{(n+1)} - 2ny^{(n)} + 2y^{(n)} = 0$

$(1-x^2)y^{n+2} + (-2nx - 2x)y^{n+1} + (-n^2 + n - 2ny + 2)y^n = 0$

$(1-x^2)y^{n+2} - (2nx + 2x)y^{n+1} - (n^2 + n - 2)y^n = 0$

at $x = 0$

$y^{(n+2)} - 0 - (n^2+n-2)y^n = 0$

$y^{(n+2)} = (y^n)_0(n^2+n-2)$

when $n = 0$

$(y^2)_0 = (y^0)_0(-2) = -2(y^0)_0$

when $n = 1$

$(y^{(3)})_0 = (y^{(1)})_0(0) = 0$

when $n = 2$

$(y^4)_0 = (y^2)_0(4) = 4\times -2(y^0)_0 = -8(y^0)_0$

when $n = 3$

$(y^5)_0 = (y^3)_0(10) = 10(y^{(3)})_0 = 10\times 0 = 0$

when $n = 4$

$(y^6)_0 = (y^4)_0(18) = 18(y^{(4)})_0 \times -8(y)_0 = -144(y)_0$

when $n = 5$

$(y^{(7)})_0 = (y^5)(28) = 28\times 0 = 0$

Using Maclaurin

$y = (y)_0 + x(y')_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^{(4)})_0$

$y = (y_0) + x(y^{(1)})_0 - x^2(y^{(0)})_0 + 0 + -\frac{x^4}{3}(y^0)_0 \cancel{+ 0 - \frac{x^6}{5}(y)_0 + 0}$

$+ 0 - \frac{x^6}{5}(y^{(0)})_0 + 0 - \frac{x^8}{7}(y^{(0)})_0 + \ldots$

$y = (y)_0\left\{1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5} - \frac{x^8}{7}\right\} + (y')_0(x)$

2i) $L[3e^{-4t} - 5e^{4t}]$

$\frac{3}{s+4} - \frac{5}{s-4} = \frac{3(s-4) - 5(s+4)}{(s+4)(s-4)}$

$= \frac{3s - 12 - 5s - 20}{s^2 - 16} = \frac{-2s - 32}{s^2 - 16}$

~~ii)~~

ii) $L[\sin 4t + \cos 4t]$

$$\frac{4}{s^2+4^2} + \frac{s}{s^2+4} = \frac{4+s}{s^2+16}$$

iii) $L[t^3 + 2t^2 - t + 4]$

$$= \frac{3!}{s^4} + \frac{2 \cdot 2!}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

$$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

$$= \frac{6 + 4s - s^2 + 4s^3}{s^4}$$

iv) $L[e^{-2t} \cos 5t]$

$$\cos 5t = \frac{s}{s^2+5^2}$$

let $s = s+2$

$$L[e^{-2t}\cos 5t] = \frac{s+2}{(s+2)^2+5} = \frac{s+2}{s^2+4s+29}$$

v) $L[t \sin 3t]$

$$L[\sin 3t] = \frac{3}{s^2+3^2} = \frac{3}{s^2+9}$$

$$-F'(s) = \frac{-d}{ds}\left[\frac{3}{s^2+9}\right]$$

$u = 3$ $\quad du/ds = 0$

$v = s^2+9$ $\quad dv/ds = 2s$

using quotient rule

$$\frac{v\,du/ds - u\,dv/ds}{v^2}$$

$$= \frac{(s^2+9)(0) - 3(2s)}{(s^2+9)^2} = \frac{0-6s}{(s^2+9)^2}$$

$$= -\frac{6s}{(s^2+9)^2}$$

vi) $\dfrac{e^{-t} - e^{-2t}}{t}$

differentiating

$$\frac{-e^{-t} + 2e^{-2t}}{1} = e^{-2t}$$

at $t = 0$

$\Rightarrow = e^{-2(0)} = 1$

vii) $L[e^{4t} \cos 2t]$

$$L[\cos 2t] = \frac{s}{s^2+2^2} = \frac{s}{s^2+4}$$

let $s = s-4$

$$L(e^{4t}\cos 2t) = \frac{s-4}{(s-4)^2+4} = \frac{s-4}{s^2-8s-12}$$

viii) $L[t \sin 2t]$

$$L[\sin 2t] = \frac{2}{s^2+2^2} = \frac{2}{s^2+4}$$

$$L[t \sin 2t] = -F'(s) = \frac{-d}{ds}\left[\frac{2}{s^2+4}\right]$$

$u = 2$ $\quad du/ds = 0$

$v = s^2+4$ $\quad dv/ds = 2s$

$$-\frac{s^2+4(0) - 2(2s)}{(s^2+4)^2} = \frac{4s}{(s^2+4)^2}$$

ix) $L[t^3 + 4t^2 + 5]$

$$= \frac{3!}{s^4} + \frac{4 \cdot 2!}{s^3} + \frac{5}{s}$$

$$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s} = \frac{6+8s+5s^3}{s^4}$$

x) $L[e^{3t}(t^2+4)]$

$L[t^2 e^{3t}] + L[4e^{3t}]$

Scanned by CamScanner

$L[t^2] = \frac{2!}{s^{2+1}} = \frac{2}{s^3}$

$s = s-3$

$L[t^2 e^{3t}] = \frac{2}{(s-3)^3}$

$L[e^{3t}(t^2+4)] = \frac{2}{(s-3)^3} + \frac{4}{s-3}$

$= \frac{2+4(s-3)^2}{(s-3)^3} = \frac{2+4s^2-24s+36}{(s-3)^3}$

$= \frac{4s^2-24s+38}{(s-3)^3}$

xi) $L[t^2 \cos t]$

$L[\cos t] = \frac{s}{s^2+1^2} = \frac{s}{s^2+1}$

$-F'(s) = -\frac{d}{ds}\left[\frac{s}{s^2+1}\right]$

$u = s \qquad du/ds = 1$

$v = s^2+1 \qquad dv/dx = 2s$

$= \frac{(s^2+1)(1) - s(2s)}{(s^2+1)^2} = \frac{s^2+1-2s^2}{(s^2+1)^2}$

$L^{-1}[t \cos t] = \frac{-s^2+1}{(s^2+1)^2} = \frac{-1(s^2+1)}{(s^2+1)^2}$

$= \frac{-1}{s^2+1}$

$L[t^2 \cos t] = -\frac{d}{ds}\left[\frac{1}{s^2+1}\right]$

$u = 1 \qquad du/ds = 0$

$v = s^2+1 \qquad dv/ds = 2s$

$\frac{(s^2+1)(0)-(1)(2s)}{(s^2+1)^2} = \frac{-2s}{(s^2+1)^2}$

3) $\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$

$A(s-4) + B(s-3) = s-5$

at $s = 4$

$B(1) = -1$

$B = -1$

at $s = 3$

$A(-1) = -2$

$A = 2$

$= \frac{2}{s-3} - \frac{1}{s-4} = 2e^{3t} - e^{4t}$

b) $\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$

$A(s-4) + B(s-2) = 2s-6$

at $s=4$

$B(2) = 2$

$B = 1$

at $s = 2$

$A(-2) = -2$

$A = 1$

$\frac{1}{s-2} + \frac{1}{s-4} = e^{2t} + e^{4t}$

c) $\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$A(s-4) + Bs = 5s-8$

at $s=0$

$A(-4) = -8$

$A = 2$

at $s = 4$

$4B = 12$

$B = 3$

$$= \frac{2}{s} + \frac{3}{s-4} \Rightarrow 2 + 3e^{4t}$$

d) $\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{A}{s-3} + \frac{B}{s-1} + \frac{C}{(s-1)^2}$

$A(s-1)^2 + B(s-3)(s-1) + C(s-3) = s^2-3s-4$

at $s=1$

$0 + 0 + C(-2) = 1^2 - 3(1) - 4 = 1-7 = -6$

$-2C = -6$

$C = 3$

at $s=3$

$A(2)^2 + 0 + 0 = 3^2 - 3(3) - 4 = 9-9-4 = -4$

$4A = -4$

$A = 1$

$A+B = 1$

$-B = 1-A = 1-(-1) = 1+1 = 2$

$$\Rightarrow \frac{-1}{s-3} + \frac{2}{s-1} + \frac{3}{(s-1)^2}$$

$$\Rightarrow -e^{3t} + 2e^{t} + 3e^{t}$$

e) $\frac{s-5}{s^2+4s+20} = \frac{s-5}{(s+2)^2+4^2}$

$$= \frac{A}{s+2} + \frac{B}{(s+2)^2} + \frac{C}{16}$$

$= A(s+2)(16) + B(16) + C(s+2)^2 = s-5$

$= (16s+32)A + 16B + (s^2+4s+4)C = s-5$

Comparing Coefficients

$C = 0$

$16A + 4C = 1$

$32A + 16B + 4C = -5$

$\therefore C = 0$

$16A + 4C = 1$

$16A + 4(0) = 1$

$A = 1/16$

$32A + 16B + 4C = -5$

$32\left(\frac{1}{16}\right) + 16B + 4(0) = -5$

$2 + 16B = -5$

$16B = -5-2$

$B = \frac{-7}{16}$

$$\Rightarrow \frac{1}{16(s+2)} - \frac{7}{16(s+2)^2} + 0$$

$$= \frac{1}{16}e^{-2t} - \frac{7}{16}te^{2t}$$