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CIVIL ENGINEERING
ENG 381 ASSIGNMENT 4

1) $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)\frac{d^2y}{dx^2} = (1-x^2)y^u$

$u = y^2 \quad u^n = y^{(n+2)}$

$v = 1-x^2 \quad v' = -2x \quad v^2 = -2 \quad v^3 = 0$

$w^n = u^n v + n u^{(n-1)} v' + \frac{n(n-1)u^{(n-2)}}{2!} v^2 + \frac{n(n-1)(n-2)u^{(n-3)}}{3!} v^3$

$= y^{(n+2)} \cdot (1+x^2) + n y^{(n+1)} \cdot -2x + \frac{n(n-1)}{\cancel{2}} y^n + 0$

$w^n = (1-x^2)y^{(n+2)} - 2xny^{(n+1)} - n(n-1)y^n$

$= (1-x^2)y^{(n+2)} - 2xny^{(n+1)} - (n^2-n)y^n$

$W_2 = -2x\frac{dy}{dx} = -2xy'$

$u = y' \quad u^n = y^{n+1}$

$v = -2x \quad v' = -2 \quad v^2 = 0$

$= y^{(n+1)} \cdot -2x + ny^n \cdot -2 + 0$

$= -2xy^{(n+1)} - 2ny^n$

$W_3 = 2y$

$u = y \quad u^n = y^n$

$v=2 \quad v'=0$

$y^{n+2}+0$

$=2y^n$

$=(1-x^2)y^{(n+2)}-2xny^{(n+1)}-2xy^{(n+1)}-(n^2-n)y^n-2ny^n+2y^n$

$=(1-x^2)y^{n+2}-2xy^{(n+1)}(n+1)-y^n(n^2-n+2n-2)$

$=(1-x^2)y^{n+2}-(n+1)2xy^{(n+1)}-(n^2+n-2)y^n$

$a+x=0$

$=(1-0)y^{(n+2)}-(n+1)2(0)y^{(n+1)}-(n^2+n-2)y^n$

$y^{(n+2)}-(n^2+n-2)y^n$

$[y^{(n+2)}]_0 = (n^2+n-2)y^n$

at $n=0$

$[y^2]_0=-2[y^0]_0$

at $n=1$

$[y^3]_0=0$

at $n=2$

$[y^4]_0=4[y^2]_0=4\times-2[y^0]_0=-8[y^0]_0$

at $n=3$

$[y^5]_0=10[y^3]_0=10\times 0=0$

at $n=4$

$[y^6]_0=18[y^4]_0=18\times-8[y^0]_0=-144[y^0]_0$

$y=y_0+x(y')_0+\frac{x^2}{2!}[y^2]_0+\frac{x^3}{3!}[y^3]_0+\ldots\frac{x^r}{r!}[y^r]_0$

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$$= y_0 + x(y')_0 + \frac{x^2}{2!} \cdot -2[y^\circ]_0 + \cancel{\frac{x^3}{3!}}\,0 + \frac{x^4}{4!} \cdot -8[y^\circ]_0 + \cancel{\frac{x^5}{5!}}\,0 +$$

$$\frac{x^6}{6!} \cdot -144[y^\circ]_0$$

$$= [y^\circ]_0 + x(y')_0 - x^2[y^\circ]_0 - \frac{x^4}{3}[y^\circ]_0 - \frac{x^6}{5}[y^\circ]_0$$

$$y_0 \left[1 - x^2 - x^4/3 - x^6/5\right] + x[y']_0$$

2) i) $L[3e^{-4t} - 5e^{4t}]$

$L[3e^{-4t}] - L[5e^{4t}]$

$$\frac{3}{s+4} - \frac{5}{s-4}$$

ii) $L[\sin 4t + \cos 4t]$

$L[\sin 4t] + L[\cos 4t]$

$$\frac{4}{s^2+16} + \frac{4}{s^2+16}$$

$$= \frac{4+s}{s^2+16}$$

iii) $L[t^3 + 2t^2 - t + 4]$

$$t^n = \frac{n!}{s^{n+1}}$$

$$= 3!/s^4 + 2\left[2!/s^3\right] - 1/s^2 + 4/s$$

$= \frac{6}{s^4} + \frac{4}{s^3} + \frac{1}{s^2} + \frac{4}{s}$

iv $L\left[e^{-2t}\cos 5t\right]$

$L[\cos 5t] = \frac{s}{s^2+25}$

$L\left[e^{-2t}\cos 5t\right] = \frac{s+2}{(s+2)^2+25}$

$= \frac{s+2}{(s+2)^2+25}$

v $t\sin 3t$

$L[\sin 3t] = \frac{3}{s^2+9}$

$L[t\sin 3t] = -1\frac{d}{ds}\left[\frac{3}{s^2+9}\right]$

$u = 3 \quad du = 0$

$v = s^2+9 \quad dv = 2s$

$-1\left[\frac{0-6s}{(s^2+9)^2}\right] = \frac{6s}{(s^2+9)^2}$

vi) $e^{-t} - e^{-2t}$

$L\left[e^{-t} - e^{-2t}\right]$

$= \frac{1}{s+1} - \frac{1}{s+2}$

$L\left[\frac{e^{-t} - e^{-2t}}{t}\right] = \int_{\sigma=s}^{\infty} \frac{1}{s+1} - \frac{1}{s+2}$

$= \int_{\sigma}^{\infty} \frac{1}{\sigma+1} - \frac{1}{\sigma+2}$

$\ln(\sigma+1) - \ln(\sigma+2)$

$= \ln\left[\frac{\sigma+1}{\sigma+2}\right]$

$= \ln\left[\frac{s+1}{s+2}\right]$

$= -\ln\left[\frac{s+1}{s+2}\right]$

$\ln\left[\frac{s+1}{s+2}\right]^{-1} = \ln\left[\frac{s+2}{s+1}\right]$

vii) $L\left[e^{4t}\cos 2t\right]$

$L[\cos 2t] = s/s^2+4$

$L\left[e^{4t}\cos 2t\right] = \frac{s-4}{(s-4)^2+4}$

viii) $t\sin 2t$

$L[\sin 2t] = 2/s^2+4$

$L[t\sin 2t] = -1\frac{ds}{ds}\left[\frac{2}{s^2+4}\right]$

$u = 2 \quad du = 0$

$v = s^2+4 \quad dv = 2s$

$$-1\left[\frac{0-4s}{(s^2+4)^2}\right] = \frac{4s}{(s^2+4)^2}$$

1x) t^3+4t^2+5

$$L[t^3] + L[4t^2] + L[5]$$

$$= \frac{3!}{s^4} + 4\left[\frac{2!}{s^3}\right] + \left[\frac{5}{s}\right]$$

$$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$$

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x) $t^2 \cos t$

$$L[\cos t] = \frac{s}{s^2+1}$$

$$L[t^2 \cos t] = (-1)^2 \frac{d^2}{ds^2}\left[\frac{s}{s^2+1}\right]$$

$$\frac{d}{ds}\left[\frac{s}{s^2+1}\right]$$

$u = s \quad du = 1$

$v = s^2+1 \quad dv = 2s$

$e^{3t}(t^2+4)$

$$L[t^2 e^{3t} + 4e^{3t}]$$

$$L[t^2 e^{3t}] = 2/s^3$$

$$= \frac{2}{(s-3)^3}$$

$$L[4e^{3t}] = \frac{4}{s-3}$$

$$\therefore e^{3t}[t^2+4] = \frac{2}{(s-3)^3} + \frac{4}{s-3}$$

$\dfrac{\sinh 2t}{t}$

$$L[\sinh 2t] = 2/s^2-4$$

$$L\left[\frac{\sinh 2t}{t}\right] = \int_{\sigma=s}^{\infty} \frac{2}{s^2-4}$$

$$= 2\int_{\sigma}^{\infty} = \frac{1}{\sigma^2-4}$$

$$= 2\left[\frac{1}{2}\frac{\tan^{-1}\sigma}{2}\right]_s^{\infty} = \left[\frac{\tan^{-1}\sigma}{2}\right]_s^{\infty}$$

$$\tan^{-1}\frac{\infty}{2} - \tan^{-1} s/2$$

$$-\tan^{-1} s/2$$

$$\left[\tan^{-1}\frac{s}{2}\right]^{-1}$$

$$= \tan^{-1}\frac{2}{s}$$

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3) convert the following to time (t) domains

i) $\frac{s-5}{(s-3)(s-4)}$

$$L^{-1}\left[\frac{A}{s-3} + \frac{B}{s-4}\right]$$

$$\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$$

$s-5 = A(s-4) + B(s-3)$

at $s = 4$

$4-5 = A(4-4) + B(4-3)$

$-1 = A(0) + B(-1)$

$-1 = 0 + B$

$B = -1$

at $s = 3$

$3-5 = A(3-4) + B(3-3)$

$-2 = -A + B(0)$

$A = 2$

$$L^{-1}\left[\frac{2}{s-3} + \frac{-1}{s-4}\right]$$

$x(t) = 2e^{3t} - e^{4t}$

ii) $\frac{2s-6}{(s-2)(s-4)}$

$$L^{-1}\left[\frac{A}{(s-2)} + \frac{B}{(s-4)}\right]$$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$$

$2s-6 = A(s-4) + B(s-2)$

at $s = 4$

$2(4)-6 = 4(4-4) + B(4-2)$

$2 = 2B$

$B = 1$

at $s = 2$

$2(2)-6 = A(2-4) + B(2-2)$

$-2 = -2A$

$A = 1$

$$x(t) = L^{-1}\left[\frac{1}{(s-2)} + \frac{1}{(s-4)}\right]$$

$x(t) = e^{2t} + e^{4t}$

iii) $\frac{5s-8}{s(s-4)}$

$$L^{-1} = \left[\frac{A}{s} + \frac{B}{s-4}\right]$$

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$$\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$5s-8 = A(s-4) + Bs$

at $s=4$

$5(4)-8 = A(4-4)+B(4)$

$12 = 4B$

$B = 12/4 = 3$

at $s=0$

$5(0)-8 = A(0-4)+B(0)$

$-8 = -4A$

$A = -8/-4 = 2$

$$L^{-1}\left[\frac{2}{s} + \frac{3}{s-4}\right]$$

$x(t) = 2 + 3e^{4t}$

iv) $$\frac{s^2-3s-4}{(s-3)(s-1)^2}$$

$$= L^{-1}\left[\frac{A}{s-3} + \frac{B}{s-1} + \frac{C}{(s-1)^2}\right]$$

$s^2-3s-4 = A(s-1)^2+B(s-3)(s-1)+C(s-3)$

$s^2-3s-4 = A(s-1)^2+B(s-3)(s-1)+C(s-3)$

at $s=3$

$(3)^2-3(3)-4 = A(3-1)^2+B(3-3)(3-1)+C(3-3)$

$-4 = 4A$

$\frac{-4}{4} = \frac{A4}{4} = -1$

at $s=1$

$1^2-3(1)-4 = A(1-1)^2+B(1-3)(1-1)+C(1-3)$

$-6 = -2C$

$C = 6/2$

$C = 3$

$s^2-3s-4 = As^2-2As+A+Cs-Cs+Bs-Bs+3B$

$A+3B-3C = -4$

$-1+3B-3(3) = -4$

$3B = 10-4$

$3B = 6$

$B = 6/3$

$B = 2$