

$$(1-x^2) \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 2y = 0$$

$$(1-x^2)y'' - 2xy' + 2y = 0$$

$$y^n = u^n v + n u^{n-1} u' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \dots$$

$$[y^{(2+n)} \cdot (1-x^2 + n(n-1)x^2) \cdot (-2x) + \frac{n(n-1)}{2!} y^{(n)} \cdot (-2)] + [y^{(n+2)} - 2x y^{(n+1)} - 2n y^n + 2y^{n+2}] = 0$$

$$(1-x^2)y^{n+2} - 2xny^{n+1} - n(n-1)y^n - 2xy^{n+1} - 2ny^n + 2y^{n+2} = 0$$

let $x=0$

$$y^{n+2} - n(n-1)y^n - 2ny^n + 2y^n = 0$$

$$y^{n+2} + y^n [-n(n-1) - 2n + 2] = 0$$

$$y^{n+2} + y^n [-n^2 + n - 2n + 2] = 0$$

$$y^{n+2} + y^n [-n^2 - n + 2] = 0$$

$$y^{n+2} = -y^n \cdot [-n^2 - n + 2]$$

$$n=0 : y^2 = y^0 \cdot 2 = -2y^0$$

$$n=1 : y^3 = -y^1 \cdot [0] = 0$$

$$n=2 : y^4 = -y^2 \cdot [-4] = 4y^2 = 4(-2y^0) = -8y^0$$

$$n=3 : y^5 = -y^3 \cdot [-10] = 10y^3 = 0$$

$$n=4 : y^6 = -y^4 \cdot [18] = 18y^4 = 18 \cdot -8y^0$$

$$n=5 : y^7 = -y^5 \cdot [28] = 28y^5 = 0$$

$$y = y^0 + xy^1 + \frac{x^2 y^3}{2!} + \frac{x^3 y^3}{3!} + \dots$$

$$y = y^0 + xy^1 + \frac{x^2}{2!} (-2)y^0 + \frac{x^3}{3!} (0) + \frac{x^4}{4!} (4)(-2)y^0 + \frac{x^5}{5!} (0) +$$

$$\frac{x^6}{6!} (18)(4)(-2)y^0 + \frac{x^7}{7!} (0)$$

$$y = y^0 \cdot [1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5}] + y^1 [x]$$

$$y = y^0 \left[1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5} \right] + y^1 [x]$$

2) $3e^{-4t} - 5e^{4t}$

$\Rightarrow L[3e^{-4t} - 5e^{4t}] \Rightarrow L[3e^{-4t}] - L[5e^{4t}]$

$\Rightarrow 3\left[\frac{1}{s-a}\right] - 5\left[\frac{1}{s-a}\right]$

$\Rightarrow 3\left[\frac{1}{s+4}\right] - 5\left[\frac{1}{s-4}\right] \Rightarrow \frac{3}{s+4} - \frac{5}{s-4}$

1) $\sin 4t + \cos 4t$

$L[\sin 4t + \cos 4t] \Rightarrow L[\sin 4t] + L[\cos 4t]$

$\Rightarrow \frac{a}{s^2+a^2} + \frac{s}{s^2+a^2} \Rightarrow \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2}$

$\Rightarrow \frac{4}{s^2+16} + \frac{s}{s^2+16} = \frac{4+s}{(s^2+16)}$

11) $t^2 + 2t^2 - t + 4$

$t^n = \frac{n!}{s^{n+1}} \Rightarrow \frac{3!}{s^{3+1}} + 2\left[\frac{2!}{s^{2+1}}\right] - \left[\frac{1!}{s^{1+1}}\right] + \frac{4}{s}$

$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

12) $e^{-2t} \cos 5t$

$L[\cos 5t] = \frac{s}{s^2+9^2}$

$\Rightarrow \frac{s}{s^2+5^2} \Rightarrow \frac{s}{s^2+25}$

$L[\cos 5t] = \frac{s+2}{(s+2)^2+25}$

v) $t \sin 3t$

$L[\sin 3t] = \frac{a}{s^2+a^2}$

$= \frac{3}{s^2+3^2} \Rightarrow \frac{3}{s^2+9}$

$L[t \sin 3t] = -F'(s)$

$u = 3 \quad \frac{du}{ds} = 0$

$v = s^2 + 9 \quad \frac{dv}{ds} = 2s$

$V \frac{du}{ds} - U \frac{dv}{ds}$

v^2

$= \frac{[s^2+9] \cdot 0 - 3(2s)}{[s^2+9]^2}$

$= \frac{-6s}{[s^2+9]^2}$

$= \frac{-6s}{[s^2+9]^2}$

~~$F'(s)$~~

$= F'(s) = -1 \left[\frac{-6s}{[s^2+9]^2} \right]$

$= \frac{6s}{[s^2+9]^2}$

$$vi) \frac{e^{-t} - e^{-2t}}{t} \Rightarrow L[F(t)] = e^{-s} - e^{-2s} \cdot \frac{1}{s+1} - \frac{1}{s+2}$$

$$= \frac{1}{s+1} - \frac{1}{s+2}$$

$$F(s) = \frac{1}{s+1} - \frac{1}{s+2}$$

$$\int F(s) L\left[\frac{F(t)}{t}\right] = \int_{-\infty}^{\infty} \frac{1}{s+1} - \frac{1}{s+2} ds$$

$$= \int_{-\infty}^{\infty} \frac{1}{s+1} ds - \int_{-\infty}^{\infty} \frac{1}{s+2} ds$$

$$\ln[(s+1)] - \ln[(s+2)]_s^{\infty}$$

$$= [\ln(s+1) - \ln(s+2)]_s^{\infty}$$

$$\Rightarrow \left[\ln \frac{s+1}{s+2} \right]_s^{\infty} = \ln \left[\frac{s+1}{s+2} \right] - \frac{s+1}{s+2}$$

$$= -\ln \left[\frac{s+2}{s+1} \right] = \ln \left[\frac{s+1}{s+2} \right]$$

$$vii) e^{4t} \cos 2t$$

$$L[\cos 2t] = \frac{s}{s^2+2^2} = \frac{s}{s^2+4}$$

$$L[e^{4t} \cos 2t] = \frac{s-4}{(s-4)^2+4}$$

$$viii) t \sin 2t$$

$$L[t \sin 2t] = \frac{-d}{ds} [F(s)]$$

$$F(s) = L[\sin 2t] = \frac{2}{s^2+2^2}$$

$$F(s) = \frac{2}{s^2+4}$$

$$u = 2 \quad \frac{du}{ds} = 0$$

$$v = s^2+4 \quad \frac{dv}{ds} = 2s$$

$$\frac{(s^2+4) \cdot 0 - 2(2s)}{(s^2+4)^2}$$

$$= \frac{-4s}{(s^2+4)^2}$$

$$L[t \sin 2t] = f'(s)$$

$$= -1 \cdot \frac{4s}{(s^2+4)^2}$$

$$= \frac{4s}{(s^2+4)^2}$$

$$ix) t^3 + 4t^2 + 5$$

$$L[t^3 + 4t^2 + 5] = \frac{3!}{s^{3+1}} + 4 \left[\frac{2!}{s^{2+1}} \right] + \frac{5}{s}$$

$$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$$

$$x) e^{3t} (t^2 + 4)$$

$$\text{let } x = t^2 + 4$$

$$L[e^{3t} x]$$

$$L[x] = L[t^2 + 4]$$

$$= L[t^2] + L[4]$$

$$= \frac{2!}{s^{2+1}} + \frac{4}{s}$$

$$= \frac{2}{s^3} + \frac{4}{s}$$

$$L[e^{3t}] = \frac{2}{[s-3]^3} + \frac{4}{[s-3]}$$

3) $\frac{s-5}{(s-3)(s-4)}$

N/D 3

$$L^{-1}\left[\frac{s-5}{(s-3)(s-4)}\right] = \frac{A}{s-3} + \frac{B}{s-4}$$

$$\frac{s-5}{(s-3)(s-4)} = \frac{A(s-4) + B(s-3)}{(s-3)(s-4)}$$

Take $s=4$

$$\Rightarrow \frac{4-5}{\cancel{(4-3)}(4-4)} = A(4-4) + B(4-3)$$

$$\Rightarrow -1 = B(1) \Rightarrow B = -1$$

Assuming $s=3$

$$\Rightarrow \frac{3-5}{(3-3)\cancel{(3-4)}} = A(3-4) + B(3-3)$$

$$\Rightarrow -2 = A(-1) \Rightarrow A = 2$$

$$\frac{s-5}{(s-3)(s-4)} = \frac{2}{s-3} + \frac{-1}{s-4} \Rightarrow 2\left[\frac{1}{s-3}\right] - \left[\frac{1}{s-4}\right]$$

$$= 2e^{3t} - e^{4t}$$

ii) $\frac{2s-6}{(s-2)(s-4)} \Rightarrow L^{-1}\left[\frac{2s-6}{(s-2)(s-4)}\right]$

$$\Rightarrow \frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4} \Rightarrow 2s-6 = A(s-4) + B(s-2)$$

Assuming $s=4$

$$2(4)-6 = A(4-4) + B(4-2) \Rightarrow 2 = 2B \Rightarrow B = 1$$

Assuming $s=2$

$$2(2)-6 = A(2-4) + B(2-2) \Rightarrow -2 = -2A \Rightarrow A = 1$$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{1}{s-2} + \frac{1}{s-4}$$

$$L^{-1}\left[\frac{2s-6}{(s-2)(s-4)}\right] = e^{2t} + e^{4t}$$

iii) $\frac{5s-8}{5(s-4)}$

$$L^{-1}\left[\frac{5s-8}{5(s-4)}\right] = \frac{A}{s} + \frac{B}{s-4}$$

$$\Rightarrow 5s-8 = A(s-4) + B(s)$$

Assuming $s=4$

$$5(4)-8 = A(4-4) + B(4) \Rightarrow 12 = 4B \Rightarrow B = 3$$

Assuming $s=0$

$$5(0)-8 = A(0-4) + B(0) \Rightarrow -8 = -4A \Rightarrow A = 2$$

$$L^{-1}\left[\frac{5s-8}{5(s-4)}\right] = \frac{2}{s} + \frac{3}{s-4}$$

$$= \frac{2}{s} + 3\left[\frac{1}{s-4}\right]$$

$$= 2 + 3e^{4t}$$

$$V) \frac{1}{s^2+4s+20}$$

$$L^{-1} \left[\frac{s-5}{s^2+4s+20} \right] \Rightarrow F(s) = \frac{s-5}{s^2+4s+20} \Rightarrow \frac{s}{(s+2)^2+16} - \frac{5}{(s+2)^2+16}$$

$$= \frac{s+2-2}{(s+2)^2+16} - \frac{5}{(s+2)^2+16} = \frac{s+2}{(s+2)^2+16} - \frac{7}{(s+2)^2+16}$$

$$= \frac{s+2}{(s+2)^2+4^2} - \frac{7}{4} \cdot \frac{4}{(s+2)^2+4^2}$$

$$f(t) = e^{-2t} (\cos 4t - \frac{7}{4} e^{-2t} \sin 4t)$$

$$\Rightarrow e^{-2t} [\cos 4t - \frac{7}{4} \sin 4t]$$

$$V) \frac{s^2 - 3s - 4}{(s-3)(s-1)^2}$$

$$\Rightarrow T(s) = \frac{A}{s-3} + \frac{B}{(s-1)^2} + \frac{C}{s-1}$$

$$A = \frac{s^2 - 3s - 4}{(s-1)^2} \Big|_{s=3} = \frac{3^2 - 3(3) - 4}{(3-1)^2} = -1$$

$$B = \frac{s^2 - 3s - 4}{(s-3)} \Big|_{s=1} = \frac{1^2 - 3(1) - 4}{1-3} = 3$$

$$C = \frac{1}{dc} \left[\frac{s^2 - 3s - 4}{s-3} \right]_{s=1} = \frac{(s-3)(2s-3) - (s^2 - 3(1) - 4)}{(s-3)^2}$$

$$= \frac{(1-3)(2(1)-3) - (1^2 - 3 - 4)}{(1-3)^2} = \frac{8}{4} = 2$$

$$F(s) = \frac{-1}{s-3} + \frac{3}{(s-1)^2} + \frac{2}{s-1}$$

$$f(t) = -e^{-3t} + 3te^t + 2e^t$$

$$\Rightarrow e^t [3t + 2] - e^{-3t}$$