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- $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)y'' - 2xy' + 2y = 0$

$w_1 (1-x^2)y'' = \omega_1$

$v = 1-x^2 \quad v' = -2x \quad v^{(2)} = -2 \quad v^{(3)} = 0$

$u = y'' , \quad u^n = y^{n+2}, \quad u^{n+1} = y^{n+1} \quad u^{n-2} = y^n$

$w_1^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + 0$

$= y^{n+2}(1-x^2) + n y^{n-1}(-2x) + n(n-1)y^n$

$= (1-x^2)y^{n+2} - 2xny^{n-1} + n(n-1)y^n$

at $w_2 = -2xy'$

$v = -2x \quad v' = -2 \quad v'' = 0$

$u = y' \quad u^n = y^{n+1}$

$u_2^n = u^n v + n u^{n-1} v' + 0$

$= y^{n+1}(-2x) + ny^n(-2)$

$= -2xy^{n+1} - 2ny^n$

$w \quad w_3 = 2y$

$v = 2 \qquad v' = 0$

$u = y \quad u^n = y^n$

$w^n = u^n v + 0 = 2y^n$

$w_2 n - (1-x^2)y^{n+2} + 2xy^{n+1}(-n-2) + y^n(n^2-3n+n)$

$w = w_1^n + w_2^n + w_3^n$

at $x = 0$

$(y^{n+2}) = y^n(n^2-3n+n) = -y^n(n^2-2n)$

when $n = 0$

$(y^2) = 0$

$n = 1 \quad (y^3)_0 = (y^1)_0$

$n = 2$

$(y^4)_0 = 0$

$n = 3$

$(y^5)_0 = (-3y^3)_0 = -3(y^{(1)})_0$

$n = 4, \ (y^4)_0 = 0$

$n = 5$

$(y^5)_0 = -15y^5 = -15(-3y)_0$

hence

$$y = y_0 + xy_0' + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^4)_0 + \frac{x^5}{5!}(y)^5_0 + \frac{x^6}{6!}(y^6)_0$$

$$+ \frac{x^7}{7!}(y^7)_0 + \left(\frac{x^8}{8!}(y^8)\right)_0$$

$$y = y_0 + x(y)_0 + \frac{x^3}{6}(y)_0 + \frac{(5y_0)x^5}{40} + \frac{(3y^6)_0 x^7}{336}$$

$$y = y_0 + (y^1)_0 \left(x + \frac{x^3}{6} + \frac{x^5}{40} + \frac{x^7}{112} + \dots\right)$$

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a) $3e^{-4t} - 5e^{4t}$

$L\{3e^{-4t} - 5e^{4t}\} = L\{3e^{-4t}\} - L\{5e^{4t}\}$

$= \frac{3}{s+4} - \frac{5}{s-4}$

$\sin 4t + \cos 4t$

$L\{\sin 4t + \cos 4t\} = L(\sin 4t) + L(\cos 4t)$

$= \frac{4}{s^2+16} + \frac{s}{s^2+16}$

$L\{t^3 + 2t^2 - t + 4\} = L(t^3) + L(2t^2) - L(t) + L(4)$

$= \frac{3!}{s^{3+1}} + \frac{2 \cdot 2!}{s^{2+1}} - \frac{1!}{s^{1+1}} + \frac{4}{s}$

$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

$L\{e^{-2t}\cos 5t\}$

$L\{\cos 5t\} = \frac{s}{s^2+5^2} = \frac{s}{s^2+25}$

$L\{e^{-2t}\cos 5t\} = \frac{(s-(-2))}{(s+2)^2+25} = \frac{s+2}{(s+2)^2+25}$

$L(4\sin 3t)$

$L(\sin 3t) = \frac{3}{s^2+9}$

$L\{t\sin 3t\} = -\frac{d}{ds}\left(\frac{3}{s^2+9}\right)$

let $u = 3$; $v = s^2+9$

$\frac{du}{ds} = 0$ $\quad \frac{dv}{ds} = 2s$

$\frac{v\frac{du}{ds} - u\frac{dv}{ds}}{v^2}$

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$$= \frac{(s^2+9)(0) - (3)(2s)}{(s^2+9)^2}$$

$$f'(s) = \frac{-6s}{(s^2+9)^2}$$

Recall

$$L\{6\sin 3t\} = -f(s)$$

$$= x \frac{-6s}{(s^2+9)^2}$$

$$L(t^3 + 4t^2 + 5)$$

$$= L(t^3) + L(4t^2) + L(5)$$

$$= \frac{3!}{s^4} + \frac{4 \cdot 2!}{s^3} + \frac{5}{s}$$

$$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$$

3) $\frac{s-3}{(s-3)(s-4)}$

$$L^{-1}\left(\frac{s-3}{(s-3)(s-4)}\right) = L^{-1}\left(\frac{A}{s-3} + \frac{B}{s-4}\right)$$

Resolving into partial fraction

$s-5 = A(s-4) + B(s-3)$

$s-5 = As - 4A + Bs - 4B$

$s-4 \; (A+B)s - 4A - 3B$

$A + B = 1 \; \text{---} \; (i) \quad -4A - 3B = -5 \; \text{---} \; (ii)$

from eqn 1

$-4(1-B) - 3B = -5$

$-4 + 4B - 3B = -5$

$B = -5 + 4 = -1$

hence $A = 1-(-1) = 1+1 = 2$

$$L^{-1}\left(\frac{s-5}{(s-3)(s-4)}\right) = L^{-1}\left(\frac{2}{s-5} + \frac{1}{s-4}\right)$$

$$L^{-1}\left(\frac{s-3}{(s-3)(s-4)}\right) = 2e^{3t} + e^{3t}$$

$$L^{-1}\left(\frac{5s-8}{s(s-4)}\right) = L^{-1}\left(\frac{A}{s} + \frac{B}{s-4}\right)$$

$5s-8 = A(s-4) + Bs$

$5s - 8 = (A+B)s - 4A$

$A + B = 4 \quad \text{———} \; (i)$

$-4A = -8 \quad \text{———} \; (ii)$

from eqn (ii)

$A = \frac{2}{5}$

$$L^{-1}\left(\frac{2}{s} + \frac{3}{s-4}\right) = 2 \; -1 \; 3e^{4t}$$