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DEPARTMENT: PETROLEUM ENGINEERING
COURSE: ENG 381
ASSIGNMENT (4)

(1.) $(1-x^2)\, d^2y/dx^2 - 2x\, dy/dx + 2y = 0$

$(1-x^2)y^2 - 2xy' + 2y = 0$

$y^n = u^n v + n u^{n-1} u' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \dots$

$[y^{(2+n)} \cdot (1-x^2) + n y^{(1+n)} \cdot (-2x) + n(n-1) y^n (-2)] + [y^{(1+n)} \cdot -2x + n y^n - 2 + [2y^n] = 0$

$(1-x^2)y^{n+2} - 2xny^{n+1} - n(n-1)y^n - 2xy^{n+1} - 2ny^n + 2y^n = 0$

let $x = 0$

$y^{n+2} - n(n-1)y^n - 2ny^n + 2y^n = 0$

$y^{n+2} + y^n[-n(n-1) - 2n + 2] = 0$

$y^{n+2} + y^n[-n^2 + n - 2n + 2] = 0$

$y^{n+2} + y^n[-n^2 - n + 2] = 0$

$y^{n+2} = -(y^n)_0[-n^2 - n + 2]$

$n = 0$: $y^2 = -y^0 \cdot 2 = -2y^0$

$n = 1$: $y^3 = -y^1 \cdot [0] = 0$

$n = 2$: $y^4 = -y^2[-4] = 4y^2 = 4(-2y^0) = -8y^0$

$n = 3$: $y^5 = -y^3[-10] = 10y^3 = 10 \cdot 0 = 0$

$n = 4$: $y^6 = -y^4[-18] = 18y^4 = 18 \cdot 4 - 2 \cdot y^0$

$n = 5$: $y^7 = y^5 \cdot [-28] = 28(y^5) = 28 \cdot 0 = 0$

$y = y^0 + xy' + \frac{x^2 y^3}{2!}(-2)y^0 + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(4)(-2)y^0 + \dots$

$\dots + \frac{x^5}{5!}(0) + \frac{x^6}{6!}(16)4(-2)y^0 + \frac{x^7}{7!}(0)$

$y = y^0 + Axy' - x^2y^0 - \frac{x^4}{3 \times 1}y^0 - \frac{x^6}{5}y^0$

$y = y^0[1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5}] + y_0'[x]$

2i) $3e^{-4t} - 5e^{4t}$

$$= L[3e^{-4t} - 5e^{4t}] = L[3e^{-4t}] - L[5e^{4t}]$$

$$= 3\left[\frac{1}{s-a}\right] - 5\left[\frac{1}{s-a}\right]$$

$$= 3\left|\frac{1}{s+4}\right| - 5\left[\frac{1}{s-4}\right]$$

$$= \frac{3}{s+4} - \frac{5}{s-4}$$

(ii) $\sin 4t + \cos 4t$

$$L[\sin 4t + \cos 4t] = L[\sin 4t] + L[\cos 4t]$$

$$= \frac{a}{s^2 + a^2} + \frac{s}{s^2 + a^2}$$

$$= \frac{4}{s^2 + 4^2} + \frac{s}{s^2 + 4^2}$$

$$= \frac{4}{s^2 + 16} + \frac{s}{s^2 + 16} = \frac{4 + s}{(s^2 + 16)}$$

(iii) $t^2 + 2t^2 - t + 4$

$$t^n = \frac{n!}{s^{n+1}} = \frac{3!}{s^{3+1}} + 2\left[\frac{2!}{s^{2+1}}\right] - \left[\frac{1!}{s^{1+1}}\right] + \frac{4}{s}$$

$$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

(iv) $e^{-2t} \cos 5t$

$$L[\cos 5t] = \frac{s}{s^2 + a^2}$$

$$= \frac{s}{s^2 + 5^2} = \frac{s}{s^2 + 25}$$

$$L[e^{-2t} \cos 5t] = \frac{s+2}{[s+2]^2 + 25}$$

(v) $t \sin 3t$

$$L[\sin 3t] = \frac{a}{s^2+a^2} = \frac{3}{s^2+3^2} = \frac{3}{s^2+a}$$

$$L[t \sin 3t] = -F'(s)$$

$u = 3 \quad du/ds = 0$ $\qquad \frac{v\,du/ds - u\,dv/ds}{v^2}$

$v = s^2+a \quad dv/ds = 2s$

$$= \frac{[s^2+a]0 - 3[2s]}{[s^2+a]^2}$$

$$= \frac{-6s}{[s^2+a]^2}$$

(vi) $\frac{e^{-t} - e^{-2t}}{t}$

$$L\left[\frac{e^{-t} - e^{-2t}}{t}\right]$$

$$L[F(t)] = e^{-t} - e^{-2t} = \frac{1}{s+a} - \frac{1}{s+a}$$

$$= \frac{1}{s+1} - \frac{1}{s+2}$$

$$F_0 = \frac{1}{\sigma+1} - \frac{1}{\sigma+2}$$

$$\int F_0 \; L\left[\frac{f(t)}{t}\right] = \int_0^\infty = s \; \frac{1}{\sigma+1} - \frac{1}{\sigma+2} \; d\sigma$$

$$= \int_0^\infty = s \; \frac{1}{\sigma+1} \; d\sigma - \int_0^\infty = s \; \frac{1}{\sigma+2} \; d\sigma$$

$$\ln[\sigma+1] - \ln[\sigma+2]_s^\infty$$

$$= [\ln(\sigma+1) - \ln(\sigma+2)]_s^\infty$$

$$= \left[\ln \frac{\sigma+1}{(\sigma+2)}\right]_s^\infty = \ln\left[\frac{\alpha+1}{(\alpha+2)} - \frac{s+1}{(s+2)}\right]$$

$$= -\ln\left[\frac{s+1}{(s+2)}\right] = \ln\left[\frac{(s+2)}{s+1}\right]$$

(viii) $e^{4t}\cos 2t$

$L[\cos 2t] = \frac{s}{s^2+2^2} = \frac{s}{s^2+4}$

$L[e^{4t}\cos 2t] = \frac{s-4}{(s-4)^2+4}$

(viii) $t \sin 2t$

$L[t \sin 2t] = \frac{-d}{ds}[F(s)]$

$F(s) = L[\sin 2t] = \frac{2}{s^2+2^2}$

$F(s) = \frac{2}{s^2+4} \quad \frac{du}{ds} = 0$

$u = 2, \ v = s^2+4 \quad dv/ds = 2s$

$$\frac{v\frac{du}{ds} - u\frac{du}{ds}}{2}$$

$$= \frac{(s^2+4)\cdot 0 - 2\cdot(2s)}{(s^2+4)^2}$$

$$= \frac{-4s}{(s^2+4)^2}$$

$L[t \sin 2t] = -f'(s)$

$$= -1\left[\frac{-4s}{(s^2+4)^2}\right]$$

$$= \frac{4s}{(s^2+4)^2}$$

(ix) $t^3 + 4t^2 + 5$

$L[t^3 + 4t^2 + 5]$

$$= \frac{3!}{s^{3+1}} + 4\left[\frac{2!}{s^{2+1}}\right] + \frac{5}{s}$$

$$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$$

x.) $e^{3t}(t^2+4)$ let $x = t^2+4$

$L[e^{3t}x]$

$L[x] = L[t^2+4] = L[t^2] + L[4]$

$= \frac{2!}{s^{2+1}} + \frac{4}{s}$

$= 2/s^3 + 4/s$

$= L[e^{3t}] = \frac{2}{(s-3)^3} + \frac{4}{(s-3)}$

(xi) $t^2 \cos t$ $L[t^2 \cos t] = t^2 L[\cos t]$

$F(s) = L[\cos t] = \frac{s}{s^2+1^2}$

$F(s) = \frac{s}{s^2+1^2}$

$F'(s) = du/ds = 1$

$u = s$ $dv/ds = 2s$

$V = s^2+1^2$

$= \frac{[s^2+1^2]1 - 2s[s]}{[s^2+1^2]^2}$

$= \cancel{s^2+1^2} = \frac{s^2+1^2-2s^2}{[s^2+1^2]^2} = \frac{-s^2+1}{[s^2+1^2]^2}$

$-f''(s) = \frac{-d}{ds}\left[\frac{s^2-1}{[s^2+1^2]^2}\right]$

$u = s^2-1$ $du/ds = 2s$

$V = [s^2+1]^2$ $dv/ds = 4s[s^2+1]$

$\frac{[s^2+1]^2 \cdot 2s - [s^2-1][4s^3+4s]}{[s^2+1]^2}$

$= \frac{-2s^5 - 4s^3 + 6s}{[(s^2+1)^2]}$

$= \frac{-2s^5 - 4s^3 + 6s}{s^4+2s^2+1} = f''(s) = \frac{-d}{ds}\left[\frac{-2s^5-4s^3+6s}{s^4+2s^2+1}\right]$

$= f''(s) = \frac{2s^5+4s^3-6s}{s^4+2s^2+1}$

(3) $\dfrac{s-5}{(s-3)(s-4)} = L^{-1}\left[\dfrac{s-5}{(s-3)(s-4)}\right] = \dfrac{A}{(s-3)} + \dfrac{B}{(s-4)}$

$\dfrac{s-5}{(s-3)(s-4)} = \dfrac{A(s-4)+B(s-3)}{(s-3)(s-4)}$

Assuming $s=4$

$4-5 = A(4-4)+B(4-3)$

$-1 = B(1)$

$B = -1$

Assuming $s=4$

$4-5 = A(4-4)+B(4-3)$

$-1 = B(1)$

$B = -1$

Assuming $s=3$

$3-5 = A(3-4)+B(3-3)$

$-2 = A(-1)$

$A = 2$

$\dfrac{s-5}{(s-3)(s-4)} = \dfrac{2}{s-3} + \dfrac{-1}{s-4}$

$= \dfrac{2}{s-3} - \dfrac{1}{s-4}$

$= 2\left[\dfrac{1}{s-3}\right] - \left[\dfrac{1}{s-4}\right]$

$= 2e^{3t} - e^{4t}$

(ii) $\dfrac{2s-6}{(s-2)(s-4)} = L^{-1}\left[\dfrac{2s-6}{(s-2)(s-4)}\right]$

$\dfrac{2s-6}{(s-2)(s-4)} = \dfrac{A}{(s-2)} + \dfrac{B}{(s-4)}$

$2s-6 = A(s-4)+B(s-2)$

Assuming $s=4$

$2(4)-6 = A(4-4)+B(4-2)$

$$8-6 = A(0) + B(2)$$
$$2 = 2B$$
$$B = 1$$

$S = 2$

$$2(2)-6 = A(2-4) + B(2-2)$$
$$4-6 = -2A + 0$$
$$-2 = -2A$$
$$A = 1$$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{1}{s-2} + \frac{1}{s-4}$$

$$L^{-1}\left[\frac{2s-6}{(s-2)(s-4)}\right] = e^{2t} + e^{4t}$$

(iii) $\frac{5s-8}{s(s-4)}$ $L^{-1}\left[\frac{5s-8}{s(s-4)}\right] = \frac{A}{s} + \frac{B}{s-4}$

$$5s-8 = A(s-4) + B(s)$$

Assuming $s = 4$

$$5(4) - 8 = A(4-4) + B(4)$$
$$20-8 = 4B$$
$$12 = 4B$$
$$B = 3$$

Assuming $s = 0$

$$5(0) - 8 = A(0-4) + B(0)$$
$$-8 = -4A$$
$$A = 2$$

$$L^{-1}\left[\frac{5s-8}{s(s-4)}\right] = \frac{2}{s} + \frac{3}{s-4}$$
$$= 2/s + 3\left[\frac{1}{s-4}\right]$$
$$= 2 + 3e^{4t}$$

$$F(s) = \frac{s-5}{s^2+4s+4+16} = \frac{s}{(s+2)^2+16} - \frac{5}{(s+2)^2+16}$$

$$= \frac{s+2-2}{(s+2)^2+16} - \frac{5}{(s+2)^2+16} = \frac{s+2}{(s+2)^2+16} - \frac{2}{(s+2)^2+16}$$

$$= \frac{s+2}{(s+2)^2+4^2} - \frac{7}{4}\cdot\frac{4}{(s+2)^2+4^2}$$

$$F(t) = e^{-2t}\cos 4t - \tfrac{7}{4}e^{-2t}\sin 4t$$

$$= e^{-2t}\left[\cos 4t - \tfrac{7}{4}\sin 4t\right]$$

(iv) $$\frac{s^2-3s-4}{(s-3)(s-1)^2} = F(s) = \frac{A}{s-3} + \frac{B}{(s-1)^2} + \frac{C}{s-1}$$

$$A: \left.\frac{s^2-3s-4}{(s-1)^2}\right|_{s=3} = \frac{3^2-3(3)-4}{(3-1)^2} = -1$$

$$B: \frac{s^2-3s-4}{s-3} \text{ at } s=1 \quad \frac{1^2-3(1)-4}{(1-3)} = 3$$

$$C: \frac{d}{ds}\left[\frac{s^2-3s-4}{s-3}\right] s=1 \Rightarrow \frac{(s-3)(2s-3)-(s^2-3s-4)}{(s-3)^2}$$

at $s=1$

$$\frac{(1-3)(2(1)-3)-(1^2-3(1)-4)}{(1-3)^2} = 2$$

$$F(s) = \frac{1}{s-3} + \frac{3}{(s-1)^2} + \frac{2}{s-1}$$

$$F(t) = -e^{3t} + 3te^{t} + 2e^{t}$$

$$= e^{t}[3t+2] - e^{3t}$$