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Medical Engineering

ENGG 381 Assignment 4

$$1) (1-x^2) \frac{dy}{dx} - 2x \frac{dy}{dx} + 2y = 0$$

$$(1-x^2)y' - 2xy' + 2y = 0$$

$$y^n = U^n \sqrt{1-x^2} + n \sqrt{1-x^2} U^{n-1} + \frac{n(n-1)}{2!} U^{n-2} \sqrt{1-x^2} + \dots$$

$$\left[y^{(2+n)} \cdot (1-x^2) + n y^{(1+n)} \cdot (-2x) + n(n-1) y^{(n)} \cdot (-2) \right] + \left[y^{(1+n)} \cdot (-2x) + n y^{(n)} \right] + 2y^{(n)} = 0$$

$$(1-x^2)y^{(2+n)} - 2xny^{(1+n)} - n(n-1)y^{(n)} - 2xy^{(1+n)} - 2ny^{(n)} + 2y^{(n)} = 0$$

let $x=0$

$$y^{(2+n)} - n(n-1)y^{(n)} - 2ny^{(n)} + 2y^{(n)} = 0$$

$$y^{(n+2)} + y^{(n)} [-n(n+1) - 2n + 2] = 0$$

$$y^{n+2} = -\frac{y^{(n)}}{n^2 - n + 2}$$

For $n=0$ $y^2 = -\frac{y^0 \cdot 2}{0^2 - 0 + 2} = -2y^0$

for $n=1$ $y^3 = 0$

for $n=2$ $y^4 = -\frac{y^2 \cdot (-4)}{2^2 - 2 + 2} = -8y^2$

$n=3$ $y^5 = 0$

$n=4$ $y^6 = -2y^4$

$n=5$ $y^7 = 0$

$$y = y^0 + xy^1 + \frac{x^2}{2!} y^2 + \frac{x^3}{3!} y^3 + \dots$$

$$y = y^0 + 2xy^1 + \frac{x^2}{2!} (-2)y^0 + \frac{x^3}{3!} (0) + \frac{x^4}{4!} (-8)y^2 + \dots$$

$$y = y^0 \left[1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5} \right] + y^1 [2x]$$

2) i) $3e^{-4t} - 5e^{4t}$

$$\mathcal{L}[3e^{-4t} - 5e^{4t}] \Rightarrow \mathcal{L}[3e^{-4t}] - \mathcal{L}[5e^{4t}]$$

$$= 3\mathcal{L}[e^{-4t}] - 5\mathcal{L}[e^{4t}]$$

$$= 3 \left[\frac{1}{s-(-4)} \right] - 5 \left[\frac{1}{s-4} \right]$$

$$= \frac{3}{s+4} - \frac{5}{s-4}$$

ii) $\sin 4t + \cos 4t$

$$\begin{aligned} L[\sin 4t] + L[\cos 4t] \\ &= \frac{4}{s^2+16} + \frac{s}{s^2+16} \\ &= \frac{4+s}{(s^2+16)} \end{aligned}$$

iii) $t^3 + 2t^2 - t + 4$

$$L[t^3] + L[2t^2] - L[t] + L[4]$$

$$t^n = n! / s^{n+1}$$

$$\begin{aligned} &= \frac{3!}{s^{3+1}} + 2 \left[\frac{2!}{s^{2+1}} \right] - \left[\frac{1!}{s^{1+1}} \right] + \frac{4}{s} \\ &= 6/s^4 + 4/s^3 - 1/s^2 + 4/s \end{aligned}$$

iv) $e^{-2t} \cos 5t$

$$L[\cos 5t] = \frac{s}{s^2+25}$$

$$= \frac{s}{s^2+25}$$

$$L[e^{-2t} \cos 5t] = (s+2) / ((s+2)^2 + 25)$$

v) $t \sin 3t$

$$L[\sin 3t] = \frac{3}{s^2+9}$$

$$= \frac{3}{s^2+9}$$

$$F(s) = \frac{3}{s^2+9}$$

$$L[t \sin 3t] = -f'(s)$$

Using quotient rule

$$\frac{v \frac{du}{ds} - u \frac{dv}{ds}}{v^2} = \frac{[s^2+9] \cdot 0 - 3 \cdot [2s]}{[s^2+9]^2}$$

$$= \frac{-6s}{[s^2+9]^2}$$

$$= \frac{6s}{[s^2+9]^2}$$

$$= \frac{6s}{[s^2+9]^2}$$

$$= \frac{6s}{[s^2+9]^2}$$

vi) $\frac{e^{-t} - e^{-2t}}{t}$

$$L \left[\frac{e^{-t} - e^{-2t}}{t} \right]$$

$$\lim_{t \rightarrow \infty} \left[\frac{e^{-t} - e^{-2t}}{t} \right] = \frac{1-1}{0} = \frac{0}{0} \Rightarrow \text{Indeterminate}$$

Applying L'Hospital's Rule

$$\lim_{t \rightarrow \infty} \left[\frac{e^{-t} - (-2)e^{-2t}}{1} \right] = \left[\frac{-1+2}{1} \right] = 1 \Rightarrow \text{determinate}$$

$$L \left[\frac{f(t)}{t} \right] = \int_{t=s}^{\infty} f(s) ds$$

$$\int s = L[f(t)]$$

$$L[f(t)] = e^{-t} - e^{-2t} = \frac{1}{s+1} - \frac{1}{s+2}$$

$$= \frac{1}{s+1} - \frac{1}{s+2}$$

$$F(s) = \frac{1}{s+1} - \frac{1}{s+2}$$

$$\int f(s) L \left[\frac{f(t)}{t} \right] = \int_{s=s}^{\infty} \left(\frac{1}{s+1} - \frac{1}{s+2} \right) ds$$

$$= \left[\ln|s+1| - \ln|s+2| \right]_s^{\infty}$$

$$= \left[\ln|s+1| - \ln|s+2| \right]_s^{\infty}$$

$$= \left[\ln \frac{s+1}{s+2} \right]_s^{\infty} = \ln \left[\frac{\infty+1}{\infty+2} - \frac{s+1}{s+2} \right]$$

$$= -\ln \left[\frac{s+1}{s+2} \right] = \ln \left[\frac{s+2}{s+1} \right]$$

vi) $e^{4t} \cos 2t$

$$L(e^{4t} \cos 2t) = e^{4t} L[\cos 2t]$$

$$L[\cos 2t] = \frac{s}{s^2+2^2}$$

$$= \frac{s}{s^2+4}$$

$$L[e^{4t} \cos 2t] = \frac{s-4}{(s-4)^2+4}$$

vii) $t \sin 2t$

$$\Rightarrow L[t \sin 2t] = -\frac{d}{ds} [f(s)]$$

$$f(s) = L[\sin 2t] = \frac{2}{s^2+2^2}$$

$$f(s) = \frac{2}{s^2+4}$$

$$f'(s) = \text{using quotient Rule}$$

$$\frac{v \frac{du}{ds} - u \frac{dv}{ds}}{v^2}$$

$$f'(s) = -4s$$

$$[s^2 + 4]^2$$

$$\therefore L[t \sin 2t] = -f'(s)$$

$$= -(-4s) = \frac{4s}{[s^2 + 4]^2}$$

$$= 4s$$

$$[s^2 + 4]^2$$

$$(2c) \cdot t^3 + 4t^2 + 5$$

$$L[t^3] + 4L[t^2] + L[5]$$

$$= \frac{3!}{s^3+1} + 4 \left[\frac{2!}{s^2+1} \right] + \frac{5}{s}$$

$$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$$

$$(x) e^{3t}(t^2+4)$$

$$\text{let } x = t^2 + 4$$

$$L[e^{3tx}]$$

$$L[x] = L[t^2 + 4]$$

$$= L[t^2] + L[4]$$

$$= \frac{2!}{s^2+1} + \frac{4}{s}$$

$$= \frac{2}{s^3} + \frac{4}{s}$$

$$L[e^{3tx}] = \frac{2}{[s-3]^3} + \frac{4}{[s-3]}$$

$$[s-3]^3 [s-3]$$

2i) $t^2 \cos t$

$$L[t^2 \cos t] = t^2 L[\cos t]$$

$$f(s) = L[\cos t] = \frac{s}{s^2 + 1^2}$$

$$f(s) = \frac{s}{s^2 + 1^2}$$

$f'(s)$ using quotient rule

$$f'(s) =$$

3) i) $\frac{s-5}{(s-3)(s-4)}$

$$(s-3)(s-4)$$

$$L^{-1} \left[\frac{s-5}{(s-3)(s-4)} \right] = \frac{A}{s-3} + \frac{B}{s-4}$$

$$\frac{s-5}{(s-3)(s-4)} = \frac{A(s-4) + B(s-3)}{(s-3)(s-4)}$$

Assuming $s=4$

$$4-5 = A(4-4) + B(4-3)$$

$$-1 = B(1)$$

$$B = -1$$

Assuming $s=3$

$$3-5 = A(3-4) + B(3-3)$$

$$-2 = -A$$

$$A = 2$$

$$\therefore \frac{s-5}{(s-3)(s-4)} = \frac{2}{s-3} + \frac{-1}{s-4}$$

$$= \underline{\underline{2e^{3t} - e^{4t}}}$$

ii) $\frac{2s-6}{(s-2)(s-4)}$

$$(s-2)(s-4)$$

$$L^{-1} \left[\frac{2s-6}{(s-2)(s-4)} \right]$$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$$

$$2s-6 = A(s-4) + B(s-2)$$

Assuming $s=4$

$$(2(4)-6) = A(4-4) + B(4-2)$$

$$8-6 = A(0) + B(2)$$

$$2 = 2B$$

$$B = 1$$

Assuming $s=2$

$$2(2)-6 = A(2-4) + B(2-2)$$

$$-2 = -2A$$

$$A = 1$$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{1}{s-2} + \frac{1}{s-4}$$

$$L^{-1} \left[\frac{2s-6}{(s-2)(s-4)} \right] = \underline{e^{2t} + e^{4t}}$$

iii) $\frac{5s-8}{s(s-4)}$

$$L^{-1} \left[\frac{5s-8}{s(s-4)} \right] = \frac{A}{s} + \frac{B}{s-4}$$

$$5s-8 = A(s-4) + B(s)$$

Assuming $s=4$

$$5(4)-8 = A(4-4) + B(4)$$

$$20-8 = 4B$$

$$12 = 4B$$

$$B = 3$$

Assuming $s=0$

$$5(0)-8 = A(0-4) + B(0)$$

$$-8 = -4A$$

$$A = 2$$

$$L^{-1} \left[\frac{5s-8}{s(s-4)} \right] = \frac{2}{s} + \frac{3}{s-4}$$

$$= \frac{2}{s} + 3 \left[\frac{1}{s-4} \right]$$

$$= \underline{2 + 3e^{4t}}$$

iv) $\frac{s-5}{s^2+4s+20}$

$$s^2+4s+20$$

$$L^{-1} \left[\frac{s-5}{s^2+4s+20} \right]$$

$$F(s) = \frac{s-5}{s^2+4s+20} = \frac{s}{(s+2)^2+16} - \frac{5}{(s+2)^2+16}$$

$$= \frac{s+2-2}{(s+2)^2+16} - \frac{5}{(s+2)^2+16} = \frac{s+2}{(s+2)^2+16} - \frac{7}{(s+2)^2+16}$$

$$= \frac{s+2}{(s+2)^2+4^2} - \frac{7 \times 4/4}{(s+2)^2+4^2}$$

$$= \frac{s+2}{(s+2)^2+4^2} - \frac{7 \cdot 4}{4((s+2)^2+4^2)}$$

$$f(t) = e^{-2t} \cos 4t - \frac{7}{4} e^{-2t} \sin 4t$$

$$= e^{-2t} \left[\cos 4t - \frac{7}{4} \sin 4t \right]$$

v) $\frac{s^2-3s-4}{(s-3)(s-1)^2}$

$$(s-3)(s-1)^2$$

$$\Rightarrow f(s) = \frac{A}{s+3} + \frac{B}{(s-1)^2} + \frac{C}{s-1}$$

$$A = \frac{s^2-3s-4}{(s-1)^2} \Big|_{s=3} = \frac{3^2-3(3)-4}{(3-1)^2} = -1$$

$$B = \frac{s^2-3s-4}{(s-3)(s-1)} \Big|_{s=1} = \frac{1^2-3(1)-4}{1-3} = 3$$

$$C = \frac{d}{ds} \left[\frac{s^2-3s-4}{s-3} \right] \Big|_{s=1} = \frac{(s-3)(2s-3) - [s^2-3s-4]}{(s-3)^2}$$

at $s=1$

$$\frac{(1-3)(2(1)-3) - [1^2 - 3(1) - 4]}{(1-3)^2} = 2$$

$$f(s) = \frac{-1}{s-3} + \frac{3}{(s-1)^2} + \frac{5}{s-1}$$

$$f(t) = -e^{-3t} + 3te^t + 2e^t$$
$$= e^t \underline{\underline{[3t+2]}} - e^{3t}$$