SUNNY OGHENERUME COLLINS

16/ENG04/070

ELECT/ELECT ENG381

ASSIGNMENT 4

(i) $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)y'' - 2xy' + 2y = 0$

let $w_1 = (1-x^2)y''$

$u = y^{(2)}$ $\quad v = (1-x^2)$

$u^n = y^{n+2}$ $\quad v' = -2x$

$u^{n-1} = y^{n+2}$ $\quad v' = -2$

let $w_2 = 2xy'$

$u = 2y'$ $\quad v = 2x$

$u^{n-1} = y^n$ $\quad v' = 2$

let $w_3 = 2y$

$u^n = 2y^n$

Using leibnitz theorem

$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v''$

For w_1

$y^{n+2}(1-x^2) + n(y^{n+2}) - 2x + \frac{n(n-1)}{2!} y^n - 2$

$y^{n+2}(1-x^2) - 2xn(y^{n+2}) - n(n-1)y^n$

For W_2

$y^{n+1}2x + ny^n 2$

For W_3

$$2y^n$$

$$y^n = (1-x^2)y^{n+2} - 2xn(y^{n+2}) - n(n-1)y^n - y^{n+1}2x$$

$$+ 2ny^n + 2y^n$$

$$y^n = y^{n+2} - x^2y^{n+2} - 2xn(y^{n+2}) - (n^2+n)y^n - y^{n+1}2x$$

$$+ 2ny^n + 2y^n$$

when $x = 0$

$$y^n = y^{n+2} - (n^2+n)y^n + 2ny^n + 2y^n$$

$$y^n = y^{n+2} - (n^2+n)y^n + y^n(2n-2) = 0$$

$$y^n = y^{n+2} - y^n n^2 - y^n n^2 - y^n n + 2ny^n + 2y^n$$

$$y^n = y^{n+2} - y^n(n^2 - n + 2) = 0$$

$$(y^{n+2})_0 = y^n(-n^2 + n + 2)$$

$n=0$, $(y^{(2)})_0 = (y^{(0)})_0\,(-2)$

$n=1$, $(y^{(3)})_0 = (y^{(1)})_0\,(0) = 0$

$n=2$, $(y^{(4)})_0 = (y^{(2)})_0\,(4)(-2)$

$n=3$, $(y^{(5)})_0 = (y^{(3)})_0 = 0$

$n=4$, $(y^{(6)})_0 = (y^{(4)})_0 = (18)(4)(-2)$

$n=5$, $(y^{(7)})_0 = (y^{(5)})_0 = 0$

$n=6$, $(y^{(8)})_0 = (y^{(6)})_0 = (18)(4)(-2)(40)$

$n=7$, $(y^{(9)})_0 = (y^{(7)})_0 = 0$

$$Y = (y^{(2)})_0\frac{x^2}{2!} + (y^{(3)})_0\frac{x^3}{3!} + (y^{(4)})_0\frac{x^4}{4!} + (y^{(5)})_0\frac{x^5}{5!} + (y^{(6)})_0\frac{x^6}{6!}$$

$$+ (y^{(7)})_0\frac{x^7}{7!} + (y^{(8)})_0\frac{x^8}{8!} + (y^{(9)})_0\frac{x^9}{9!}$$

$$Y = (y^{(0)})(-2)\frac{x^2}{2!} + \cancel{(-2)x^4}(y^{(2)})_0(4)(-2)\frac{x^4}{4!} + \frac{x^6}{6!}(y^{(4)})_0$$

$$(18)(4)(-2) + (y^{(8)})_0(18)(-4)(-2)(40)\frac{x^8}{8!}$$

(2) (i) $3e^{-4t} - 5e^{4t}$

$$L[3e^{-4t} - 5e^{4t}]$$

$$L[3e^{-4t}] - L[5e^{4t}]$$

$$\frac{3}{s+4} - \frac{5}{s-4}$$

(ii) $\sin 4t + \cos 4t$

$$L[\sin 4t + \cos 4t]$$

$$L(\sin 4t) + L(\cos 4t)$$

$$\frac{4}{s^2+16} + \frac{s}{s^2+16}$$

(iii) $t^3 + 2t^2 - t + 4$

$$L(t^3 + 2t^2 - t + 4]$$

$$L(t^3) + L(2t^2) - L(t) + L(4)$$

$$\frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

~~(iv)~~

(iv) $e^{-2t}\cos 5t$

$L(e^{-2t} \cos 5t)$

$= \frac{s-2}{(s-2)^2+25}$

(v) $t \sin 3t$

$L(t \sin 3t)$

$\frac{3}{s^2+9}$

(vi) $\frac{e^{-t}-e^{-2t}}{t}$

$L(e^{-t}-e^{-2t})$

$(e^{-t})-L(e^{-2t})$

$\frac{1}{s+1}-\frac{1}{s+2}$

$= \int_s^\infty \left(\frac{1}{\sigma+1}-\frac{1}{\sigma+2}\right) d\sigma$

$\int_s^\infty \frac{1}{\sigma+1} d\sigma - \int_s^\infty \frac{1}{\sigma+2} d\sigma$

$\left[\ln(\sigma+1)-\ln(\sigma+2)\right]_s^\infty$

$$\left[\frac{\ln(s+1)}{(s+2)}\right]_s^{\infty}$$

$$\left[\ln\frac{(\infty+1)}{(\infty+2)} - \ln\left(\frac{(s+1)}{(s+2)}\right)\right]$$

$$0 - \ln\frac{(s+1)}{(s+2)}$$

$$-1 \quad \ln\frac{s+2}{s+3}$$

(vii) $e^{4t}\cos 2t$

$$L(e^{4t}\cos 2t)$$

$$\frac{s-4}{(s-4)^2+4}$$

(viii) $t \sin 2t$

$$\frac{2}{s^2+4}$$

(ix) $t^3 + \cancel{4}t^2 + 5$

$$L(t^3+4t^2+5)$$

$$L(t^3) + L(4t^2) + L(5)$$

$$\frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$$

(x) $e^{3t}(t^2+4)$

$$\frac{1}{s-3}\left(\frac{2}{s^3}+\frac{4}{s}\right)$$

(xi) $t^2 \cos t$

$$\frac{s-1}{(s-1)^2+1}$$

(xii) $$\frac{\sinh 2t}{t} = L\left(\frac{\sinh 2t}{t}\right)$$

$$= \frac{\tan^{-1}(2)}{s}$$

(3) (i) $$\frac{s-5}{(s-3)(s-4)} = \frac{A}{(s-3)} + \frac{B}{(s-4)}$$

$$\frac{A(s-4)+B(s-3)}{(s-3)(s-4)}$$

$s-5 = As - 4A + Bs - 3B$

$s-5 = As + Bs - 4A - 3B$

$s-5 = s(A+B) - 4A - 3B$

$A+B = 1 \times -4$

$-4A - 3B = -5 \times 1$

$-4A - 4B = -4$

$-4A - 3B = -5$

$-B = 1$

$B = -1$

$A + B = 1$

$A - 1 = 1$

$A = 2$

$\therefore \dfrac{2}{(s-3} - \dfrac{1}{(s-4)}$

$L^{-1}\left[\dfrac{2}{(s-3)} - \dfrac{1}{(s-4)}\right]$

$2e^{3t} - e^{4t}$

(iv) $\dfrac{2s-6}{(s-2)(s-4)}$

$\dfrac{A}{(s-2)} + \dfrac{B}{(s-4)}$

$\dfrac{A(s-4) + B(s-2)}{(s-2)(s-4)}$

$2s - 6 = As - 4A + sB - 2B$

$2s - 6 = As + sB - 4A - 2B$

$2s - 6 = s(A+B) - 4A - 2B$

$A+B=2 \quad \times -4$

$-4A-2B=-6 \quad \times 1$

$-4A+4B=-8$

$-4A-2B=-6$

$-2B=-2$

$B=1$

$A+B=2$

$A+1=2$

$A=1$

$\therefore \dfrac{1}{(S-2)} + \dfrac{1}{(S-4)}$

$L^{-1}\left(\dfrac{1}{(S-2)} + \dfrac{1}{(S-4)}\right)$

$= e^{2t} + e^{4t}$

(iii) $\dfrac{5S-8}{S(S-4)} = \dfrac{A}{S} + \dfrac{B}{S-4}$

$\dfrac{5S-8}{S(S-4)} = \dfrac{A(S-4)+BS}{S(S-4)}$

$5S-8 = A(S-4)+BS$

$5S-8 = AS-4A+BS$

$5S-8 = AS+BS-4A$

$$5s - 8 = s(A+B) - 4A$$

$$-4A = -8$$

$$A = 2$$

$$A + B = 5$$

$$2 + B = 5$$

$$B = 3$$

$$L^{-1}\left(\frac{2}{s} + \frac{B}{s-3}\right) = 2 + e^{3t}$$

(iv) $$\frac{s^2 - 3s - 4}{(s-3)(s-1)^2}$$

$$\frac{A}{(s-3)} + \frac{B}{(s-1)} + \frac{C}{(s-1)^2}$$

$$s^2 - 3s - 4 = A(s-1)^2 + B(s-3)(s-1) + C(s-3)$$

$$(s-1) = 0$$

$$C = -2, \quad A = 1, \quad B = -3$$

$$\frac{1}{(s-3)} - \frac{3}{(s-1)} - \frac{2}{(s-1)^2}$$

$$L^{-1}\left[\frac{1}{(s-3)}\right] - L^{-1}\left[\frac{3}{(s-1)}\right]$$

$$= e^{3t} - 3e^{t}$$