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16/ENG06/086
Mechanical engineering

① $(1-x^2)\dfrac{d^2y}{dx^2} - 2x\dfrac{dy}{dx} + 2y = 0$

$(1-x^2)d'' - 2xy' + 2y = 0$

taking First term

$(1-x^2)d''$

$\Rightarrow v = (1-x^2) \qquad u = y''$

$v' = (-2x) \qquad u^n = y^{n+2}$

$v'' = -2$

$v''' = 0$

${}^nC_0 y^{n+2}(1-x^2) + {}^nC_1 y^{n+1}(-2x) + {}^nC_2 y^n(-2)$

$y^{n+2}(1-x^2) + ny^{n+1}(-2x) + \frac{n(n-1)}{2}y^n - x$

$y^{n+2}(1-x^2) + ny^{n+1}(-2x) - n(n-1)y^n$

For module 2

$-2x\dfrac{dy}{dx} \quad \Rightarrow -2y'$

$v = -2 \qquad u = y'$

$v' = 0 \qquad u^n = y^{n+1}$

${}^nC_0 y^{n+1}(-2)$

$-2y^{n+1}$

For module 3

$2y$

$u^n = y^n$

$v = 2$

${}^nC_0 y^n 2$

$2y^n$

$y^{n+2}(1-x^2) + ny^{n+1}(-2x) - n(n-1)y^n - 2y^{n+1} + 2y^n = 0$

y^n

$0 = -2C$

$C = 3$

when $s = 3$

$3^2 - 3(3) - 4 = A(3-1)^2$

$(1-x^2)y^{(n+2)} - 2xny^{(n+1)} - 2n(n-1)y^{(n)} - 2xy^{(n+1)}$
$-2ny^{(n)} + 2y^{(n)} = 0$

$(1-x^2)y^{(n+2)} - 2x(ny^{(n+1)} + y^{(n+1)}) + y^{(n)}(-2n + 2) = 0$ —— (6)

Now at $x = 0$, eqn 6 becomes

$y^{(n+2)} + 2y^{(n)}(-n+1) = 0$

$y^{(n+2)} = -(-n+1)2y^{(n)}$

$y^{(n+2)} = (n-1)2y^{(n)}$ —— (7)

eqn(7) gives the recurrence equation.

now at $n = 0$

$y^2 = (0-1)2y^{(0)}$

$(y^2)_0 = -2y_0$ —— (7)

now at $n = 1$

$(y^3)_0 = 0$ —— (8)

now at $n = 2$

$(y^4)_0 = 2(y^2)_0 = 2(-2y_0) = -4y_0$ —— (9)

now at $n = 3$

$(y^5)_0 = 4(y^{(3)})_0 = 4(0) = 0$ —— (10)

now at $n = 4$

$(y^6)_0 = 6y^{(4)} = 6(-4y_0) = -24y_0$ —— (11)

now at $n = 5$

$(y^7)_0 = 0$ —— (12)

now at $n = 6$

$(y^8)_0 = 10(y^{(6)})_0 = 10(-24y_0)$
$= -240y_0$ —— (13)

Recall the Maclaurin method

$y(x) = y_0(x) + xy'_0(x) + \frac{x^2}{2!}y''_0(x) + \frac{x^3}{3!}y'''_0(x) + \frac{x^4}{4!}y^4_0(x) + \frac{x^5}{5!}y^5_0(x) + \frac{x^6}{6!}y^6_0(x) + \cdots \frac{x^n}{n!}y^n_0(x)$ —— (14)

now substituting eqn (7, 8, 9, 10, 11, 12 & 13) into eqn (14) we have;

$y(x) = y_0(x) + xy'_0 + \frac{x^2}{2!}(-2y_0) + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(-4y_0) + \frac{x^5}{5!}(0) + \frac{x^6}{6!}(-24y_0) + \frac{x^7}{7!}(0) + \frac{x^8}{8!}(-240y_0)$

$y(x) = y_0\left(-2\frac{x^2}{2!} - \frac{4x^4}{4!} - \frac{24x^6}{6!} - \frac{240x^8}{8!}\right) + y'_0x + 0$

$y(x) = y_0\left(-x^2 - \frac{x^4}{6} - \frac{x^6}{30} - \frac{x^8}{168}\right) + y'_0x$

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(2) Transform each of the following Function into Laplace domain

$3e^{-4t} - 5e^{4t}$

$\dfrac{3}{(4+s)} - \dfrac{5}{(4-s)}$

(b) $\sin 4t + \cos 4t$

$\dfrac{a}{a^2+s^2} + \dfrac{s}{a^2+s^2}$

$\Rightarrow \dfrac{4}{4^2+s^2} + \dfrac{s}{s^2+4^2}$

$\dfrac{4}{s^2+16} + \dfrac{s}{s^2+16}$

(c) $t^3 + 2t^2 - t + 4$

$\dfrac{3!}{t^4} + 2\dfrac{2!}{t^3} - \dfrac{1}{t^2} + \dfrac{1}{t}$

$\dfrac{3!}{s^4} + \dfrac{2\cdot 2!}{s^3} - \dfrac{1}{s^2} + \dfrac{4}{s}$

$\dfrac{6}{s^4} + \dfrac{4}{s^3} - \dfrac{1}{s^2} + \dfrac{4}{s}$

$\dfrac{1}{s^4}(4s^3 - s^2 + 4s + 6)$

$\dfrac{(s+2)(s+2)}{s^2+4s+4}$

$e^{-2t}\cos 5t$

using shift method

$\cos t = \dfrac{s}{s^2+a^2}$

$= \dfrac{(s+2)}{s^2+5^2}$ when $s = \dfrac{1}{(s+2)}$

$\dfrac{(s+2)}{(s+2)^2+25}$

$\dfrac{(s+2)}{s^2+4s+4+25}$

$\dfrac{s+2}{s^2+4s+29}$

(i) $t\sin 3t$

$\sin t = \dfrac{a}{s^2+a^2}$ $t = \dfrac{1}{s}$

$\dfrac{3}{(s^2+3^2)}$

$= \dfrac{3}{s^2+9}$ $\cdot t\left(\dfrac{3}{s^2+9}\right) \Rightarrow -\dfrac{d}{ds}\left(\dfrac{3}{s^2+9}\right) = \dfrac{6s}{(s^2+9)^2}$

(h) $\dfrac{e^{-t} - e^{-2t}}{t} \Rightarrow \dfrac{\left(\dfrac{1}{s+1} - \dfrac{1}{s+2}\right)}{t}$

$(s+1)(s-2)$ $(s+1)(s+2)$

$s^2+2s-s-2$

s^2-3s-2

$(s-4)(s-4)$

$s^2-4s-4s+16$

$s^2-8s+16$

$\frac{1}{2}\ln\left(\frac{1}{s+1}-\frac{1}{s+2}\right)$

$\ln\left(\frac{1}{s+1}-\frac{1}{s+2}\right)^{-1/2}$

$= \ln\left(\frac{(s+2)-(s+2)}{s^2-3s-2}\right)^{-1/2}$

$\ln\sqrt{\cancel{\frac{(s+1)-(s+2)}{1}}} = \ln\sqrt{\frac{s^2-3s-2}{(s+2)-(s+2)}}$

$e^{4t}\cos 4t$

$\frac{s}{s^2+9^2}$ $\quad s=e^{4t}$ $\quad \Rightarrow \frac{1}{s-4}$

$\frac{s-4}{(s-4)^2+4^2}$

$\frac{s-4}{s^2-8s+16+16} \Rightarrow \frac{s-4}{s^2-8s+32}$

$t\sin 2t$

$\sin 2t = \frac{a}{s^2+a^2} \Rightarrow \frac{2}{s^2+4}$

$t\left(\frac{2}{s^2+4}\right) \Rightarrow \frac{4s}{(s^2+4)^2}$ $\quad \frac{4s}{(s^2+4)^2}$

$(s^2$

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Mechanical Engineering

①

t^3+4t^2+5

$= \frac{3!}{s^4}+4\frac{2!}{s^3}+\frac{5}{s}$

$\frac{1}{s^4}(5s^3+8s+6)$

$e^{3t}(t^2+4)$

$\frac{1}{s-3}\left(\frac{2!}{s^3}+\frac{4}{s}\right)$

$\frac{1}{s-3}\left(\frac{2s+4s^2}{s^4}\right)$

$\frac{4s^2+2s}{s^4(s-3)}$

$t^2\cos t$

$\left(\frac{s}{s+1}\right)t)t$

$(s^2+1)(s^2+1)$

$s^4 + s^2 + s^2 + 1$

$s^4 + 2s^2 + 1$

$t^2 \cos t$

$\frac{s}{s^2+1}$

$t\ \frac{(s^2+1)(1) - s(2s)}{(s^2+1)^2}$

$\frac{-d}{ds}\left(\frac{s^2+1 - 2s^2}{(s^2+1)^2}\right]$

$-\left[\frac{s^2+1 - 2s^2}{(s^2+1)^2}\right]$

$\frac{-s^2 - 1 + 2s^2}{(s^2+1)^2}$

$\frac{2s^2 - s^2 - 1}{(s^2+1)^2}$

now $t\left(t \frac{-d}{ds}\left(t^2 \cos t\right)\right)$

For second $\cdot t$ $\quad$ $vdu - udv$

$-\frac{d}{ds}\left(\frac{2s^2 - s^2 - 1}{(s^2+1)^2}\right)$

$-\left[\frac{(s^2+1)^2(4s^3 - 2s) - (2s^2 - s^2 - 1)(4s^3 + 4s)}{(s^2+1)^3}\right.$

$-\left[\frac{(s^4+2s^2+1)(4s - 2s) - (2s^2 - s^2 - 1)(4s^3 + 4s)}{(s^2+1)^8}\right.$

$\frac{4s - 2s + 8s^3 - \cancel{4s^3} + 4s - 2s) - 8s^5 - 8s^3 + \cancel{4s^5} + \cancel{4s^3} + \cancel{4s^3} + \ldots}{(s^2+1)^3}$

$-\cancel{2s^5}(2s^5 + 2s^3 + 2s - 8s^5 - 8s^3 + 4s^6 + 8s^3 + \ldots$

$-(-2s^5 + 2s^3 + 6s)$

$\Rightarrow \frac{2s^5 - 2s^3 + 6s}{(s^2+1)^3}$

$\frac{\sinh 2t}{t}$

$\Rightarrow \left(\frac{2}{s^2-4}\right)$

$\frac{1}{2} \ln\left(\frac{2}{s^2-4}\right)$

$\ln\left(\frac{2}{s^2-4}\right)^{-1/2}$

$\ln\left(\frac{s^2-4}{2}\right)^{1/2}$

$\ln\sqrt{(s^2-2)}$

(3) Convert the following function to time (t) domain

(i) $\dfrac{s-5}{(s-3)(s-4)}$

$= \dfrac{A}{(s-3)} \; \dfrac{(B)}{(s-4)}$

$\dfrac{s-5}{(s-3)(s-4)} = A(s-4) + B(s-3)$

~~whereas~~ $s-5 = A(s-4) + B(s-3)$

when $s = 4$

$4-5 = A(4-4) + B(4-3)$

$-1 = B$

$B = -1$

when $s = 3$

$3-5 = A(3-4)$

$A = \dfrac{-2}{-1}$

$A = 2$

$\dfrac{2}{s-3} - \dfrac{1}{s-4}$

$2e^{3t} - e^{4t}$

$\dfrac{2s-6}{(s-2)(s-4)} \Rightarrow \dfrac{A}{(s-2)} + \dfrac{B}{(s-4)}$

$2s-6 = A(s-4) + B(s-2)$

when $s = 4$

$2(4)-6 = B(4-2)$

$2 = 2B$

$B = 1$

when $s = 2$

$2(2)-6 = A(2-4)$

$-2 = -2A$

$A = 1$

$\dfrac{1}{s-2} + \dfrac{1}{s-4}$

$e^{2t} + e^{4t}$

$\dfrac{5s-8}{s(s-4)} \Rightarrow \dfrac{A}{s} + \dfrac{B}{s-4}$

~~when~~

$5s-8 = A(s-4) + B(s)$

when $s = 4$

$5(4)-8 = B(4)$

$12 = 4B$

$B = 3$

when $s = 0$

$s(0) - 8 = A(0-4)$

$-8 = -4A$

$A = 2$

$\frac{A}{s} + \frac{B}{s-4}$

$\frac{2}{s} + \frac{3}{s-4}$

$2t + 3e^{4t}$

$\frac{s^2 - 3s - 4}{(s-3)(s-1)^2}$

$s^2 - 3s - 4 = \frac{A}{s-3} + \frac{B}{(s-1)} + \frac{C}{(s-1)^2}$

$s^2 - 3s - 4 = A(s-1)^2 + B(s-1)(s-3) + C(s-3)$

when $s = 1$

$1^2 - 3(1) - 4 = C(1-3)$

$-6 = -2C$

$C = 3$

when $s = 3$

$3^2 - 3(3) - 4 = A(3-1)^2$

$(s^2 + 4s + 20)$

$\frac{A}{(s+2-j4)} + \frac{B}{(s+2+4j)} = \frac{s-5}{(s+2-j4)(s+2+4s)}$

$A/s = -2+4j = \frac{(-2+4j)-5}{(-\cancel{2}+4j+\cancel{2}+4j)} = \frac{-7+4j}{8j} \times \frac{j}{j}$

$= \frac{7j-4}{-8} = \frac{7j+4}{8}$

$B/s = -2-4j = \frac{-2-4j-5}{(-\cancel{2}-4j+\cancel{2}-4j)} = \frac{-7-4j}{-8j} \times \frac{j}{j}$

$B/s = -2-4j = \frac{-7j+4}{8}$

$F(s) = \frac{7j+4}{8(s+2-4s)} = \frac{7j+4}{8(s+2+4s)}$

$F(t) = L^{-1}[F(s)]$

$F(t) = \frac{7j+4}{8} e^{(-2+4j)t} - \frac{7j+4}{8} e^{(-2-4j)t}$

$F(t) = \frac{7j+4}{8}\left(e^{(-2+4j)t} - e^{(-2-4j)t}\right)$