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15/EN 906/039 MECHANICAL

ASSIGNMENT

1 $(1-x^2)\frac{d^2y}{dx^2} - \frac{2xdy}{dx} + 2y = 0$

$(1-x^2)\frac{d^2y}{dx} = (1-x^2)y''$

$u = y^2 \quad u^n = y^{(n+2)}$

$v = 1-x^2 \quad v' = -2x \quad v^2 = -2$

$v^3 = 0$

$w^n = u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v^2 + \frac{n(n-1)(n-2)}{3!} u^{n-3} v^3$

$= y^{(n+2)} \cdot (1-x^2) + n y^{(n+1)} \cdot -2x + \frac{n(n-1)}{2} y^n + 0$

$w^n = (1-x^2) y^{(n+2)} - 2xny^{(n+1)} - n(n-1)y^n$

$= (1-x^2)y^{(n+2)} - 2xny^{(n+1)} - (n^2-n)y^n$

$w^2 = -\frac{2x\,dy}{dx} = -2xy'$

$u = y' \quad u^n = y^{n+1}$

$v = -2x \quad v' = -2 \quad v^2 = 0$

$= y^{(n+1)} \cdot -2x + ny^n \cdot -2 + 0$

$= -2xy^{(n+1)} - 2ny^n$

$w_2^3 = 2y$

$u = y \quad u^n = y^n$

$v = 2 \quad v' = 0$

$y^n \cdot 2 + 0$

$= 2y^n$

$(1-x^2)y^{(n+2)} - 2xny^{(n+1)} - 2xy^{(n+1)} - (n^2-n)y^n - 2ny^n + 2y^n$

$= (1-x^2)y^{(n+2)} - 2xy^{(n+1)}(n+1) - y^n(n^2-n+2n-2)$

$= (1-x^2)y^{(n+2)} - (n+1)2xy^{(n+1)} - (n^2+n-2)y^n$

at $x = 0$

$= (1-0^2)y^{(n+2)} - (n^2+n-2)y^n$

$\left[y^{(n+2)}\right]_0 = (n^2+n-2)y^n$

$$y = y^{(0)} + xy^{(1)} + \frac{x^2}{2!} y^{(0)}(-2) + 0 \frac{x^4}{4!}(8y^{(0)}) + 0 - 8(18) y^{(0)} \frac{x^6}{6!} \cdots$$

$$y = y^{(0)} + xy^{(1)} - x^2 y^{(0)} - \frac{x^4}{3} y^{(0)} - \frac{x^6}{5} y^{(0)} \cdots$$

$$y = \cancel{y^{(0)}} \; xy^{(1)} + y^{(0)}\left(1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5}\right)$$

2.) (i) $x(t) = 3e^{-4t} - 5e^{4t}$

$x(s) = L[x(t)] = 3L[e^{-4t}] - 5L[e^{4t}]$

$$x(s) = \frac{3}{s+4} - \frac{5}{s-4}$$

(ii) $x(t) = \sin 4t + \cos 4t$

$$x(s) = \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2}$$

(iii) $x(t) = t^3 + 2t^2 - t + 4$

$$x(s) = \frac{3!}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

$$x(s) = \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

(iv) $x(t) = e^{-2t} \cos 5t$

$$L[\cos 5t] = \frac{s}{s^2+5^2}$$

$$x(s) = \frac{s+2}{(s+2)^2+25}$$

(v) $x(t) = t \sin 3t$

$$L[\sin 3t] = \frac{3}{s^2+9}$$

$$x(s) = -\frac{d}{ds}\left(\frac{3}{s^2+9}\right)$$

$$x(s) = -\frac{d}{ds}\left[3(s^2+9)^{-1}\right]$$

$$x(s) = \cancel{-\frac{d}{ds}\left(-6s\right.} \quad \frac{6s}{(s^2+9)^2}$$

(vi) $x(t) = \frac{e^{-t} - e^{-2t}}{t}$

$\lim_{t \to 0} \frac{e^{-0} - e^{-0}}{0}$ = undefined

Using L'Hopital's rule

$\lim_{t \to 0} \frac{-e^{-t} + 2e^{-2t}}{1}$ = real ans.

$L[e^{-t} - e^{-2t}] = \frac{1}{s+1} - \frac{1}{s+2}$

$L\left[\frac{e^{-t} - e^{-2t}}{t}\right] = \int_{\sigma=s}^{\infty} \left(\frac{1}{\sigma+1} - \frac{1}{\sigma+2}\right)$

$= \ln(\sigma+1)\Big|_s^{\infty} - \ln(\sigma+2)\Big|_s^{\infty}$

$= \left[\ln\left(\frac{\sigma+1}{\sigma+2}\right)\right]_s^{\infty}$

$= \ln\frac{\infty+1}{\infty+2} - \ln\frac{s+1}{s+2}$

$= \ln 1 - \ln\frac{s+1}{s+2}$

$= 0 - \ln\frac{s+1}{s+2}$

$= -\ln\frac{s+1}{s+2}$

$= \ln\frac{s+2}{s+1}$

(vii) ~~$e^{4t}\cos 2t$~~ $x(t) = e^{4t}\cos 2t$

$L[\cos 2t] = \frac{s}{s^2+4}$

$x(s) = \frac{s-4}{(s-4)^2+4}$

(viii) $x(t) = t \sin 2t$

$L[\sin 2t] = \frac{2}{s^2+4}$

$x(s) = \frac{-d}{ds}\left(\frac{2}{s^2+4}\right)$

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$$x(s) = \frac{4s}{(s^2+4)^2}$$

(ix) $x(t) = t^3 + 4t^2 + 5$

$$x(s) = \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$$

(x) $x(t) = e^{3t}(t^2+4)$

$$L[t^2+4] = \frac{2}{s^3} + \frac{4}{s}$$

$$x(s) = \frac{2}{(s-3)^3} + \frac{4}{s-3}$$

(xi) $x(t) = t^2 \cos t$

$$L[\cos t] = \frac{s}{s^2+1}$$

$$x(s) = \frac{d^2}{ds^2}\left(\frac{s}{s+1}\right)$$

$$x(s) = \frac{d}{ds}\left(\frac{d}{ds}\left(\frac{s}{s+1}\right)\right)$$

$$x(s) = \frac{d}{ds}\left(\frac{1}{s+1} - \frac{s}{(s+1)^2}\right)$$

$$x(s) = -\frac{1}{(s+1)^2} - \frac{1}{(s+1)^2} + \frac{2s}{(s+1)^3}$$

$$x(s) = \frac{2s}{(s+1)^3} - \frac{2}{(s+1)^2}$$

(xii) $x(t) = \frac{\sinh 2t}{t}$

$$L[\sinh 2t] = \frac{2}{s^2-4}$$

$$x(s) = \int_s^\infty \frac{2}{s^2-4}$$

$$x(s) = \int_s^\infty \frac{2}{(s-2)(s+2)}$$

$$\frac{2}{(s-2)(s+2)} = \frac{A}{s-2} + \frac{B}{s+2}$$

$$2 = A(s+2) + B(s-2)$$

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$2 = (A+B)s + 2A - 2B$

Comparing both sides

$2A - 2B = 2$

$A - B = 1$ - - - ①

$A + B = 0$ - - - ②

$2A = 1$

$A = \frac{1}{2}$

Subst. into ①

$\frac{1}{2} - B = 1$

$B = -\frac{1}{2}$

$\therefore x(s) = \int_s^{\infty} \frac{1}{2(s-2)} - \frac{1}{2(s+2)}$

$x(s) = \frac{1}{2} \ln (s-2) \Big|_s^{\infty} - \frac{1}{2} \ln (s+2) \Big|_s^{\infty}$

$x(s) = \frac{1}{2} \left[\ln \frac{s-2}{s+2} \right]_s^{\infty}$

$x(s) = \frac{1}{2} \left(\ln \frac{\infty - 2}{\infty + 2} - \ln \frac{s-2}{s+2} \right)$

$x(s) = -\frac{1}{2} \ln \frac{s-2}{s+2}$

$x(s) = \ln \sqrt{\frac{s+2}{s-2}}$

3 (i) $x(s) = \frac{s-5}{(s-3)(s-4)}$

$x(s) = \frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$

$s - 5 = A(s-4) + B(s-3)$

$s - 5 = (A+B)s - 4A - 3B$

Comparing both sides

$A + B = 1$ - - - ①

$-4A - 3B = -5$ - - - ②

From ①, $A = 1 - B$

So, $-4(1-B) - 3B = -5$

$-4 + 4B - 3B = -5$

$B - 4 = -5$

$B = -1$

$A = 1-(-1)$

$A = 2$

$\therefore x(s) = \frac{2}{s-3} - \frac{1}{s-4}$

$x(t) = 2e^{3t} - e^{4t}$

(ii) $x(s) = \frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$

$2s-6 = A(s-4) + B(s-2)$

$2s-6 = (A+B)s - 4A - 2B$

$\therefore A+B = 2$

$A = 2-B$ --- ①

$-4A - 2B = -6$ --- ②

$-4(2-B) - 2B = -6$

$-8 + 4B - 2B = -6$

$2B - 8 = -6$

$2B = 2$

$B = 1$

~~$x(s) =$~~ $A = 2-1$

$A = 1$

$x(s) = \frac{1}{s-2} + \frac{1}{s-4}$

$x(t) = e^{2t} + e^{4t}$

(iii) $x(s) = \frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$5s-8 = A(s-4) + Bs$

$5s-8 = (A+B)s - 4A$

$\therefore -8 = -4A$

$A = 2$

$A+B = 5$

$B = 5-A$

$B = 5-2$

$B = 3$

$x(s) = \frac{2}{s} + \frac{3}{s-4}$

$x(t) = 2 + 3e^{4t}$

(iv) $x(s) = \dfrac{s^2-3s-4}{(s-3)(s-1)^2} = \dfrac{A}{s-3} + \dfrac{B}{s-1} + \dfrac{C}{(s-1)^2}$

$s^2-3s-4 = A(s-1)^2 + B(s-1)(s-3) + C(s-3)$

let $s = \cancel{+}1$

$1-3-4 = -2C$

$-6 = -2C$

$C = 3$

let $s = 3$

$9-9-4 = 4A$

$A = -1$

$s^2-3s\cancel{-3s}-4 = -1(s-1)^2 + B(s-1)(s-3) + 3(s-3)$

$s^2-3s-4 = -s^2+2s-1+B(s^2-3s-s+3)+3s-9$

$s^2-3s-4 = -s^2+2s-1+Bs^2-4Bs+3B+3s-9$

Comparing both sides

$B-1 = 1$

$B = 2$

$\therefore x(s) = \dfrac{-1}{s-3} + \dfrac{2}{s-1} + \dfrac{3}{(s-1)^2}$

$x(t) = -e^{3t} + 2e^{t} + 3te^{t}$

$x(t) = 3te^{t} + 2e^{t} - e^{3t}$

(v) $x(s) = \dfrac{s-5}{s^2+4s+20} = \dfrac{s-5}{s^2+4s+16+4}$

$= \dfrac{s-5}{(s+2)^2+4}$

$= \dfrac{s-5+2-2}{(s+2)^2+4}$

$= \dfrac{s+2}{(s+2)^2+4} - \dfrac{7}{(s+2)^2+4}$

$= \dfrac{s+2}{(s+2)^2+2^2} - \dfrac{7}{(s+2)^2+2^2}$

$x(t) = e^{-2t}\cos 2t - 7e^{-2t}$