QUESTION I :

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$$

$$(1-x^2)y'' - 2xy' + 2y = 0.$$

SUB 1

$u = y''$ $\quad u^n = y^{(n+2)}$

$v = 1-x^2$ $\quad v' = -2x$ $\quad v'' = -2$

$$y^{(n)} = y^{(n+2)} \cdot 1-x^2 + ny^{(n+1)} \cdot -2x + \frac{n(n-1)}{2} y^n \cdot -2$$

$$y^{(n)} = (1-x^2)y^{(n+2)} - 2nxy^{(n+1)} - n(n-1)y^n$$

SUB 2

$u = y'$ $\quad u^n = y^{(n+1)}$

$v = -2x$ $\quad v' = -2$

$$y^{(n)} = -2x\, y^{(n+1)} + ny^{(n)} \cdot -2$$

$$-2xy^{(n+1)} - 2ny^{(n)}.$$

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$$y^{(n)} \quad (1-x^2)y^{(n+2)} - 2nxy^{(n+1)} - n(n-1)y^n$$

SUB 2

$u = y'$ $\quad u^n = y^{(n+1)}$

$v = -2x$ $\quad v' = -2$

$$y^{(n)} = -2x\, y^{(n+1)} + ny^{(n)} \cdot -2$$

$$-2xy^{(n+1)} - 2ny^{(n)}$$

SUB 3 :

$u = y \qquad u^n = y^n$

$v = 2$

$y^{(n)} = 2 \cdot y^{(n)} = 2y^{(n)}$

Combinatia !

$(1-x^2)y^{(n+2)} - 2nxy^{(n+1)} - n(n-1)y^{(n)} - 2xy^{(n+1)} - 2ny^{(n)} + 2y^{(n)}$

$(1-x^2)y^{(n+2)} + (-2nx - 2x)y^{(n+1)} + (n^2 + n - 2n + 2)y^{(n)} = 0$

$(1-x^2)y^{(n+2)} - (2nx + 2x)y^{(n+1)} - (n^2 + n - 2)y^{(n)} = 0$

at $x = 0$

$y^{(n+2)} - 0 - (n^2 + n - 2)y^{(n)} = 0$

$y^{(n+2)} = (y^n)_0 (n^2 + n - 2)$

when $n = 0$

$(y^{(2)})_0 = (y^{0})_0 (-2) = -2(y^{(0)})_0$

when $n=1$

$$(y^{(3)})_0 = (y^{(1)})_0 (0) = 0$$

when $n=2$

$$(y^{(4)})_0 = (y^{(2)})_0 (4) = 4 \times -2(y^{(0)})_0 = -8(y^{(0)})_0$$

when $n=3$,

$$(y^{(5)})_0 = (y^{(3)})_0 (10) = 10(y^{(3)})_0 = 10 \times 0 = 0$$

when $n=4$,

$$(y^{(6)})_0 = (y^{(4)})_0 (18) = 18(y^{(4)})_0 = 18 \times -8(y^{(0)})_0$$
$$= -144(y^{(0)})_0$$

when $n=5$,

$$(y^{(7)})_0 = (y^{(5)})(28) = 28 \times 0 = 0$$

when $n=6$,

$$(y^{(8)})_0 = (y^{(6)})_0 (40) = 40 \times -144(y^{(0)})_0 = -5760(y^{(0)})_0.$$

$$y = (y)_0 + x(y')_0 + \frac{x^2}{2!}(y^{(2)})_0 + \frac{x^3}{3!}(y^{(3)})_0 + \frac{x^4}{4!}(y^{(4)})_0 + \cdots$$

$$y = (y)_0 + x(y^{(1)})_0 - x^2(y^{(0)})_0 + 0 + -\frac{x^4}{3}(y^{(0)})_0 + 0$$
$$- \frac{x^6}{5}(y^{(0)})_0 + 0 - \frac{x^8}{7}(y^{(0)})_0 + \cdots$$

$$y = (y)_0 \left[1 - x^2 - x^4/3 - x^6/5 - x^8/7\right] + (y')_0 (x)$$

Q: $L\{3e^{-4t} - 5e^{4t}]$

$$\frac{3}{s+4} - \frac{5}{s-4} = \frac{3(s-4) - 5(s+4)}{(s+4)(s-4)}$$

$$= \frac{3s - 12 - 5s - 20}{\cancel{s+4}\ s^2 - 16} = \frac{-2s - 32}{s^2 - 16}$$

ii $L[\sin 4t + \cos 4t]$

$$\frac{4}{s^2+4^2} + \frac{s}{s^2+4^2} = \frac{4+s}{s^2+16}$$

iii $L[t^3 + 2t^2 - t + 4]$

$$= \frac{3!}{s^4} + \frac{2 \cdot 2!}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

iii. $L[e^{-2t}\cos 5t]$

$\cos 5t = \frac{s}{s^2+5^2}$

let $s = s+a$

$L[e^{-2t}\cos 5t] = \frac{s+2}{(s+2)^2+5^2} = \frac{s+2}{s^2+4s+29}$

v. $L[t\sin 3t]$

$L[\sin 3t] = \frac{3}{s^2+3^2} = \frac{3}{s^2+9}$

$-F'(s) = -\frac{d}{ds}\left[\frac{3}{s^2+9}\right]$

$u = 3$ $\quad du/ds = 0$

$v = s^2+9$ $\quad dv/ds = 2s$

Using quotient rule

$\frac{v\,du/ds - u\,dv/ds}{v^2}$

$= \frac{(s^2+9)(0) - 3(2s)}{(s^2+9)^2} = \frac{0-6s}{(s^2+9)^2} = \frac{-6s}{(s^2+9)^2}$

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vi. $\dfrac{e^{-t} - e^{-2t}}{t}$

differentiating separately :

$\dfrac{-e^{-t} + 2e^{-2t}}{1} = e^{-2t}$

at $t = 0$

$\Rightarrow = e^{-2(0)} = 1$.

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xi: $t^2 \cos t$

$L[\cos t] = \frac{s}{s^2+1^2} = \frac{s}{s^2+1}$

$-f'(s) = \frac{-d}{ds}\left[\frac{s}{s^2+1}\right]$

$u = s \qquad du/ds = 1$

$v = s^2+1 \qquad dv/ds = 2s$

$\frac{v\, du/ds - u\, dv/ds}{v^2}$

$-\left(\frac{(s^2+1)(1) - s(2s)}{(s^2+1)^2}\right) = \left(\frac{s^2+1-2s^2}{(s^2+1)^2}\right)$

$L^{-1}[t\cos t] = \frac{-s^2+1}{(s^2+1)^2} = \frac{-1(s^2+1)}{(s^2+1)^2} = \frac{-1}{s^2+1}$

$= 1/s^2+1$

Diff Eindrat $L[t^2\cos t] = \frac{-d}{ds}\left[\frac{1}{s^2+1}\right]$

$u = 1 \qquad du/ds = 0$

$v = s^2+1 \qquad dv/ds = 2s$

$v\,du/ds \quad \frac{(s^2+1)(0) - (1)(2s)}{(s^2+1)^2} = \frac{-2s}{(s^2+1)^2}$

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Ex: $t^2 \cos t$

$$L[\cos t] = \frac{s}{s^2+1^2} = \frac{s}{s^2+1}$$

$$-f'(s) = \frac{-d}{ds}\left[\frac{s}{s^2+1}\right]$$

$u = s$ $\quad du/ds = 1$

$v = s^2+1$ $\quad dv/ds = 2s$

$$\frac{v\,du/ds - u\,dv/ds}{v^2}$$

$$-\left(\frac{(s^2+1)(1) - s(2s)}{(s^2+1)^2}\right) = \left(\frac{s^2+1-2s^2}{(s^2+1)^2}\right)$$

$$L^{-1}[t\cos t] = \frac{-s^2+1}{(s^2+1)^2} = \frac{-1(s^2+1)}{(s^2+1)^2} = \frac{-1}{s^2+1}$$

$$= 1/s^2+1$$

Ex: $\frac{\cancel{\cos 2t}}{t}$ $\quad L[t^2\cos t] = \frac{-d}{ds}\left[\frac{1}{s^2+1}\right]$

$u = 1$ $\quad du/ds = 0$

$v = s^2+1$ $\quad dv/ds = 2s$

$$\cancel{v\,du/ds} \quad \frac{(s^2+1)(0) - (1)(2s)}{(s^2+1)^2} = \frac{-2s}{(s^2+1)^2}$$

QUESTION 3.

1. $\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$

$A(s-4) + B(s-3) = s-5$

at $s = 4$

$B(1) = 4-5 = -1$

$B = -1$

at $s = 3$

$A(-1) + 0 = -2$

$-A = -2$

$A = 2$

$= \frac{2}{s-3} - \frac{1}{s-4}$

$\Rightarrow 2e^{3t} - e^{4t}$

$\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$

$A(s-4) + B(s-2) = 2s-6$

v. $\frac{s-5}{s^2+4s+20} = \frac{s-5}{(s+2)^2+4^2}$

$= \frac{A}{(s+2)} + \frac{B}{(s+2)^2} + \frac{C}{16^2}$

$= A(s+2)(16) + B(16) + C(s+2)^2 = s-5$

let s:

$= (16s+32)A + 16B + (s^2+4s+4)C = s-5$

$s^2 C = 0$

$s(16A+4C) = 1$

$32A + 16B + 4C = -5$

$\therefore C = 0$

$16A + 4C = 1$

$16A + 4(0) = 1$

$16A = 1$

$A = 1/16$

$32A + 16B + 4C = 5$

$32(1/16) + 16B + 4(0) = -5$

$2 + 16B = 5$

$16B = -7$

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$$= \frac{1}{16(s+2)} - \frac{7}{16}\frac{1}{(s+2)^2} + 0$$

$$= \frac{1}{16}e^{-2t} - \frac{7}{16}te^{2t}$$