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15/GroQ O4/054

ELECT - ELEC

ENG 381

(1) $3e^{-4t} - 5e^{4t}$

Soln;

$L(3e^{-4t}) = \frac{3}{s+4}$

$L(-5e^{4t}) = -5/_{s-4}$

$L[3e^{-4t} - 5e^{4t}] = 3/_{s+4} - 5/_{s-4}$

$\Rightarrow \frac{3(s-4) - 5(s+4)}{(s+4)(s-4)} =$

$\Rightarrow \frac{3s-12-5s-20}{(s+4)(s-4)} = \frac{-2s-32}{(s+4)(s-4)}$

$\Rightarrow \frac{-2(s+16)}{(s+4)(s-4)}$

(2) $\sin 4t + \cos 4t$

$L[\sin 4t] = \frac{4}{s^2+4^2} = \frac{4}{s^2+16}$

$L[\cos 4t] = \frac{s}{s^2+4^2} = \frac{s}{s^2+16}$

$L[\sin 4t + \cos 4t] = \frac{4}{s^2+16} + \frac{s}{s^2+16} = \frac{4+s}{s^2+16}$

(ii) $t^3 + 2t^2 - t + 4$

$L[t^3] = \frac{3!}{s^{3+1}} = \frac{6}{s^4}$

$L[2t^2] = \frac{2 \times 2!}{s^{2+1}} = \frac{4}{s^3}$

$L[-t] = \frac{-1!}{s^{1+1}} = -1/s^2$

$L[4] = 4/s$

$L[t^3 + 2t^2 - t + 4] = 6/s^4 + 4/s^3 - 1/s^2 + 4/s$

$= \frac{6 + 4s - s^2 + 4s^3}{s^4} = \frac{4s^3 - s^2 + 4s + 6}{s^4}$

(iii) $e^{-2t} \cos t$

$L[e^{-2t} \cos t] = 1/s+2$

$L[\cos 5t] = \frac{s}{s^2+5^2} = \frac{s}{s^2+25}$

$L[e^{-2t} \cos 5t] = \frac{s+2}{(s+2)^2+25}$

(iv) $t \sin 3t$

$L[\sin 3t] = \frac{3}{s^2+3^2} = \frac{3}{s^2+9}$

$L[t \sin 3t] = (-1) \, d/ds \left[\frac{3}{s^2+9}\right]$

$\Rightarrow -1 \left[\frac{(s^2+9)(0) - 3(2s)}{(s^2+9)^2}\right]$

$\Rightarrow -1 \left[\frac{-6s}{(s^2+9)^2}\right] = \frac{6s}{(s^2+9)^2}$

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(iv) $e^{4t} \cos 2t$

$$L[\cos 2t] = \frac{s}{s^2+2^2} = \frac{s}{s^2+4}$$

$$\Rightarrow \frac{s-4}{(s-4)^2+4}$$

(v) $t \sin 2t$

$$L[\sin 2t] = \frac{2}{s^2+2^2} = 2/s^2+4$$

$$L[t \sin 2t] = (-1) \, d/ds \, (2/s^2+4)$$

Using Quotient rule

$$\Rightarrow -1 \left[\frac{(s^2+4)(0) - 2(2s)}{(s^2+4)^2}\right]$$

$$\Rightarrow -1 \left(\frac{4s}{-(s^2+4)^2}\right) = \frac{4s}{(s^2+4)^2}$$

(vi) $t^3 + 4t^2 + 5$

$$L[t^3] = \frac{3!}{s^{3+1}} = 6/s^4$$

$$L[4t^2] = \frac{4 \times 2!}{s^{2+1}} = 8/s^3$$

$$L[5] = 4/s$$

$$L[t^3+4t^2+5] = 6/s^4 + 8/s^2 + 4/s$$

$$\Rightarrow \frac{6+8s+4s^3}{s^4} = \frac{4s^2+8s+6}{s^4}$$

(x) $e^{3t}(t^2+4)$

$L[t^2] = \frac{2!}{s^{2+1}} = 2/s^3$

$L[4] = 4/s$

$L[t^2+4] = 2/s^3 + 4/s$

$\Rightarrow \frac{2+4s^2}{s^3} = \frac{4s^2+2}{s^3}$

$L[e^{3t}[t^2+4]] = \frac{4(s-3)^2+2}{(s-3)^3}$

(x) $t^2 \cos t$

$L[\cos t] = s/s^2+1^2 = s/s^2+1$

$L[t^2 \cos t] = (-1)^2 \, d/ds \, [s/s^2+1]$

Using quotient rule

$= \left[\frac{(s^2+1)(1) - s(2s)}{(s^2+1)^2}\right]$

$\Rightarrow \frac{s^2+1-2s^2}{(s^2+1)^2} = \frac{1-s^2}{(s^2+1)^2}$

(xi) $\frac{\sinh 2t}{t} = t^{-1} \sinh 2t$

$L(\sinh 2t] = 2/s^2-2^2 = 2/s^2-4$

$L\left[\frac{\sinh 2t}{t}\right] = (-1)^1 \, d/ds \, [2/s^2-4]$

$\Rightarrow -1\left[\frac{(s^2-4)(0) - 2(2s)}{(s^2-4)^2}\right]$

$$\Rightarrow -1\left[\frac{-4s}{(s^2-4)^2}\right] = \frac{4s}{(s^2-4)^2}$$

(vi) $\dfrac{e^{-t}-e^{-2t}}{t} \Rightarrow t^{-1}\left(e^{-t}-e^{-2t}\right)$

$$L[e^{-t}] = 1/s+2$$

$$L[-e^{-2t}] = -1/s+2$$

$$L[e^{t}-e^{-2t}) = 1/s+1 - 1/s+2 = \frac{s+2-3-1}{(s+1)(s+2)}$$

$$\Rightarrow \frac{1}{(s+1)(s+2)}$$

$$\Rightarrow \frac{1}{s^2+3s+2}$$

$$L\left(\frac{e^{t}-e^{-2t}}{t}\right] = (-1)^{-1}\, d/ds\left(\frac{1}{s^2+3s+2}\right)$$

Using quotient rule.

$$\Rightarrow -1\left[\frac{(s^2+3s+2)(0) - (2s+3)(1)}{(s^2+3s+2)^2}\right]$$

$$\Rightarrow -1\left[\frac{-(2s+3)}{(s^2+3s+2)^2}\right]$$

$$\Rightarrow \frac{2s+3}{(s^2+3s+2)^2}$$

$\frac{s-5}{s^2+4s+20}$

$s^2+4s+20$

Using Quadratic Equation

$$s = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$s = \frac{-4 \pm \sqrt{4^2-4(20\times1)}}{2(1)}$$

$$s = \frac{-4 \pm \sqrt{16-80}}{2}$$

$$s = \frac{-4 \pm \sqrt{-64}}{24}$$

$$s = \frac{-4 \pm \sqrt{8}}{2}$$

$s = -2 \pm j4$

$s = -2 + j4$

$(s_1 - 2) = j4$

$s_2 = -2 - j4$

$(s_2 - 2) = -j4$

$$\frac{s-5}{s^2+4s+20} = \frac{A_A + B}{s^2+4s+20}$$

$s - 5 = As + B$

$A = 1$, $B = -5$

$$\frac{s-5}{s^2+4s+20} = \frac{s-5}{s^2+4s+20} = \frac{s-5}{(s+4)^2+4^2}$$

$$L^{-1}\left\{\frac{s-5}{s^2+4s+2}\right\} = e^{-5t}\cos 4t.$$

(11) $\dfrac{5s-8}{s(s-4)}$

$$\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$A = (s)F(s)\Big|_{s=0} \quad \frac{\cancel{s}(5s-8)}{\cancel{s}(s-4)}\Big|_{s=0}$$

$$= \frac{5s-8}{s-4} = \frac{5(0)-8}{(0)-4}$$

$$\Rightarrow \cancel{8}/4 = 2$$

$$B = (s-4)F(s)\Big|_{s=4} \quad \frac{\cancel{(s-4)}(5s-8)}{s\cancel{(s-4)}}\Big|_{s=4}$$

$$= \frac{5s-8}{s} = \frac{5(4)-8}{4}$$

$$\Rightarrow \frac{20-8}{4} = 12/4$$

$$= 3$$

$$\Rightarrow \frac{5s-8}{s(s-4)} = \frac{2}{s} + \frac{3}{s-4}$$

$$L^{-1}\left[\frac{5s-8}{s(s-4)}\right] = 2+3e^{4t}$$

$$= 3e^{4t}+2$$

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