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15/ENG01/003
CHEMICAL ENGINEERING
Assignment 4

1 $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x)^2 y'' - 2xy' + 2y = 0$

let $w_1 = (1-x^2)y''$

$u = y^2$ $\quad v = 1-x^2$

$u^n = y^{n+2}$ $\quad v' = -2x$

$u^{n-1} = y^{n+1}$ $\quad v'' = -2$

$v''' = 0$

$$u^n v + nu^{n-1}v' + \frac{n(n-1)}{2!}u^{n-1}v''$$

$$= y^{n+2}(1-x^2) + n(y^{n+1})(-2x) + \frac{n(n-1)}{2!}y^n(-2)$$

$$= (1-x)^2 y^{n+2} - 2xny^{n+1} + (-n^2+n)y^n$$

let $w_2 = -2xy'$

$u = y'$ $\quad v = -2x$

$u^n = y^{n+1}$ $\quad v' = -2$

$v'' = 0$

$= u^n v + nu^{n-1}v'$

$= y^{n+1}(-2x) + ny^n(-2)$

$= -2xy^{n+1} - 2ny^n$

let $w_3 = 2y$

$u = y$ $\quad v = 2$

$u^n = \cancel{y}y^n$ $\quad v' = 0$

$= 2y^n$

$y^n = (1-x^2)y^{n+2} - 2xny^{n+1} + (n-n^2)y^n - 2xy^{n+1} - 2ny^n + 2y^n$

$(1-x)^2 y^{n+2} + \cancel{y}y^{n+1})(-2xn-2x) + y^n(n-n^2-2n+2)$

$(1-x^2)y^{n+2} - 2xy^{n+1}(n+1) + y^n(-n^2-n+2) = 0$

At $x=0$

$(1-0)y^{n+2} + y^n(-n^2-n+2)=0$

$(y^{n+2})_0 = -(y^n)_0(-n^2-n+2)=0$

$(y^{n+2})_0 = (y^n)_0(n^2+n-2)$

At $n=0$

$(y^2)_0 = -(y^0)_0(-0-0+2)$

$= -(y^0)_0(2)$

$= -2(y)_0$

At $n=1$

$(y^3)_0 = -(y')_0(-1-1+2)$

$= -y'_0(0)$

$=0$

At $n=2$

$(y^4)_0 = (y^2)_0(2^2+2-2$

$=4(y^2)_0 = 4(-2)(y)_0$

At $n=3$

$(y)^5_0 = (y^3)_0(9+3-2)$

$= y^3)_0(10)$

$=0(10)=0$

At $n=4$

$(y^6)_0 = (y^4)_0(16+4-2)$

$= (y^4)_0(18)$

$=18(4)(-2)(y)_0$

At $n=5$

$(y^7)_0 = 0$

$$y^n = (y)_0 + x(y')_0 + \frac{x^2}{2!}(y^n)_0 + \frac{x^3}{3!}(y''')_0 + \frac{x^4}{4!}(y^{(4)})_0 + \frac{x^5}{5!}(y^{(5)})_0 +$$

$$y^n = (y)_0 + x(y')_0) + \left[\frac{x^2}{2}(-2)(y)_0\right] + \left[\frac{x^3}{3!}(0)\right] + \left[\frac{x^4}{4\times3\times2} \times (4)(-2)(y)_0\right]$$

$$+ \left[\frac{x^5}{5!}(0)\right] + \left[\frac{x^6}{6\times5\times4\times3\times2}(18)(4)(-2)(y_0)\right]$$

$$y^n = (y)_0 + x(y')_0 + \left[-2x^2(y)_0\right] + 0 + \left[\frac{-x^4}{3}(y)_0\right] + 0 + \left[\frac{-x^6}{5}(y)_0\right]$$

$$y^n = (y)_0 + x(y')_0 - 2x^2(y)_0 - \frac{x^4}{3}(y)_0 - \frac{x^6}{5}(y)_0$$

$$y^n = (y)_0\left[1 - 2x^2 - \frac{x^4}{3} - \frac{x^6}{5} + \dots\right] + x(y')_0$$

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2 $3e^{-4t} - 5e^{4t}$

$= \frac{3}{s+4} - \frac{5}{s-4}$

$= \frac{3(s-4) - 5(s+4)}{(s+4)(s-4)}$

$= \frac{3s-12-5s-20}{(s+4)(s-4)}$

$= \frac{-2s-32}{s^2-16}$

b $\sin 4t + \cos 4t$

$= \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2}$

$= \frac{4}{s^2+16} + \frac{s}{s^2+16}$

$= \frac{4+s}{s^2+16}$

c $t^3 + 2t^2 - t + 4$

$t^n = \frac{n!}{s^{n+1}}$

$= \frac{3!}{s^4} + \frac{2(2!)}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

$= \frac{6+4s-s^2+4s^3}{s^4}$

d $e^{-2t}\cos 5t$

$= \frac{s+2}{(s+2)^2+25}$

e $t\sin 3t$

$= \frac{(-1)^1 \, dF(s)}{ds^2}$

$F(s) = \frac{3}{s^2+3^2} = \frac{3}{s^2+9}$

$\frac{dF(s)}{ds^2}$ using quotient rule

$u = 3 \quad v = s^2+9$

$du = 0 \quad dv = 2s$

$\frac{v^{du}/dx - u^{dv}/dx}{v^2}$

$= \frac{s^2+4(0) - 3(2s)}{(s^2+9)^2}$

$= \frac{-3(2s)}{(s^2+9)^2}$

$\frac{dF(s)}{ds} = \frac{-6s}{(s^2+9)^2}$

$(-1)\frac{dF(s)}{ds} = -\left(\frac{-6s}{(s^2+9)^2}\right)$

v.) $\frac{e^{-t} - e^{-2t}}{t}$

$= \frac{e^{-t}}{t} - \frac{e^{-2t}}{t}$

For $L\left\{\frac{e^{-t}}{t}\right\}$

taking limit at $t=0$

$= \frac{e^{-0}}{0}$ = undefined

applying L'hopital rule

$= \frac{-e^{-t}}{1} = \frac{-e^{0}}{1} = \frac{0}{1} = 0$

$\int_0^\infty f(s)$

$f(s) = \frac{1}{s+1}$

$\int_0^\infty \frac{1}{\sigma+1} \, d\sigma$

$= \ln(\sigma+1)\Big|_{s}^{\infty}$

$= \ln(\infty+1) - \ln(s+1)$

$\ln\infty - \ln(s+1)$

$= \ln \frac{\infty}{s+1}$

Also for $\frac{e^{-2t}}{t}$

$F(s) = \frac{1}{s+2}$

$\int_{\sigma=s}^{\infty} \frac{1}{\sigma+2}\, d\sigma$

$= \ln(\sigma+2)\Big|_{\sigma=s}^{\infty}$

$= \ln(\infty+2) - \ln(s+2)$

$= \ln\infty - \ln(s+2$

$= \ln\left(\frac{\infty}{s+2}\right)$

$\therefore L\left\{\frac{e^{-t}}{t} - \frac{e^{-2t}}{t}\right\}$

$= \ln\left(\frac{\infty}{s+1}\right) - \ln\left(\frac{\infty}{s+2}\right)$

$= \ln\left(\frac{\infty}{s+1} \div \frac{\infty}{s+2}\right)$

$= \ln\left(\frac{\infty}{s+1} \times \frac{s+2}{\infty}\right)$

$= \ln\left(\frac{s+2}{s+1}\right)$

vii $e^{4t}\cos 2t$

$= \frac{(s-4)}{(s-4)^2+2^2}$

$= \frac{s-4}{s^2-8s+16+4} \quad = \frac{s-4}{s^2-8s+20}$

viii $t\sin 2t$

$= (-1)\frac{dF(s)}{ds^2}$

$F(s) = \frac{2}{s^2+2^2} = \frac{2}{s^2+4}$

using quotient rule

$u = 2 \qquad v = s^2+4$

$du = 0 \qquad dv = 2s$

$\frac{v\,du/dx - u\,dv/dx}{v^2}$

$= \frac{(s^2+4)(0) - 2(2s)}{(s^2+4)^2}$

$= \frac{-4s}{(s^2+4)^2}$

$(-1)\frac{dF(s)}{ds^n} = -\left(\frac{-4s}{(s^2+4)^2}\right)$

$= \frac{4s}{(s^2+4)^2}$

ix) $t^3 + 4t^2 + 5$

$= \frac{3!}{s^4} + \frac{4(2!)}{s^3} + \frac{5}{s}$

$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$

$= \frac{6+8s+5s^3}{s^4}$

$= \frac{1}{s^4}(5s^3+8s+6)$

x) $e^{3t}(t^2+4)$

$= t^2e^{3t} + 4e^{3t}$

$= \frac{2}{(s-3)^3} + \frac{4}{s-3}$

$= \frac{2+4(s-3)^2}{(s-3)^3}$

xi) $t^2\cos t$

$= (-1)^2\frac{d^2F(s)}{ds^2}$

$F(s) = \frac{s}{s^2+1^2} = \frac{s}{s^2+1}$

$\frac{df(s)}{ds}$ using quotient rule

let $u = s$ $\quad v = s^2+1$

$du = 1$ $\quad dv = 2s$

$= \frac{s^2+1(1) - s(2s)}{(s^2+1)^2}$

$= \frac{s^2+1-2s^2}{(s^2+1)^2}$

$= \frac{-s^2+1}{(s^2+1)^2}$

$\frac{d^2 f(s)}{ds^2}$

let $u = -s^2+1$ $\quad v = u^2$

$du = -2s$ $\quad dv = 2u$

$dv = \frac{du}{ds} \times \frac{dv}{du}$

$= 2s(2u)$

$= 4su$

$dv = 4s(s^2+1)$

$\therefore \frac{(s^2+1)^2(-2s) - (-s^2+1)(s^2+1)4s}{(s^2+1)^4}$

$= \frac{\cancel{(s^2+1)}[s^2+1(-2s) - (-s^2+1)4s]}{(s^2+1)^{\cancel{4}3}}$

$= \frac{(-2s)(s^2+1) - 4s(-s^2+1)}{(s^2+1)^3}$

$= \frac{-2s^3 - 2s + 4s^3 - 4s}{(s^2+1)^3}$

$= \frac{2s^3 - 6s}{(s^2+1)^3}$

$= \frac{2s(s^2-3)}{(s^2+1)^3}$

$L\{t^2 \cos t\} = \frac{2s(s^2-3)}{(s^2+1)^3}$

i) $\frac{\sinh 2t}{t}$

using L'hopital rule

$\frac{2\cosh 2t}{1}$

taking limit as $t \to 0$

$\frac{2\cosh 0}{1} = 2$

$= \int_{\sigma=s}^{\infty} f(s)$

$f(s) = \frac{2}{s^2-2^2} = \frac{2}{s^2-4}$

$= \int_{\sigma=s}^{\infty} \frac{2}{\sigma^2-4}$

$= 2\int_{\sigma=s}^{\infty} \frac{1}{\sigma^2-4}\, d\sigma$

$= 2\left(\frac{1}{2(2)}\right) \ln\left[\frac{\sigma-2}{\sigma+2}\right]$

$= \frac{1}{2} \ln\left[\frac{\sigma-2}{\sigma+2}\right]\Bigg|_{\sigma=s}^{\infty}$

$= \frac{1}{2}\ln\left(\frac{\infty-2}{\infty+2}\right) - \frac{1}{2}\ln\left[\frac{s-2}{s+2}\right]$

$= \frac{1}{2}\ln\left(\frac{\infty}{\infty}\right) - \frac{1}{2}\ln\left[\frac{s-2}{s+2}\right]$

$= \ln 1 - \frac{1}{2}\ln\left[\frac{s-2}{s+2}\right]$

$= \ln\left[\frac{s+2}{s-2}\right]^{-1/2}$

3) Convert the following functions to time domain

a) $\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$

$s-5 = A(s-4) + B(s-3)$

At $s=4$

$4-5 = A(0) + B(1)$

$-1 = B$

At $s=3$

$3-5 = A(3-4) + B(3-3)$

$-2 = A(-1)$

$A = 2$

$\therefore \frac{2}{s-3} - \frac{1}{s-4}$

$= 2e^{3t} - e^{-4t}$

ii) $\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$

$2s-6 = A(s-4) + B(s-2)$

$2s-6 = As - 4A + Bs - 2B$

$A + B = 2 \quad --- ①$

$-4A - 2B = -6 \quad --- ②$

from ①

$A = 2 - B \quad --- ③$

substituting ③ into ②

$-4(2-B) - 2B = -6$

$-8 + 4B - 2B = -6$

$-8 + 2B = -6$

$2B = 2$

$B = 1$

substituting $B=1$ into eqn ③

$A = 2 - B$

$= 2 - 1$

$A = 1$

$= \frac{1}{s-2} + \frac{1}{s-4}$

$= e^{2t} + e^{4t}$

iii) $\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$5s-8 = A(s-4) + Bs$

At $s=0$

$-8 = A(-4)$

$A = 2$

At $s=4$

$20-8 = 4B$

$12 = 4B$

$B = 3$

$= \frac{2}{s} + \frac{3}{s-4}$

$= 2 + 3e^{4t}$

iv) $\frac{s^2 - 3s - 4}{(s-3)(s-1)^2} = \frac{A}{s-3} + \frac{B}{s-1} + \frac{C}{s-1}$

$s^2 - 3s - 4 = A(s-1)^2 + B(s-3)(s-1) + C(s-3)$

At $s=1$

$1 - 3(1) - 4 = A(1-1)^2 + B(1-3)(1-1) + C(1-3)$

$1-3-4 = -2C$

$-6 = -2C$

$\cancel{C = 1/2} \quad C = 3$

At $s = 3$

$3(3) - 4 = A(3-1)^2 + B(3-3)(3-1) + C(3-3)$

$9-9-4 = A(2)^2$

$-4 = 4A$

$\cancel{A = 5/4}$

$A = -1$

$s^2 - 3s - 4 = A(s^2 - 2s + 1) + B(s^2 - 4s + 3) + C(s-3)$

$s^2 - 3s - 4 = As^2 - 2As + A + Bs^2 - 4Bs + 3B + Cs - 3C$

$A + B = 1 \quad --- ①$

$-2A - 4B + C = -3$

$A + 3B - 3C = -4$

$= 1-(-1)$

$= 2$

$$= \frac{-1}{s-3} + \frac{2}{s+1} + \frac{3}{(s+1)^2}$$

$$= -e^{3t} + 2e^{-t} + 3e^{-t}$$

v $$\frac{s-5}{s^2+4s+20} = \frac{s-5}{s^2+4s+4+16}$$

$$= \frac{s-5}{(s+2)^2+16}$$

$$= \frac{s-5}{(s+2)^2+4^2}$$

$$= \frac{s-5+2-2}{(s+2)^2+4^2}$$

$$= \frac{s+2}{(s+2)^2+4^2} + \left[\frac{-5-2}{(s+2)^2+4^2}\right]$$

$$= \frac{s+2}{(s+2)^2+4^2} - \frac{7}{(s+2)^2+4^2}$$

$$= \frac{s+2}{(s+2)^2+4^2} - \left[\frac{\cancel{7}}{(s+2)^2+4^2} \times \frac{7 \times 4}{4 \cancel{7}}\right]$$

$$= \frac{s+2}{(s+2)^2+4^2} - \left[\frac{4}{(s+2)^2+4^2} \times \frac{7}{4}\right]$$

$$= \frac{s+2}{(s+2)^2+4^2} - \frac{7}{4}\left[\frac{4}{(s+2)^2+4^2}\right]$$

$$= e^{-2t}\cos 4t - \frac{7}{4} e^{-2t} \sin 4t$$

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