

15/ENG04/002

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ELECT/ELECT ENG

ENG381 ASS

$$1.) (1-x^2) \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 2y = 0$$

$$(1-x^2) y^{(2)} - 2x y^{(1)} + 2y^{(0)} = 0$$

$$\text{let } w_1 = (1-x^2) y^{(2)}$$

$$u = y^{(2)} \quad v = (1-x^2)^2$$

$$u^{(n)} = y^{(n+2)} \quad v^{(1)} = -2x$$

$$u^{(n-1)} = y^{(n+1)} \quad v^{(2)} = -2$$

$$u^{(n-2)} = y^{(n)}$$

$$w_1^{(n)} = (1-x^2) y^{(n+2)} + n(y^{(n+1)})(-2x) + \frac{n(n-1)}{2} (y^{(n)}) (-2)$$

$$w_1^{(n)} = (1-x^2) y^{(n+2)} - 2xn y^{(n+1)} - (n^2-n) y^{(n)}$$

$$w_2 = -2x y^{(1)}$$

$$u = y^{(1)} \quad v = x$$

$$u^{(n)} = y^{(n+1)} \quad v^{(1)} = 1$$

$$u^{(n-1)} = y^{(n)}$$

$$w_2^{(n)} = x y^{(n+1)} + n y^{(n)}$$

$$w_3 = y^{(0)}$$

$$w_3^{(n)} = y^{(n)}$$

$$\therefore (1-x^2) y^{(n+2)} - 2xn y^{(n+1)} - (n^2-n) y^{(n)} - 2x y^{(n+1)} - 2n y^{(n)} + 2y^{(n)} = 0$$

$$y^{(n+2)} (1-x^2) - 2x y^{(n+1)} (n+1) - y^{(n)} (n^2-n+2n-2) = 0$$

$$y^{(n+2)} (1-x^2) - 2x y^{(n+1)} (n+1) - y^{(n)} (n^2+n-2) = 0$$

$$\text{let } x = 0$$

$$\therefore y^{(n+2)} - y^{(n)} (n^2+n-2) = 0$$

$$y^{(n+2)} = y^{(n)} (n^2+n-2)$$

$$n=0: y^{(2)} = y^{(0)} (-2)$$

$$n=1: y^{(3)} = y^{(1)} (0) = 0$$

$$n=2: y^{(4)} = y^{(2)} (4) = -8(y^{(0)})$$

$$n=3: y^{(5)} = y^{(3)} (10) = 0$$

$$n=4: y^{(6)} = y^{(4)} (18) = -8(18) y^{(0)}$$

$$y = y^{(0)} + xy^{(1)} + \frac{x^2}{2!} y^{(2)}(-2) + 0 \frac{x^4}{4!} (8y^{(4)}) + 0 \frac{x^6}{6!} (-8(18)y^{(6)}) - \dots$$

$$y = y^{(0)} + xy^{(1)} - x^2 y^{(2)} - \frac{x^4}{3} y^{(4)} - \frac{x^6}{5} y^{(6)} - \dots$$

$$y = xy^{(1)} + y^{(0)} \left(1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5}\right)$$

2.) (i) $x(t) = 3e^{-4t} - 5e^{4t}$

$$x(s) = L[x(t)] = 3L[e^{-4t}] - 5L[e^{4t}]$$

$$x(s) = \frac{3}{s+4} - \frac{5}{s-4}$$

(ii) $x(t) = \sin 4t + \cos 4t$

$$x(s) = \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2}$$

(iii) $x(t) = t^3 + 2t^2 - t + 4$

$$x(s) = \frac{3!}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

$$x(s) = \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

(iv) $x(t) = e^{-2t} \cos 5t$

$$L[\cos 5t] = \frac{s}{s^2+5^2}$$

$$x(s) = \frac{s+2}{(s+2)^2+25}$$

(v) $x(t) = t \sin 3t$

$$L[\sin 3t] = \frac{3}{s^2+9}$$

$$x(s) = -\frac{d}{ds} \left(\frac{3}{s^2+9} \right)$$

$$x(s) = -\frac{d}{ds} [3(s^2+9)^{-1}]$$

$$x(s) = -\frac{d}{ds} \left(\frac{-6s}{(s^2+9)^2} \right)$$

$$(vi) \frac{e^{-t} - e^{-2t}}{t}$$

$$\lim_{t \rightarrow 0} \frac{e^{-t} - e^{-2t}}{t} = \text{undefined}$$

Using L'Hopital's rule

$$\lim_{t \rightarrow 0} \frac{-e^{-t} + 2e^{-2t}}{1} = \text{real ans.}$$

$$L[e^{-t} - e^{-2t}] = \frac{1}{s+1} - \frac{1}{s+2}$$

$$L\left[\frac{e^{-t} - e^{-2t}}{t}\right] = \int_{\sigma=s}^{\infty} \left(\frac{1}{\sigma+1} - \frac{1}{\sigma+2}\right)$$

$$= \ln(\sigma+1) \Big|_s^{\infty} - \ln(\sigma+2) \Big|_s^{\infty}$$

$$= \left[\ln\left(\frac{\sigma+1}{\sigma+2}\right) \right]_s^{\infty}$$

$$= \ln \frac{\infty+1}{\infty+2} - \ln \frac{s+1}{s+2}$$

$$= \ln 1 - \ln \frac{s+1}{s+2}$$

$$= 0 - \ln \frac{s+1}{s+2}$$

$$= -\ln \frac{s+1}{s+2}$$

$$= \ln \frac{s+2}{s+1}$$

$$(vii) \frac{e^{4t} \cos 2t}{\cos 2t} \times (t) = e^{4t} \cos 2t$$

$$L[\cos 2t] = \frac{s}{s^2+4}$$

$$x(s) = \frac{s-4}{(s-4)^2+4}$$

$$(viii) x(t) = t \sin 2t$$

$$L[\sin 2t] = \frac{2}{s^2+4}$$

$$x(s) = -\frac{d}{ds} \left(\frac{2}{s^2+4} \right)$$

$$x(s) = \frac{4s}{(s^2+4)^2}$$

$$(i) x(t) = t^3 + 4t^2 + 5$$

$$x(s) = \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$$

$$(x) x(t) = e^{3t} (t^2 + 4)$$

$$L[t^2 + 4] = \frac{2}{s^3} + \frac{4}{s}$$

$$x(s) = \frac{2}{(s-3)^3} + \frac{4}{s-3}$$

$$(xi) x(t) = t^2 \cos t$$

$$L[\cos t] = \frac{s}{s^2+1}$$

$$x(s) = \frac{d^2}{ds^2} \left(\frac{s}{s+1} \right)$$

$$x(s) = \frac{d}{ds} \left(\frac{d}{ds} \left(\frac{s}{s+1} \right) \right)$$

$$x(s) = \frac{d}{ds} \left(\frac{1}{s+1} - \frac{s}{(s+1)^2} \right)$$

$$x(s) = \frac{-1}{(s+1)^2} - \frac{1}{(s+1)^2} + \frac{2s}{(s+1)^3}$$

$$x(s) = \frac{2s}{(s+1)^3} - \frac{2}{(s+1)^2}$$

$$(xii) x(t) = \frac{\sinh 2t}{t}$$

$$L[\sinh 2t] = \frac{2}{s^2-4}$$

$$x(s) = \int_s^\infty \frac{2}{s^2-4}$$

$$x(s) = \int_s^\infty \frac{2}{(s-2)(s+2)}$$

$$\frac{2}{(s-2)(s+2)} = \frac{A}{s-2} + \frac{B}{s+2}$$

$$2 = A(s+2) + B(s-2)$$

$$2 = (A+B)s + 2A - 2B$$

Comparing both sides

$$2A - 2B = 2$$

$$A + B = 1 \quad \text{--- (1)}$$

$$A + B = 0 \quad \text{--- (2)}$$

$$2A = 1$$

$$A = \frac{1}{2}$$

Subst. into (1)

$$\frac{1}{2} + B = 1$$

$$B = -\frac{1}{2}$$

$$\therefore x(s) = \int_s^{\infty} \frac{1}{2(s-2)} - \frac{1}{2(s+2)}$$

$$x(s) = \frac{1}{2} \ln(s-2) \Big|_s^{\infty} - \frac{1}{2} \ln(s+2) \Big|_s^{\infty}$$

$$x(s) = \frac{1}{2} \left[\ln \frac{s-2}{s+2} \right]_s^{\infty}$$

$$x(s) = \frac{1}{2} \left(\ln \frac{\infty-2}{\infty+2} - \ln \frac{s-2}{s+2} \right)$$

$$x(s) = -\frac{1}{2} \ln \frac{s-2}{s+2}$$

$$x(s) = \ln \sqrt{\frac{s+2}{s-2}}$$

$$3(i) \quad x(s) = \frac{s-5}{(s-3)(s-4)}$$

$$x(s) = \frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$$

$$s-5 = A(s-4) + B(s-3)$$

$$s-5 = (A+B)s - 4A - 3B$$

Comparing both sides

$$A+B = 1 \quad \text{--- (1)}$$

$$-4A - 3B = -5 \quad \text{--- (2)}$$

From (1), $A = 1 - B$

So, $-4(1-B) - 3B = -5$

$$-4 + 4B - 3B = -5$$

$$B - 4 = -5$$

$$B = -1$$

$$A_2 = -(-1)$$

$$A_2 = 2$$

$$\therefore x(s) = \frac{2}{s-3} - \frac{1}{s-4}$$

$$x(t) = 2e^{3t} - e^{4t}$$

$$(iv) \frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$$

$$2s-6 = A(s-4) + B(s-2)$$

$$2s-6 = (A+B)s - 4A - 2B$$

$$\therefore A+B = 2$$

$$A = 2-B \quad \text{--- (1)}$$

$$-4A - 2B = -6 \quad \text{--- (2)}$$

$$-4(2-B) - 2B = -6$$

$$-8 + 4B - 2B = -6$$

$$2B - 8 = -6$$

$$2B = 2$$

$$B = 1$$

$$A = 2 - 1$$

$$A = 1$$

$$x(s) = \frac{1}{s-2} + \frac{1}{s-4}$$

$$x(t) = e^{2t} + e^{4t}$$

$$(v) x(s) = \frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$5s-8 = A(s-4) + Bs$$

$$5s-8 = (A+B)s - 4A$$

$$\therefore -8 = -4A$$

$$A = 2$$

$$A+B = 5$$

$$B = 5 - A$$

$$B = 5 - 2$$

$$B = 3$$

$$x(s) = \frac{2}{s} + \frac{3}{s-4}$$

$$x(t) = 2 + 3e^{4t}$$

$$(iv) \quad x(s) = \frac{s^2 - 3s - 4}{(s-3)(s-1)^2} = \frac{A}{s-3} + \frac{B}{s-1} + \frac{C}{(s-1)^2}$$

$$s^2 - 3s - 4 = A(s-1)^2 + B(s-1)(s-3) + C(s-3)$$

$$\text{let } s = 1$$

$$1 - 3 - 4 = -2C$$

$$-6 = -2C$$

$$C = 3$$

$$\text{let } s = 3$$

$$9 - 9 - 4 = 4A$$

$$A = -1$$

$$s^2 - 3s - 4 = -1(s-1)^2 + B(s-1)(s-3) + 3(s-3)$$

$$s^2 - 3s - 4 = -s^2 + 2s - 1 + B(s^2 - 3s - s + 3) + 3s - 9$$

$$s^2 - 3s - 4 = -s^2 + 2s - 1 + Bs^2 - 4Bs + 3B + 3s - 9$$

Comparing both sides

$$B - 1 = 1$$

$$B = 2$$

$$\therefore x(s) = \frac{-1}{s-3} + \frac{2}{s-1} + \frac{3}{(s-1)^2}$$

$$x(t) = -e^{3t} + 2e^t + 3te^t$$

$$x(t) = 3te^t + 2e^t - e^{3t}$$

$$(v) \quad x(s) = \frac{s-5}{s^2+4s+20} = \frac{s-5}{s^2+4s+16+4}$$

$$= \frac{s-5}{(s+2)^2+4}$$

$$= \frac{s-5+2-2}{(s+2)^2+4}$$

$$= \frac{s+2}{(s+2)^2+4} - \frac{7}{(s+2)^2+4}$$

$$= \frac{s+2}{(s+2)^2+2^2} - \frac{7}{(s+2)^2+2^2}$$

$$= \frac{s+2}{(s+2)^2+2^2} - \frac{7}{(s+2)^2+2^2}$$

$$= \frac{s+2}{(s+2)^2+2^2} - \frac{7}{(s+2)^2+2^2}$$

$$x(t) = e^{-2t} \cos 2t - 7e^{-2t}$$