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 75/ENH05/004
 MECHATRONICS.

$$① (1-x^2) \frac{d^2y}{dx^2} - 2xy \frac{dy}{dx} + 2y = 0.$$

$$(1-x^2)y'' - 2xy' + 2y = 0.$$

$$y = (1-x^2)y''$$

$$v = (1-x^2) \quad v' = -2x \quad v'' = -2 \quad v''' = 0.$$

$$u = y'' \quad u^n = y^{n+2}.$$

$$y^n = u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2} u^{(n-2)} v'' + \dots$$

$$= y^{n+2} \cdot (1-x^2) + (n+2) y^{n+1} \cdot (-2x) + \frac{(n+2)(n+1)}{2} y^n \cdot (-2).$$

$$= (1-x^2)y^{n+2} - 2x(n+2)y^{n+1} + \frac{(-2)(n+2)(n+1)}{2!} y^n.$$

$$= y = -2xy'$$

$$v = -2x \quad v' = -2 \quad v'' = 0.$$

$$u = xy' \quad u^n = y^{n+1}$$

$$y^n = y^{(n+1)} \cdot (-2x) + (n+1) y^n \cdot (-2).$$

$$= -2xy^{n+1} - 2(n+1)y^n.$$

$$= y = 2y$$

$$v = 2 \quad v' = 0$$

$$u = y \quad u^n = y^n.$$

$$y^n = 2y^n.$$

$$y^n = (1-x^2)y^{n+2} - 2x(n+2)y^{n+1} - \frac{2(n+2)(n+1)}{2} y^n - 2xy^{n+1} - 2(n+1)y^n = 0.$$

at $x=0$

$$0 = y^{n+2} - (n^2 - n)y^n - 2ny^n + 2y^n.$$

$$y^{n+2} = [n^2 - n + 2n - 2]y^n.$$

$$y^{n+2} = [n^2 + n - 2]y^n.$$

$$n = 1, 2, 3, 4, \dots$$

$$n = 1$$

$$y^3 = y^1 = [1^2 - 1 + 2 - 2]y^1 = 0.$$

$$n = 2.$$

$$y^4 = [2^2 + 2 - 2]y^2 = 4y^2$$

$$n = 3$$

$$y^5 = [3^2 + 3 - 2]y^3 = 10y^3 = 0.$$

$$n = 4$$

$$y^6 = y^4 = [4^2 + 4 - 2]y^4 = 16y^4 = 16y^4 = 16[4y^2].$$

$$n=6$$

$$y^6 \rightarrow y^{(6)} = [s^2 + 6 - 2] y^{(6)} \rightarrow 4y^{(6)} \rightarrow [4 \times 16 \times 4] y^{(6)}$$

$$y = 1 + y^2 + y' + \frac{y''}{2!} + \frac{y'''}{3!} + \frac{16y^{(4)}}{4!} + \frac{40[16 \times 4] y^{(5)}}{5!} + \frac{40[16 \times 4] y^{(6)}}{6!}$$

$$y = 1 + y^2 + y' + y'' \cdot \left[\frac{1}{2} + \frac{1}{6} + \frac{1}{10} + \frac{1}{14} \right]$$

$$\begin{aligned} 2) i) \quad L[3e^{-4t} - 5e^{4t}] &= 3L[e^{-4t}] - 5L[e^{4t}] \\ &= 3 \left[\frac{1}{s+4} \right] - 5 \left[\frac{1}{s-4} \right] \\ &= \frac{3}{s+4} - \frac{5}{s-4} \end{aligned}$$

$$\begin{aligned} ii) \quad L[\sin 4t + \cos 4t] &= L[\sin 4t] + L[\cos 4t] \\ &= \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2} \\ &= \frac{4}{s^2+16} + \frac{s}{s^2+16} \end{aligned}$$

$$\begin{aligned} iii) \quad L[t^3 + 2t^2 - t + 4] &= L[t^3] + 2L[t^2] - L[t] + L[4] \\ &= \frac{3}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s} \end{aligned}$$

$$\begin{aligned} iv) \quad L[e^{-2t} \cos 5t] &= L[\cos 5t] = \frac{s}{s^2+25} \\ L[e^{-2t} \cos 5t] &= \frac{s}{(s+2)^2+25} \end{aligned}$$

$$\begin{aligned} v) \quad L[t \sin 3t] &= L[\sin 3t] = \frac{3}{s^2+9} \\ L[t \sin 3t] &= -\frac{d}{ds} \left[\frac{3}{s^2+9} \right] \\ u &= 3 \quad v = s^2+9 \\ du &= 0 \quad dv = 2s \\ -\frac{d}{ds} \left[\frac{3}{s^2+9} \right] &= - \left[\frac{vdu - u dv}{v^2} \right] = - \left[\frac{0 - 3(2s)}{(s^2+9)^2} \right] = \frac{+6s}{(s^2+9)^2} \end{aligned}$$

$$vi) L \left[\frac{e^{-t} - e^{-2t}}{t} \right]$$

$$L[e^{-t} - e^{-2t}] = \frac{1}{s+1} - \frac{1}{s+2}$$

$$L \left[\frac{e^{-t} - e^{-2t}}{t} \right] = \int_{s=0}^{\infty} F(s) ds$$

$$= \int_0^{\infty} \left(\frac{1}{s+1} - \frac{1}{s+2} \right) ds$$

$$= \int_0^{\infty} \frac{1}{s+1} ds - \int_0^{\infty} \frac{1}{s+2} ds$$

$$= \left[\ln(s+1) - \ln(s+2) \right]_0^{\infty}$$

$$= \ln \left[\frac{s+1}{s+2} \right] \Big|_0^{\infty}$$

$$= 0 - \ln \frac{1}{2}$$

$$vii) L[e^{4t} \cos 2t]$$

$$L[\cos 2t] = \frac{s}{s^2+4}$$

$$L[e^{4t} \cos 2t] = \frac{(s-4)}{(s-4)^2+4}$$

$$viii) L[t \sin 2t]$$

$$L[\sin 2t] = \frac{2}{s^2+4}$$

$$L[t \sin 2t] = -\frac{d}{ds} \left[\frac{2}{s^2+4} \right]$$

$$u = 2 \quad v = s^2 + 4$$

$$du = 2s \quad dv = 2s$$

$$-\frac{d}{ds} \left[\frac{2}{s^2+4} \right] = - \left[\frac{v du - u dv}{v^2} \right] = - \left[\frac{0 - 4s}{(s^2+4)^2} \right] = \frac{4s}{(s^2+4)^2}$$

$$ix) L[e^2 + 4e^2 + 5] = L[e^2] + L[4e^2] + L[5]$$

$$= \frac{2}{s} + \frac{4}{s} + \frac{5}{s}$$

$$x) L[t^2 \cos t]$$

$$L[\cos t] = \frac{s}{s^2+1} = L[t^2 \cos t] = -\frac{d^2}{ds^2} \left[\frac{s}{s^2+1} \right]$$

$$u = s$$

$$du = 1$$

$$v = s^2 + 1$$

$$dv = 2s$$

$$\rightarrow \frac{v du - u dv}{v^2} = \frac{(s^2 + 1) - 2s^2}{(s^2 + 1)(s^2 + 1)}$$

$$u = (s^2 + 1) - 2s^2 \quad du = 2s - 4s = -2s$$

$$v = (s^2 + 1)(s^2 + 1) \quad dv = 4s^3 + 4s$$

$$\begin{aligned} \therefore -\frac{d^2}{ds^2} \left[\frac{s}{s^2 + 1} \right] &= \frac{-(s^2 + 1)^2(-2s) - (1 - s^2)(4s^3 + 4s)}{(s^4 + 2s^2 + 1)^2} \\ &= \frac{-(s^4 + 2s^2 + 1)(-2s) - (1 - s^2)(4s^3 + 4s)}{(s^4 + 2s^2 + 1)^2} \\ &= \frac{-(2s^5 + 4s^4 - 6s)}{(s^4 + 2s^2 + 1)^2} \end{aligned}$$

$$xii) \quad L \left[\frac{\sinh 2t}{t} \right]$$

$$L[\sinh 2t] = \frac{2}{s^2 - 4}$$

$$L \left[\frac{\sinh 2t}{t} \right] = \int_0^{\infty} f(s) ds = \int_0^{\infty} \frac{2}{s^2 - 4} ds$$

$$= 2 \int_0^{\infty} \frac{1}{s^2 - 4} ds = 2 \ln(s^2 - 4)$$

$$x) \quad L[e^{3t}(t^2 + 4)]$$

$$L[(t^2 + 4)] = \frac{2}{s^3} + \frac{4}{s}$$

$$L[e^{3t}(t^2 + 4)] = \frac{2}{(s-3)^2} + \frac{4}{(s-3)}$$

$$\text{at } s-3=0, s=3$$

Sub. in (v).

$$3-5 = A(3-4) + 0.$$

$$-2 = -A.$$

$$A = 2.$$

$$\text{at } s-4=0, s=4$$

$$4-5 = 0 + B(4-3).$$

$$-1 = B$$

$$B = -1$$

$$\Rightarrow \mathcal{L}^{-1} \left[\frac{2}{s-3} + \frac{(-1)}{(s-4)} \right]$$
$$= 2e^{3t} - e^{4t}$$

$$(ii) \frac{2s-6}{(s-2)(s-4)} \Rightarrow \frac{A}{s-2} + \frac{B}{s-4} \Rightarrow \frac{A(s-4) + B(s-2)}{(s-2)(s-4)}.$$

$$2s-6 = A(s-4) + B(s-2).$$

$$\text{at } s-4=0, s=4.$$

$$2(4) - 6 = 0 + B(4-2)$$

$$2 = 2B$$

$$B = 1$$

$$\text{at } s-2=0, s=2$$

$$2(2) - 6 = A(2-4) + 0.$$

$$-2 = -2A$$

$$A = 1.$$

$$\mathcal{L}^{-1} \left[\frac{1}{s-2} + \frac{1}{s-4} \right]$$
$$= e^{2t} + e^{4t}$$

$$(iii) \frac{5s-8}{s(s-4)} \Rightarrow \frac{A}{s} + \frac{B}{s-4} \Rightarrow \frac{A(s-4) + Bs}{s(s-4)}.$$

$$5s - 8 = A(s-4) + Bs$$

$$\text{at } s=0$$

$$0 - 8 = A(-4)$$

$$A = 2$$

$$\text{at } s-4=0 \quad s=4$$

$$5(4) - 8 = B(4)$$

$$12 = 4B$$

$$B = 3$$

$$L^{-1} \left[\frac{2}{s} + \frac{3}{s-4} \right]$$

$$= 2 + 3e^{4t}$$

$$(iv) \frac{s^2 - 3s - 4}{(s-3)(s-1)^2} \Rightarrow \frac{A}{s-3} + \frac{B}{s-1} + \frac{C}{(s-1)^2}$$

$$\frac{A(s-1)(s-1)^2 + B(s-3)(s-1)^2 + C(s-3)(s-1)}{(s-3)(s-1)(s-1)^2} = \frac{s^2 - 3s - 4}{(s-3)(s-1)^2}$$

$$s^2 - 3s - 4 = A(s-1)^2 + B(s-1)^2 + B(s-3)(s-1) + C(s-3)$$

$$8^2 - 3(2) - 4 = A(8-1)^2 \Rightarrow A = -1$$

$$1^2 - 3(1) - 4 = C(1-3) \Rightarrow C = 3$$

$$s^2 - 3s - 4 = [s^2 - 2s + 1]A + [s^2 - 4s + 3]B + (s-3)C$$

$$= -2A - 4B + C = -3$$

$$-2(-1) - 4(B) + 3 = -3$$

$$-4B = -3 - 3 - 2$$

$$B = 2$$

$$L^{-1} \left[\frac{-1}{s-3} + \frac{2}{s-1} + \frac{3}{(s-1)^2} \right]$$

$$= -e^{3t} + 2e^t + 3te^t$$

$$(v) \frac{s-5}{s^2+4s+20} = \frac{s-5}{s^2+4s+16+4} = \frac{s-5}{(s+2)^2+4}$$

$$\frac{s-5}{(s+2)^2+4} \Rightarrow (e^{2t} - 7) \cos 2t$$