

QUESTION 1:

$$(1-x^2)^{n/2} \left(\frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 2y \right) = 0$$

$$(1-x^2)y'' - 2xy' + 2y = 0$$

Sol 1

$$u = y''$$

$$u'' = y^{(n+2)}$$

$$v = 1-x^2$$

$$v' = -2x$$

$$v'' = -2$$

$$y^{(n+2)} = y^{(n+2)} \cdot (1-x^2) + n y^{(n+1)} \cdot (-2x) + \frac{n(n-1)}{2} y^{(n)} \cdot (-2)$$

$$y^{(n+2)} = (1-x^2)y^{(n+2)} - 2nxy^{(n+1)} - n(n-1)y^{(n)}$$

Sol 2

$$u = y'$$

$$u'' = y^{(n+1)}$$

$$v = -2x$$

$$v' = -2$$

$$y^{(n+1)} = -2x y^{(n+1)} + n y^{(n)} \cdot (-2)$$
$$= -2xy^{(n+1)} - 2ny^{(n)}$$

QUESTION 1

$$(1-x^2)^{n/2} \frac{dy}{dx} - 2x \frac{dy}{dx} + 2y = 0$$

$$(1-x^2)y'' - 2xy' + 2y = 0$$

SUB 1

$$u = y''$$

$$u'' = y^{(n+2)}$$

$$v = 1-x^2$$

$$v' = -2x$$

$$v'' = -2$$

$$y^{(n+2)} = y^{(n+2)} \cdot 1-x^2 + n y^{(n+1)} \cdot (-2x) + \frac{n(n-1)}{2} y^{(n)} \cdot (-2)$$

$$y^{(n+2)} (1-x^2) - 2nxy^{(n+1)} - n(n-1)y^{(n)}$$

SUB 2

$$u = y'$$

$$u'' = y^{(n+3)}$$

$$v = -2x$$

$$v' = -2$$

$$y^{(n+3)} = -2xy^{(n+3)} + n y^{(n+2)} \cdot (-2)$$

SUB 3:

$$u = y$$

$$v = 2$$

$$u' = y'$$

$$y^{(n)} = 2 \cdot y^{(n)} = 2y^{(n)}$$

Combinatorics!

$$(1-x^2)y^{(n+2)} - 2nxy^{(n+1)} - n(n-1)y^{(n)} - 2xy^{(n+1)}$$
$$2ny^{(n)} + 2y^{(n)}$$

$$(1-x^2)y^{(n+2)} + (-2nx - 2x)y^{(n+1)} + (n^2 + n - 2n)y^{(n)} = 0$$

$$(1-x^2)y^{(n+2)} - (2nx + 2x)y^{(n+1)} + (n^2 + n - 2n)y^{(n)} = 0$$

$$\text{at } x=0$$

$$y^{(n+2)} - 0 - (n^2 + n - 2n)y^{(n)} = 0$$

$$y^{(n+2)} = (y^{(n)})_0 (n^2 + n - 2)$$

when $n=0$

$$(y^{(2)})_0 = (y^{(0)})_0 (-2) = -2(y^{(0)})_0$$

when $n = 1$

$$(y^{(3)})_0 = (y^{(1)})_0 (0) = 0$$

when $n = 2$

$$(y^{(4)})_0 = (y^{(2)})_0 (4) = 4x - 2(y^{(2)})_0 = -8(y^{(0)})_0$$

when $n = 3$

$$(y^{(5)})_0 = (y^{(3)})_0 (10) = 10(y^{(1)})_0 = 10x = 0$$

when $n = 4$

$$(y^{(6)})_0 = (y^{(4)})_0 (18) = 18(y^{(2)})_0 = 18x - 8(y^{(0)})_0 = -144(y^{(0)})_0$$

when $n = 5$

$$(y^{(7)})_0 = (y^{(5)})_0 (28) = 28x = 0$$

when $n = 6$

$$(y^{(8)})_0 = (y^{(6)})_0 (40) = 40x - 144(y^{(0)})_0 = -5760(y^{(0)})_0$$

$$y = (y)_0 + x(y')_0 + \frac{x^2}{2!} (y^{(2)})_0 + \frac{x^3}{3!} (y^{(3)})_0 + \frac{x^4}{4!} (y^{(4)})_0 + \dots$$

$$y = (y)_0 + x(y')_0 + \frac{x^2}{2} (y^{(2)})_0 + 0 + \frac{-x^4}{3} (y^{(4)})_0 + 0 - \frac{x^6}{5} (y^{(6)})_0 + \dots$$

$$y = (y)_0 \left[1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5} - \frac{x^8}{7} \right] + (y')_0 (x)$$

$$a.) \{ 3e^{-4t} - 5e^{4t} \}$$

$$\frac{3}{s+4} - \frac{5}{s-4} = \frac{3(s-4) - 5(s+4)}{(s+4)(s-4)}$$

$$= \frac{3s - 12 - 5s - 20}{s^2 - 16} = \frac{-2s - 32}{s^2 - 16}$$

$$iv.) \{ \sin 4t + \cos 4t \}$$

$$\frac{4}{s^2 + 4^2} + \frac{s}{s^2 + 4^2} = \frac{4+s}{s^2 + 16}$$

$$v.) \{ t^3 + 2t^2 - t + 4 \}$$

$$= \frac{3!}{s^4} + \frac{2 \cdot 2!}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

$$ii. \mathcal{L}[e^{-2t} \cos 3t]$$

$$\cos 3t = \frac{s}{s^2 + 9}$$

$$\text{let } s = s + 2$$

$$\mathcal{L}[e^{-2t} \cos 3t] = \frac{s+2}{(s+2)^2 + 9} = \frac{s+2}{s^2 + 4s + 29}$$

$$iii. \mathcal{L}[t \sin 3t]$$

$$\mathcal{L}[\sin 3t] = \frac{3}{s^2 + 9} = \frac{3}{s^2 + 9}$$

$$-F'(s) = -\frac{d}{ds} \left[\frac{3}{s^2 + 9} \right]$$

$$u = 3$$

$$v = s^2 + 9$$

$$\frac{du}{ds} = 0$$

$$\frac{dv}{ds} = 2s$$

Using quotient rule

$$\frac{v \frac{du}{ds} - u \frac{dv}{ds}}{v^2}$$

$$= \frac{(s^2 + 9)(0) - 3(2s)}{(s^2 + 9)^2} = \frac{0 - 6s}{(s^2 + 9)^2} = \frac{-6s}{(s^2 + 9)^2}$$

vi. $\frac{e^{-t} - e^{-2t}}{t}$

t

differentiating separately:

$$\frac{-e^{-t} + 2e^{-2t}}{1} = e^{-2t}$$

1

at $t=0$

$$\Rightarrow e^{-2(0)} = 1$$

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = \frac{e^0 - 1}{0} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = \lim_{x \rightarrow 0} \frac{e^x}{1} = \frac{e^0}{1} = \frac{1}{1} = 1$$

$$\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{x} = \frac{e^{2 \cdot 0} - 1}{0} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{x} = \lim_{x \rightarrow 0} \frac{2e^{2x}}{1} = \frac{2e^{2 \cdot 0}}{1} = \frac{2 \cdot 1}{1} = 2$$

$$\lim_{x \rightarrow 0} \frac{e^{3x} - 1}{x} = \frac{e^{3 \cdot 0} - 1}{0} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{e^{3x} - 1}{x} = \lim_{x \rightarrow 0} \frac{3e^{3x}}{1} = \frac{3e^{3 \cdot 0}}{1} = \frac{3 \cdot 1}{1} = 3$$

$$\lim_{x \rightarrow 0} \frac{e^{4x} - 1}{x} = \frac{e^{4 \cdot 0} - 1}{0} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{e^{4x} - 1}{x} = \lim_{x \rightarrow 0} \frac{4e^{4x}}{1} = \frac{4e^{4 \cdot 0}}{1} = \frac{4 \cdot 1}{1} = 4$$

$$\lim_{x \rightarrow 0} \frac{e^{5x} - 1}{x} = \frac{e^{5 \cdot 0} - 1}{0} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{e^{5x} - 1}{x} = \lim_{x \rightarrow 0} \frac{5e^{5x}}{1} = \frac{5e^{5 \cdot 0}}{1} = \frac{5 \cdot 1}{1} = 5$$

$$\lim_{x \rightarrow 0} \frac{e^{6x} - 1}{x} = \frac{e^{6 \cdot 0} - 1}{0} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{e^{6x} - 1}{x} = \lim_{x \rightarrow 0} \frac{6e^{6x}}{1} = \frac{6e^{6 \cdot 0}}{1} = \frac{6 \cdot 1}{1} = 6$$

$$\lim_{x \rightarrow 0} \frac{e^{7x} - 1}{x} = \frac{e^{7 \cdot 0} - 1}{0} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{e^{7x} - 1}{x} = \lim_{x \rightarrow 0} \frac{7e^{7x}}{1} = \frac{7e^{7 \cdot 0}}{1} = \frac{7 \cdot 1}{1} = 7$$

$$\lim_{x \rightarrow 0} \frac{e^{8x} - 1}{x} = \frac{e^{8 \cdot 0} - 1}{0} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{e^{8x} - 1}{x} = \lim_{x \rightarrow 0} \frac{8e^{8x}}{1} = \frac{8e^{8 \cdot 0}}{1} = \frac{8 \cdot 1}{1} = 8$$

$$\lim_{x \rightarrow 0} \frac{e^{9x} - 1}{x} = \frac{e^{9 \cdot 0} - 1}{0} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{e^{9x} - 1}{x} = \lim_{x \rightarrow 0} \frac{9e^{9x}}{1} = \frac{9e^{9 \cdot 0}}{1} = \frac{9 \cdot 1}{1} = 9$$

$$\lim_{x \rightarrow 0} \frac{e^{10x} - 1}{x} = \frac{e^{10 \cdot 0} - 1}{0} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{e^{10x} - 1}{x} = \lim_{x \rightarrow 0} \frac{10e^{10x}}{1} = \frac{10e^{10 \cdot 0}}{1} = \frac{10 \cdot 1}{1} = 10$$

$$\lim_{x \rightarrow 0} \frac{e^{11x} - 1}{x} = \frac{e^{11 \cdot 0} - 1}{0} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{e^{11x} - 1}{x} = \lim_{x \rightarrow 0} \frac{11e^{11x}}{1} = \frac{11e^{11 \cdot 0}}{1} = \frac{11 \cdot 1}{1} = 11$$

$$\lim_{x \rightarrow 0} \frac{e^{12x} - 1}{x} = \frac{e^{12 \cdot 0} - 1}{0} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{e^{12x} - 1}{x} = \lim_{x \rightarrow 0} \frac{12e^{12x}}{1} = \frac{12e^{12 \cdot 0}}{1} = \frac{12 \cdot 1}{1} = 12$$

$$\lim_{x \rightarrow 0} \frac{e^{13x} - 1}{x} = \frac{e^{13 \cdot 0} - 1}{0} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{e^{13x} - 1}{x} = \lim_{x \rightarrow 0} \frac{13e^{13x}}{1} = \frac{13e^{13 \cdot 0}}{1} = \frac{13 \cdot 1}{1} = 13$$

$$\lim_{x \rightarrow 0} \frac{e^{14x} - 1}{x} = \frac{e^{14 \cdot 0} - 1}{0} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{e^{14x} - 1}{x} = \lim_{x \rightarrow 0} \frac{14e^{14x}}{1} = \frac{14e^{14 \cdot 0}}{1} = \frac{14 \cdot 1}{1} = 14$$

$$\lim_{x \rightarrow 0} \frac{e^{15x} - 1}{x} = \frac{e^{15 \cdot 0} - 1}{0} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{e^{15x} - 1}{x} = \lim_{x \rightarrow 0} \frac{15e^{15x}}{1} = \frac{15e^{15 \cdot 0}}{1} = \frac{15 \cdot 1}{1} = 15$$

$$\lim_{x \rightarrow 0} \frac{e^{16x} - 1}{x} = \frac{e^{16 \cdot 0} - 1}{0} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{e^{16x} - 1}{x} = \lim_{x \rightarrow 0} \frac{16e^{16x}}{1} = \frac{16e^{16 \cdot 0}}{1} = \frac{16 \cdot 1}{1} = 16$$

$$\lim_{x \rightarrow 0} \frac{e^{17x} - 1}{x} = \frac{e^{17 \cdot 0} - 1}{0} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{e^{17x} - 1}{x} = \lim_{x \rightarrow 0} \frac{17e^{17x}}{1} = \frac{17e^{17 \cdot 0}}{1} = \frac{17 \cdot 1}{1} = 17$$

$$\lim_{x \rightarrow 0} \frac{e^{18x} - 1}{x} = \frac{e^{18 \cdot 0} - 1}{0} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{e^{18x} - 1}{x} = \lim_{x \rightarrow 0} \frac{18e^{18x}}{1} = \frac{18e^{18 \cdot 0}}{1} = \frac{18 \cdot 1}{1} = 18$$

$$\lim_{x \rightarrow 0} \frac{e^{19x} - 1}{x} = \frac{e^{19 \cdot 0} - 1}{0} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{e^{19x} - 1}{x} = \lim_{x \rightarrow 0} \frac{19e^{19x}}{1} = \frac{19e^{19 \cdot 0}}{1} = \frac{19 \cdot 1}{1} = 19$$

$$\lim_{x \rightarrow 0} \frac{e^{20x} - 1}{x} = \frac{e^{20 \cdot 0} - 1}{0} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{e^{20x} - 1}{x} = \lim_{x \rightarrow 0} \frac{20e^{20x}}{1} = \frac{20e^{20 \cdot 0}}{1} = \frac{20 \cdot 1}{1} = 20$$

xi. $e^{\cos t}$

$$L[\cos t] = \frac{s}{s^2+1} - \frac{s}{s^2-1}$$

$$-f'(s) = \frac{-d}{ds} \left[\frac{s}{s^2+1} \right]$$

$$u = s \quad \frac{du}{ds} = 1$$

$$v = s^2+1 \quad \frac{dv}{ds} = 2s$$

$$\frac{d}{ds} \frac{u \frac{dv}{ds} - v \frac{du}{ds}}{v^2}$$

$$= \frac{(s^2+1)(1) - s(2s)}{(s^2+1)^2} = \frac{(s^2+1 - 2s^2)}{(s^2+1)^2}$$

$$L^{-1}[\cos t] = \frac{-s^2+1}{(s^2+1)^2} = -\frac{1(s^2+1)}{(s^2+1)^2} = \frac{-1}{s^2+1} = \frac{1}{s^2+1}$$

xii. $t e^{\cos t}$

$$L[t e^{\cos t}] = \frac{-d}{ds} \left[\frac{1}{s^2+1} \right]$$

$$u = 1$$

$$\frac{du}{ds} = 0$$

$$v = s^2+1$$

$$\frac{dv}{ds} = 2s$$

$$\frac{d}{ds}$$

$$\frac{(s^2+1)(0) - (1)(2s)}{(s^2+1)^2} = \frac{-2s}{(s^2+1)^2}$$

$$x: t^2 \cos t$$

$$L[\cos t] = \frac{s}{s^2+1} \quad \therefore \frac{s}{s^2+1}$$

$$-f'(s) = \frac{-d}{ds} \left[\frac{s}{s^2+1} \right]$$

$$u = s \quad \frac{du}{ds} = 1$$

$$v = s^2+1 \quad \frac{dv}{ds} = 2s$$

$$d \frac{v \frac{du}{ds} - u \frac{dv}{ds}}{v^2}$$

$$- \left(\frac{(s^2+1)(1) - s(2s)}{(s^2+1)^2} \right) = \left(\frac{s^2+1 - 2s^2}{(s^2+1)^2} \right)$$

$$L^{-1}[\cos] = \frac{-s^2+1}{(s^2+1)^2} = - \frac{1(s^2+1)}{(s^2+1)^2} = \frac{-1}{s^2+1} = \frac{1}{s^2+1}$$

$$x: t \cos t \quad L[t^2 \cos t] = \frac{-d}{ds} \left[\frac{1}{s^2+1} \right]$$

$$u = 1$$

$$v = s^2+1$$

~~$\frac{du}{ds}$~~

$$\frac{du}{ds} = 0$$

$$\frac{dv}{ds} = 2s$$

$$\frac{(s^2+1)(0) - (1)(2s)}{(s^2+1)^2} = \frac{-2s}{(s^2+1)^2}$$

QUESTION 3.

$$1. \frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$$

$$A(s-4) + B(s-3) = s-5$$

$$\text{at } s=4$$

$$B(1) = 4-5 = -1$$

$$B = -1$$

$$\text{at } s=3$$

$$A(-1) + 0 = -2$$

$$-A = -2$$

$$A = 2$$

$$= \frac{2}{s-3} - \frac{1}{s-4}$$

$$\Rightarrow \Rightarrow 2e^{3t} - e^{4t}$$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$$

$$A(s-4) + B(s-2) = 2s-6$$

$\frac{1}{x^2} = x^{-2}$
 $\frac{d}{dx} x^{-2} = -2x^{-3}$
 $= -2x^{-3}$
 $= -\frac{2}{x^3}$

$\frac{d}{dx} x^{-2} = -2x^{-3}$
 $= -\frac{2}{x^3}$
 $\frac{d}{dx} x^{-2} = -\frac{2}{x^3}$
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$\frac{d}{dx} x^{-2} = -2x^{-3}$
 $= -\frac{2}{x^3}$
 $\frac{d}{dx} x^{-2} = -\frac{2}{x^3}$
 $\frac{d}{dx} x^{-2} = -\frac{2}{x^3}$
 $\frac{d}{dx} x^{-2} = -\frac{2}{x^3}$

$$v. \quad \frac{s-5}{s^2+4s+20} = \frac{s-5}{(s+2)^2+4^2}$$

$$= \frac{A}{(s+2)} + \frac{B}{(s+2)^2} + \frac{C}{16}$$

$$= A(s+2)(16) + B(16) + C(s+2)^2 = s-5$$

Let $s =$

$$-(16s+32)A + 16B + (s^2+4s+4)C = s-5$$

$$s^2 C = 0 \quad \therefore C = 0$$

$$s(16A+4C) = 1 \quad \therefore 16A+4C = 1$$

$$32A+16B+4C = -5$$

$$\therefore C = 0$$

$$16A+4C = 1$$

$$16A+4(0) = 1$$

$$16A = 1$$

$$A = \frac{1}{16}$$

$$32A+16B+4C = -5$$

$$32\left(\frac{1}{16}\right) + 16B + 4(0) = -5$$

$$-2 + 16B = -5$$

$$16B = -3$$

$$= \frac{1}{16(s+2)} - \frac{7}{16(s+2)^2} + 0$$

$$= \frac{1}{16} e^{-2t} - \frac{7}{16} t e^{-2t}$$