ENG 381
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Elect/Elect

1) $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$$\underbrace{(1-x^2)y''}_{w_1} - \underbrace{2xy'}_{w_2} + \underbrace{2y}_{w_3} = 0$$

$$w_1 = (1-x^2)y^{(n+2)} - 2xny^{(n+1)} - 2\frac{n(n-1)}{2!}y^{(n)}$$

$$(1-x^2)y^{(n+2)} - 2xny^{(n+1)} - n(n-1)y^{(n)}$$

$$w_2 = -2xy^{(n+1)} - 2ny^{(n)}$$

$$w_3 = 2y^{(n)}$$

$$= (1-x^2)y^{(n+2)} - 2xny^{(n+1)} - n(n-1)y^{(n)} - 2xy^{(n+1)} - 2ny^{(n)} + 2y^{(n)}$$

$$= (1-x^2)y^{(n+2)} - (2n+1)xy^{(n+1)} - [n(n-1) - 2n + 2]y^{(n)}$$

$$= (1-x^2)y^{(n+2)} - (2n+1)xy^{(n+1)} - [n^2 - n - 2n + 2]y^{(n)}$$

$$= (1-x^2)y^{(n+2)} - (2n+1)xy^{(n+1)} - [n^2 - 3n + 2]y^{(n)}$$

at $x = 0$

$$(1-x^2)y^{(n+2)} - [n^2 - 3n + 2]y^{(n)} = 0$$

$$(1-x^2)y^{(n+2)} = [n^2 + 3n - 2]y^{(n)}$$

$n = 0$

$$y^{(2)} = [0^2 - 3(0) - 2]y^{(0)} = -2y^{(0)}$$

$n = 1$

$$y^{(3)} = [1^2 + 3(1) - 2]y^{(1)} = 2y^{(1)}$$

$n = 2$

$$y^{(4)} = [2^2 + 3(2) - 2]y^{(2)} = 8y^{(2)} = (8)(-2)y^{(0)}$$

$n = 3$

$$y^{(5)} = [3^2 + 3(3) - 2]y^{(3)} = 16y^{(3)} = (16)(2)y^{(1)}$$

$$= y^{(0)} + xy^{(1)} + \frac{x^2}{2!}(-2y^{(0)}) + \frac{x^3}{3!}(2y^{(1)}) + \frac{x^4}{4!}(8)(-2)y^{(0)} + \frac{x^5}{5!}(16)(2)y^{(1)} + \dots$$

$$= y^{(0)} + x y^{(1)} - x^2 y^{(0)} + \frac{x^3}{3} y^{(1)} - 2 y^{(0)} \frac{x^4}{3} +$$

$$4 y^{(1)} \frac{x^5}{15}$$

$$= y^{0} \left[1 + x^2 - \frac{2x^4}{3} \right] + y^{(1)} \left[x + \frac{x^3}{3} + \frac{4x^5}{15} \right]$$

3. 2) i) $3e^{-4t} - 5e^{4t}$

$L[3e^{-4t}] - L[5e^{4t}]$

$= \frac{3}{s+4} - \frac{5}{s-4}$

ii) $\sin 4t + \cos 4t$

$L[\sin 4t] + L[\cos 4t]$

$= \frac{4}{s^2+16} + \frac{s}{s^2+16}$

iii) $t^3 + 2t^2 - t + 4$

$L[t^3] + L[2t^2] - L[t] + L[4]$

$\frac{3!}{s^{3+1}} + 2 \cdot \frac{2!}{s^{2+1}} - \frac{1}{s^2} + \frac{4}{s}$

$\frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

iv) $e^{-2t}\cos 5t$

$L[\cos 5t] = \frac{s}{s^2+25}$ $\quad L[e^{-2t}] = \frac{1}{s+2}$

$= \frac{s+2}{(s+2)^2+25}$

$=$

v) $t\sin 3t$

$L[\sin 3t] = \frac{3}{s^2+9}$

$-\frac{d}{ds}\left(\frac{3}{s^2+9}\right) \begin{matrix} u \\ v \end{matrix}$

$u = 3 \quad v = s^2+9$

$du = 0 \quad dv = 2s$

$= \frac{v\,du}{ds} - \frac{u\,dv}{ds}$ using quotient rule

$$\frac{(s^2+9)\,0 - 3(2s)}{(s^2+9)^2} = \frac{6s}{(s^2+9)^2}$$

vi) $\frac{e^{-t} - e^{-2t}}{t}$

$L[e^{-t} - e^{-2t}] = \frac{1}{s+1} - \frac{1}{s+2}$

$$L\left[\frac{e^{-t} - e^{-2t}}{t}\right] = \int_{a=s}^{\infty}\left(\frac{1}{s+1} - \frac{1}{s+2}\right) ds$$

$$= \int_s^{\infty} \frac{1}{s+1}\, ds - \int_s^{\infty} \frac{1}{s+2}\, ds$$

$$[\ln(s+1) - \ln(s+2)]_s^{\infty}$$

$$= \ln\left[\frac{(s+1)}{s+2}\right]_s^{\infty}$$

$$0 - \ln\frac{s+1}{s+2}$$

vii) $e^{4t}\cos 2t$

$L[\cos 2t] = \frac{s}{s^2+4}$

$L[e^{4t}(\cos 2t)] = \cancel{\frac{s+4}{s}} \quad \frac{s-4}{(s-4)^2+4}$

viii)

$t \sin 2t$

$L[\sin 2t] \quad \frac{2}{s^2+4}$

$$-\frac{d}{ds}\left[\frac{2}{s^2+4}\right] \quad \frac{u}{v}$$

$du = 0 \qquad dv = 2s$

$u = 2 \qquad v = s^2+4$

using quotient rule

$$\frac{v\frac{du}{ds} - u\frac{dv}{ds}}{v^2}$$

$$\frac{(s^2+4)\cdot 0 - 2(2s)}{(s^2+4)^2}$$

$$= \frac{4s}{(s^2+4)^2}$$

1x)

$t^2 + 4t^2 + 5$

$L[t^2] + L[4t^2] + L[5]$

$$\frac{2!}{s^{2+1}} + 4\cdot\frac{2!}{s^{2+1}} + \frac{5}{s}$$

$$\frac{2}{s^3} + \frac{8}{s^3} + \frac{5}{s}$$

x) $t^2 \cos t$

$$L(\cos t) = \frac{s}{s^2+1}$$

$$L[t^2 \cos t] = \frac{d^2}{ds^2}\left[\frac{s}{s^2+1}\right]$$

$u = s \qquad v = s^2+1$

$du = 1 \qquad dv = 2s$

using quotient rule

$$\frac{(s^2+1)\cdot 1 - s(2s)}{(s^2+1)}$$

$$\frac{(s^2+1) - 2s^2}{(s^2+1)^2}$$

$$\frac{-s^2+1+2s^2}{(s^2+1)^2} = s^2 - 1$$

$$\frac{-s^2+1}{(s^2+1)^2}$$

$= (s^2+1).$

$\frac{\sinh 2t}{t}$ =

$L[\sinh 2t] = \frac{2}{s^2 - 4}$

$L\left[\frac{\sinh 2t}{t}\right] = \int$

11) $e^{3t}(t^2+4)$

$L(t^2+4)$

$\frac{2}{s^3} + \frac{4}{s}$

$L(e^{3t}(t^2+4)) = \frac{2}{(s-3)^3} + \frac{4}{(s-3)}$

3) i) $\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$

$\frac{A(s-4) + B(s-3)}{(s-3)(s-4)}$

$s-5 = A(s-4) + B(s-3)$

at $s-3=0$ $s=3$

$3-5 = A(3-4) + 0$

$-2 = -A$

$A = 2$

at $s-4=0$ $s=4$

$4-5 = 0 + B(4-3)$

$-1 = B$

$B = -1$

$L^{-1}\left[\frac{2}{(s-3)} + \frac{(-1)}{(s-4)}\right]$

$= 2e^{3t} - e^{4t}$

ii) $\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$

$\frac{A(s-4) + B(s-2)}{(s-2)(s-4)}$

$2s-6 = A(s-4) + B(s-2)$

at $s-2=0$ $s=2$

$2(2)-6 = A(2-4) + 0$

$4-6 = -2A$

$-2 = -2A$

$A = 1$

at $s-4=0$ $s=4$

$2(4)-6 = 0 + B(4-2)$

$8-6 = 2B$

$2 = 2B$

$B = 1$

$L^{-1}\left[\frac{1}{s-2} + \frac{1}{s-4}\right]$

$e^{2t} + e^{4t}$

iii) $\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$\frac{A(s-4)+B(s)}{s(s-4)}$

$5s-8 = A(s-4) + B(s)$

at $s-4=0 \quad s=4$

$5(4)-8 = 0 + 4B$

$20-8 = 4B$

$12 = 4B$

$B = 3$

at $s=0$

$-8 = -4A + 0$

$4A = 8$

$A = 2$

$L^{-1}\left[\frac{2}{s} + \frac{3}{s-4}\right]$

$2 + 3e^{4t}$

v $\frac{s^2-3s-4}{(s-3(s-1)^2} = \frac{A}{s-3} + \frac{B}{s-1} + \frac{C}{(s-1)^2}$

$\frac{A(s-1)(s-1)^2 + B(s-3)(s-1^2) + C(s-3)(s-1)}{(s-3)(s-1)(s-1)^2}$

$s^2-3s-4 = A(s-1)^2 + B(s-1)^2 + B(s-3)(s-1) + C(s-3)$

$3^2-3(3)-4 = A(3-1)^2$

$A = -1$

$1^2-3(1)-4) = C(1-3)$

$C=3$

$s^2-3s-4 = [s^2-2s+1]A + s^2-4s+3)B + (s-3)C$

$= -2A - 4B + C = -3$

$-2(-1) - 4(B) + 3 = -3$

$-4B = -3-3-2$

$-4B = -8$

$B - 2$

$$L^{-1}\left[\frac{-1}{s-3}+\frac{2}{s-1}+\frac{3}{(s-1)^2}\right]$$

$$-e^{3t}+2e^{t}+3te^{t}$$