[illegible]
11/ENG04/068

Assignment 5

① $\frac{dy}{dt} + 3y = e^{-2t}$ given that at $t=0, y=2$

$L\{\frac{dy}{dt}\} = sY(s) - y(0)$

$L\{3y\} = 3Y(s)$

$L\{e^{-2t}\} = \frac{1}{s+2}$

$sY(s) - y(0) + 3Y(s) = \frac{1}{s+2}$

$sY(s) + 3Y(s) - 2 = \frac{1}{s+2}$

$Y(s)(s+3) = \frac{1}{s+2} + 2$

$Y(s) = \frac{1 + 2(s+2)}{s+2} = \frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

$2s+5 = A(s+3) + B(s+2)$

$A + B = 2$ --- ①
$3A + 2B = 5$ --- ②
$B = 1$
$A = 1$

$\frac{1}{s+2} + \frac{1}{s+3}$

$L^{-1} = e^{-2t} + e^{-3t}$ //

② $3\frac{dy}{dt} - 6y = \sin 2t$ ~~when~~ $t=0, y=1$

~~L~~ $3sY(s) - 6Y(s) - 3 = \frac{2}{s^2+2^2}$

$$Y(s)(3s-6) = \frac{2}{(s+2)^2} + 3.$$

$$Y(s) = \frac{2+3(s+2)^2}{(s+2)^2(3s-6)} = \frac{A}{(s+2)} + \frac{B}{(s+2)^2} + \frac{C}{(3s-6)}$$

$$2+3(s+2)^2 = A(s+2)(3s-6) + B(3s-6) + C(s+2)^2.$$

$$2+3s^2+12s+12 = A3s^2 - 12A + 3Bs - 6B + Cs^2 + 4Cs + 4C$$

$$3A + C = 3 \quad \text{------} \ 1$$

$$3B + 4C = 12 \quad \text{----} \ 2$$

$$-12A - 6B + 4C = 14 \quad \text{----} \ 3$$

$$3A = 3 - C$$

$$A = \frac{3-C}{3}$$

$$3B + 4C = 12$$

$$-12\left(\frac{3-C}{3}\right) - 6B + 4C = 14$$

$$-12 + 4C - 6B + 4C = 14$$

$$-6B + 8C = 28 \quad \text{----} \ 4$$

$$-18B - 24C = -72$$

$$-18B + 24C = 84$$

$$-48C = -156$$

$$C = 13/4$$

$$3B = 12 - 13$$

$$B = \frac{-1}{3}$$

$$A = \frac{3 - 13/4}{3} = \frac{-1}{12}$$

$$\frac{2 + 3(s+2)^2}{(s+2)^2(3s-6)} = \frac{-1/12}{(s+2)} - \frac{1/3}{(s+2)^2} + \frac{13/4}{(3s-6)}$$

$$L^{-1}\{Y(s)\} = -\tfrac{1}{12}e^{-2t} - \tfrac{1}{3}te^{-2t} + \tfrac{13}{12}e^{3t}$$

$$y = -\tfrac{1}{12}\left(e^{-2t} + 4te^{-2t} - 13e^{3t}\right)$$

(b) $dy/dt - 4y = 8$

$$sY(s) - y(0) - 4Y(s) = 8/s$$

$$Y(s)(s-4) - 2 = 8/s$$

$$Y(s) = \frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$8 + 2s = A(s-4) + Bs = As - 4A + Bs$$

$A + B = 2$

$-4A = 8$

$A = -2$

$B = 2 + 2 = 4$

$$\frac{-2}{s} + \frac{4}{s-4}$$

$$L^{-1}\{Y(s)\} = -2 + 4e^{4t}$$

(14) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$ at $t=0$, $y=2$, $y'=1$

$L\{d^2y/dt^2\} = s^2Y(s) - sY(0) - Y'(0)$

$L\{-2\, dy/dt\} = -2sY(s) + 2Y(0)$

$L\{5y\} = 5Y(s)$

$L\{e^{2t}\} = \frac{1}{s-2}$

$s^2Y(s) - sY(0) - Y'(0) - 2sY(s) + 2Y(0) + 5Y(s) = \frac{1}{s-2}$

$s^2Y(s) - 2sY(s) + 5Y(s) - 2s - 1 + 4 = \frac{1}{s-2}$

$Y(s)\,(s^2-2s+5) = \frac{1}{s-2} + 2s - 3.$

$$Y(s) = \frac{1+2s^2-7s+6}{(s-2)(s^2-2s+5)} = \frac{2s^2-7s+7}{(s-2)(s^2-2s+5)}.$$

$$= \frac{A}{(s-2)} + \frac{Bs+C}{(s^2-2s+5)}$$

$2s^2-7s+7 = As^2-2As+5A+Bs^2-2Bs+Cs-2C$

$A+B=2$ ----- (1)

$-2A-2B+C = -7$ --- (2)

$5A-2C = 7$ ---- (3).

$B = 2 - A$

$-2A - 2(2 - A) + C = -7$

$C = -3.$

$5A - 2(-3) = 7$

$A = 1/5$

$A + B = 2$

$\frac{1}{5} + B = 2$

$B = 9/5$

$$\frac{2s^2 - 7s + 7}{(s-2)(s^2-2s+5)} = \frac{\frac{1}{5}}{(s-2)} + \frac{9/5\, s - 3}{(s^2-2s+5)}$$

$$\frac{2s^2 - 7s + 7}{(s-2)(s^2-2s+5)} = \frac{\frac{1}{5}}{s-2} + \frac{9/5\, s}{(s^2-2s+5)} - \frac{3}{(s^2-2s+5)}$$

$$= \frac{\frac{1}{5}}{(s-2)} + \frac{9}{5}\left(\frac{s-1+1}{s^2-2s+5}\right) - \frac{3 \times 2/2}{(s^2-2s+5)}$$

$$= \frac{\frac{1}{5}}{(s-2)} + 9/5\left(\frac{s-1+1}{(s-1)^2+4}\right) - \frac{3}{2}\left(\frac{2}{(s-1)^2+4}\right)$$

$$L^{-1}\{Y(s)\} = \frac{1}{5}e^{2t} + 9/5\left[e^t \cos 2t + \frac{1}{2}(e^t \sin 2t)\right] - \frac{3}{2}\left(e^t \sin 2t\right.$$

(5) $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$ $t=0,\ y=0,\ y'=2.$

$L\{d^2y/dt^2\} = s^2Y(s) - sY(0) - Y'(0)$

$L\{-6\,dy/dt\} = -6sY(s) + 6Y(0)$

$L\{8y\} = 8Y(s)$

$L\{e^{3t}\} = \frac{1}{s-3}$

$s^2Y(s) - sY(0) - Y'(0) - 6sY(s) + 6Y(0) + 8Y(s) = \frac{1}{s-3}$

$s^2Y(s) - 6sY(s) + 8Y(s) - 2 = \frac{1}{s-3}$

$Y(s)\,(s^2 - 6s + 8) = \frac{1}{s-3} + 2$

$Y(s)(s^2 - 6s + 8) = \frac{1 + 2(s-3)}{(s-3)}$

$Y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{A}{(s-3)} + \frac{Bs+C}{s^2-6s+8}$

$2s-5 = A(s^2-6s+8) + (Bs+C)(s-3)$

$2s-5 = As^2 - 6As + 8A + Bs^2 - 3Bs + Cs - 3C$

$A+B=0$ ..... (1)

$-6A - 3B + C = 2$ ..... (2)

$8A - 3C = -5$ ..... (3)

$B = -A$

$-6A + 3A + C = 2$

$-3A + C = 2 \quad \cdots \text{④} \times 5$

$8A - 3C = -5 \quad \cdots \text{⑤}$

$9A - 3C = -6$

$8A - 3C = -5$

$A = -1$

$B = 1$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{-1}{(s-3)} + \frac{s-1}{(s^2-6s+8)}$$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{-1}{(s-3)} + \frac{s-1}{(s-2)(s-4)}$$

$$\frac{s-1}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$$

$s-1 = A(s-4) + B(s-2)$

$A + B = 1 \quad \cdots \text{①}$

$-4A - 2B = -1 \quad \cdots \text{②}$

$-2B = -3$

$B = 3/2 \quad , A = -1/2$

$$\frac{s-1}{(s-2)(s-4)} = \frac{-1/2}{s-2} + \frac{3/2}{s-4}$$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{-1}{s-3} + \frac{-1/2}{(s-2)} + \frac{3/2}{s-4}$$

$$\mathcal{L}^{-1}(Y(s)) = -e^{3t} - \tfrac{1}{2}e^{2t} + \tfrac{3}{2}e^{4t}$$
$$= -\tfrac{1}{2}\left(2e^{3t} + e^{2t} - 3e^{4t}\right)$$