1) $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)y^2 - 2xy' + 2y = 0$

$y^n = u^n v + n u^{n-1} u' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \dots n$

$[y^{(2+n)} \cdot (1-x^2 + n y^{(1+n)} \cdot (-2x) + \frac{n(n-1)}{2!} y^n \cdot (-2)] + [y^{(1+n)} \cdot -2x + n y^n \cdot -2 +$

$[2y^n] = 0$

$(1-x^2)y^{n+2} - 2xny^{n+1} - n(n-1)y^n - 2xy^{n+1} - 2ny^n + 2y^n = 0$

let $x = 0$

$y^{n+2} - n(n-)y^n - 2ny^n + 2y^n = 0$

$y^{n+2} + y^n[-n(n-1) - 2n + 2] = 0$

$y^{n+2} + y^n[-n^2 + n - 2n + 2] = 0$

$y^{n+2} + y^n[-n^2 - n + 2] = 0$

$y^{n+2} = -(y^n)_0 [-n^2 - n + 2]$

$n = 0 : y^2 = y^0 \cdot 2 = -2y^0$

$n = 1 : y^3 = -y' \cdot [0] = 0$

$n = 2 : y^4 = -y^2[-4] = 4y^2 = 4(-2y^0) = -8y^0$

$n = 3 : y^5 = -y^3 \cdot [-10] = 10y^3 = 0$

$n = 4 : y^6 = -y^4(-18) = 18y^4 = 18 \cdot -18y^0$

$n = 5 : y^7 = -y^5 \cdot [-28) = 28y^5 = 0$

$y = y^0 + xy' + \frac{x^2y^3}{2!} + \frac{x^3y^3}{3!} + \dots n$

$y = y^0 + xy' + \frac{x^2}{2!}(-2)y^0 + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(4)(-2)y^0 + \frac{x^5}{5!}(0) +$

$\frac{x^6}{6!}(18)4(-2)y^0 + \frac{x^7}{7!}(0)$

$y = y^0 \cdot \cancel{d} xy' - x^2y^0 - \frac{x^4}{3}y^0 - \frac{x^6}{5}y^0$

$y = y^0[1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5}] + y_0'[x]$

2i) $3e^{-4t} - 5e^{4t}$

$\Rightarrow L[3e^{-4t}-5e^{4t}] \Rightarrow L[3e^{-4t}] - L[5e^{4t}]$

$\Rightarrow 3\left[\frac{1}{s-a}\right] - 5\left[\frac{1}{s-a}\right]$

$\Rightarrow 3\left[\frac{1}{s+4}\right] - 5\left[\frac{1}{s-4}\right] \Rightarrow \frac{3}{s+4} - \frac{5}{s-4}$

ii) $\sin 4t + \cos 4t$

$L[\sin 4t + \cos 4t] \Rightarrow L[\sin 4t] + L[\cos 4t]$

$\Rightarrow \frac{a}{s^2+a^2} + \frac{s}{s^2+a^2} \Rightarrow \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2}$

$\Rightarrow \frac{4}{s^2+16} + \frac{s}{s^2+16} = \frac{4+s}{(s^2+16)}$

iii) $t^2 + 2t^2 - t + 4$

$t^n = \frac{n!}{s^{n+1}} \Rightarrow \frac{3!}{s^{3+1}} + 2\left[\frac{2!}{s^{2+1}}\right] - \left[\frac{1!}{s^{1+1}}\right] + \frac{4}{5}$

$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{5}$

iv) $e^{-2t} \cos 5t$

$L[\cos 5t] = \frac{s}{s^2+a^2}$

$\Rightarrow \frac{s}{s^2+5^2} \Rightarrow \frac{s}{s^2+25}$

$L[\cos 5t] = \frac{s+2}{[s+2]^2+25}$

v) $t \sin 3t$

$L[\sin 3t] = \frac{a}{s^2+a^2}$

$= \frac{3}{s^2+3^2} \Rightarrow \frac{3}{s^2+9}$

$L[t \sin 3t] = -F'(s)$

$u = 3$ $\quad \frac{du}{ds} = 0$

$v = s^2 + 9$ $\quad \frac{dv}{ds} = 2s$

$$\frac{v\frac{du}{ds} - u\frac{dv}{ds}}{v^2}$$

$$= \frac{[s^2+9]\cdot 0 - 3(2s)}{[s^2+9^2]}$$

$$= \frac{-6s}{[s^2+9]^2}$$

$= F'(s) = -1\left[\frac{-6s}{[s^2+9]^2}\right]$

$$= \frac{6s}{[s^2+9]^2}$$

$$L\left[\frac{e^{-t} - e^{-2t}}{t}\right] \Rightarrow L[f(t)] = e^{-t} - e^{-2t} = \frac{1}{s+a} - \frac{1}{s+a}$$

$$= \frac{1}{s+1} - \frac{1}{s+2}$$

$$F\sigma = \frac{1}{\sigma+1} - \frac{1}{\sigma+2}$$

$$\int F\sigma \; L\left[\frac{f(t)}{t}\right] = \int_{a=s}^{\infty} \frac{1}{\sigma+1} - \frac{1}{\sigma+2}\, d\sigma$$

$$= \int_{\sigma=2}^{\infty} \frac{1}{\sigma+1}\, d\sigma - \int_{\sigma=s}^{\infty} \frac{1}{\sigma+2}\, d\sigma$$

$$\ln[(\sigma+1) - \ln[\sigma+2)]_s^{\infty}$$

$$= [\ln(\sigma+1) - \ln(\sigma+1)]_s^{a}$$

$$\Rightarrow \left[\ln\frac{\sigma+1}{(\sigma+2)}\right]_s^{\infty} = \ln\left[\frac{\alpha+1}{(\alpha+2)} - \frac{s+1}{s+2}\right]$$

$$= -\ln\left[\frac{s+1}{s+2}\right] = \ln\left[\frac{s+2}{s+1}\right]$$

(vii) $e^{4t}\cos 2t$

$$L(\cos 2t] = \frac{s}{s^2+2^2} = \frac{s}{s^2+4}$$

$$L[e^{4t}\cos 2t] = \frac{s-4}{[s-4]^2+4}$$

(viii) $t\sin 2t$

$$L[t\sin 2t] = \frac{-d}{ds}[F(s)]$$

$$F(s) = L[\sin 2t] = \frac{2}{s^2+2^2}$$

$$F(s) = \frac{2}{s^2+4}$$

$u = 2 \quad \frac{du}{ds} = 0$

$v = s^2+4 \quad \frac{dv}{ds} = 2s$

$$\frac{(s^2+4)\cdot 0 - 2(2s)}{(s^2+4)^2}$$

$$= \frac{-4s}{(s^2+4)^2}$$

$$L[t\sin 2t] = f'(s)$$

$$= -1\cdot\left[\frac{-4s}{(s^2+4)^2}\right]$$

$$= \frac{4s}{[s^2+4]^2}$$

(ix) $t^3 + 4t^2 + 5$

$$L[t^3 + 4t^2 + 5]$$

$$= \frac{3!}{s^{3+1}} + 4\left[\frac{2!}{s^{2+1}}\right] + \frac{5}{s}$$

$$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$$

(x) $e^{3t}(t^2+4)$

let $x = t^2+4$

$$L[e^{3t}x]$$

$$L[x] = L[t^2+4]$$

$$= L[t^2] + L[4]$$

$$= \frac{2!}{s^{2+1}} + \frac{4}{s}$$

$$= \frac{2}{s^3} + \frac{4}{s}$$

$$L[e^{3t}] = \frac{2}{[s-3]^3} + \frac{4}{[s-3]}$$

No 3

3) $\frac{s-5}{(s-3)(s-4)}$

$L^{-1}\left[\frac{s-5}{(s-3)(s-4)}\right] = \frac{A}{(s-3)} + \frac{B}{(s-4)}$

$\frac{s-5}{(s-3)(s-4)} = \frac{A(s-4)+B(s-3)}{(s-3)(s-4)}$

Take $s=4$

$\Rightarrow (4-5) = A(4-4) + B(4-3)$

$\Rightarrow -1 = B(1) \Rightarrow B = -1$

Assuming $s=3$

$\Rightarrow (3-5) = A(3-4) + B(3-3)$

$\Rightarrow -2 = A(-1) \Rightarrow A = 2$

$\frac{s-5}{(s-3)(s-4)} = \frac{2}{s-3} + \frac{-1}{s-4} \Rightarrow \frac{2}{s-3} - \frac{-1}{s-4} \Rightarrow 2\left[\frac{1}{s-3}\right] - \left[\frac{1}{s-4}\right]$

$= 2e^{3t} - e^{4t}$

ii) $\frac{2s-6}{(s-2)(s-4)} \Rightarrow L^{-1}\left[\frac{2s-6}{(s-2)(s-4)}\right]$

$\Rightarrow \frac{2s-6}{(s-2)(s-4)} = \frac{A}{(s-2)} + \frac{B}{(s-4)} \Rightarrow 2s-6 = A(s-4) + B(s-2)$

Assuming $s=4$

$2(4)-6 = A(4-4) + B(4-2) \Rightarrow 2 = 2B \Rightarrow B = 1$

Assuming $s=2$

$2(2)-6 = A(2-4) + B(2-2) \Rightarrow -2 = -2A \Rightarrow A = 1$

$\frac{2s-6}{(s-2)(s-4)} = \frac{1}{s-2} + \frac{1}{s-4}$

$L^{-1}\left[\frac{2s-6}{(s-2)(s-4)}\right] = e^{2t} + e^{4t}$

iii) $\frac{5s-8}{s(s-4)}$

$L^{-1}\left[\frac{5s-8}{s(s-4)}\right] = \frac{A}{s} + \frac{B}{s-4} \Rightarrow 5s-8 = A(s-4) + B(s)$

Assuming $s=4$

$5(4)-8 = A(4-4) + B(4) \Rightarrow 20-8 = 4B \Rightarrow 12 = 4B \Rightarrow B = 3$

Assuming $s=0$

$5(0)-8 = A(0-4) + B(0) \Rightarrow -8 = -4A \Rightarrow A = 2$

$L^{-1}\left[\frac{5s-8}{s(s-4)}\right] = \frac{2}{s} + \frac{3}{s-4}$

$= \frac{2}{s} + 3\left[\frac{1}{s-4}\right]$

$= 2 + 3e^{4t}$

V) $\frac{s-5}{s^2+4s+20}$

$L^{-1}\left[\frac{s-5}{s^2+4s+20}\right] \Rightarrow F(s) = \frac{s-5}{s^2+4s+4+6} \Rightarrow \frac{s}{(s+2)^2+16} - \frac{5}{(s+2)^2+16}$

$= \frac{s+2-2}{(s+2)^2+16} - \frac{5}{(s+2)^2+16} = \frac{s+2}{(s+2)^2+6} - \frac{2}{(s+2)^2+6}$

$= \frac{s+2}{(s+2)^2+4^2} - \frac{7}{4} \cdot \frac{4}{(s+2)^2+4^2}$

$T(t) = e^{-2t}\cos 4t - \frac{7}{4} e^{-2t}\sin 4t$

$\Rightarrow e^{-2t}\left[\cos 4t - \frac{7}{4}\sin 4t\right]$

V) $\frac{s^2-3s-4}{(s-3)(s-1)^2}$

$\Rightarrow T(s) = \frac{A}{s-3} + \frac{B}{(s-1)^2} + \frac{C}{s-1}$

$A = \left.\frac{s^2-3s-4}{(s-1)^2}\right|_{s=3} = \frac{3^2-3(3)-4}{(3-1)^2} = -1$

$B = \left.\frac{s^2-3s-4}{(s-3)}\right|_{s=1} = \frac{1^2-3(1)-4}{1-3} = 3$

$C = \frac{1}{dc}\left[\frac{s^2-3s-4}{s-3}\right]_{s=1} = \frac{(s-3)(2s-3)-(s^2-3(1)-4)}{(s-3)^2}$

$= \frac{(1-3)(2(1)-3)-(1^2-3-4)}{(1-3)^2} = \frac{8}{4} = 2$

$F(s) = \frac{-1}{s-3} + \frac{3}{(s-1)^2} + \frac{2}{s-1}$

$F(t) = -e^{3t} + 3te^{t} + 2e^{t}$

$\Rightarrow e^{t}[3t+2] - e^{3t}$