Giwa Abdussamd
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Mechanical Engr

1) $(1-x^2)\frac{dy}{dx} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)y^2 - 2xy^1 + 2y = 0$

$y^n = u^n v + n u^{n-1} v^1 + \frac{n(n-1)}{2b} u^{n-2} v^2 + --$

$\left[y^{(2+n)} \cdot (1-x^2) + ny^{(1+n)} \cdot (2x) + \frac{n(n-1)y^n}{2b} (-2)\right] + \left[y^{(1+n)} \cdot -2x + ny^n \cdot -2 + [2y^n] = 0\right.$

$(1-x^2)y^{(2n+)} - 2xny^{(+n)} - n(n-1)y^n - 2xy^{(1+n)} - 2ny^{(n)} + 2y^{(n)} = 0$

let $x = 0$

$y^{(2+n)} - n(n-1)y^{(n)} - 2ny^{(n)} + 2y^n = 0$

$y^{n+2} + y^n[-n(n-1) - 2n + 2] = 0$

$y^{n+2} = -(y^n)_0 [n^2 - n + 2]$

$n = 0 \quad y^2 = -y^0 \cdot 2 = -2y^0$

$n = 1 \quad y^3 = -y^1[0] = 0$

$n = 2 \quad y^4 = -y^2[-4] = 4y^2 = 4(-2y^0) = -8y^0$

$n = 3 \quad y^5 = -y^3[-10] = 10y^3 = 10 \cdot 0 = 0$

$n = 4 \quad y^6 = -y^4[-18] = 18y^4 = 18 \cdot 4 = -2y^0$

$n = 5 \quad y^7 = -y^5[-28] = 28y^5 = 28 \cdot 0 = 0$

$y = y^0 + xy^1 + \frac{x}{2b}y^3 + \frac{x^3}{3b}y^3 + --$

$+y^n$

$y^2 + yx$

$+1)y^n$

0

(ABUAD), The Road to Intellectualism, Quality and Excellence

~~[illegible]~~

$$y = y^0 + xy' + \frac{x^2}{2b}(-2)y^0 + \frac{x^3}{3b}(0) + \frac{x^4}{4b}(4)(-2)y^0 + \cdots$$

$$+ \frac{x^5}{5b}(0) + \frac{x^6}{6b}(18) + (-2)y^0 + \frac{x^7}{7b}(0)$$

$$y = y^0 + xy' - x^2y^0 - \frac{x^4}{3}y^0 - \frac{x^6}{7b}(0)$$

$$y = y^0 + xy' - x^2y^0 - \frac{x^4}{3}y^0 - \frac{x^6}{5}y^0$$

2) Trans for each of the following function into laplace domain

i) $3e^{-4t} - 5e^{4t}$

Recall $L\{f(t) \pm G(t)\}$

$$= L\{3e^{-4t} - 5e^{4t}\} \Rightarrow L\{3e^{-4t}\} - L\{5e^{4t}\}$$

$$= 3\left\{\frac{1}{s-a}\right\} - 5\left\{\frac{1}{s-a}\right\}$$

$$= 3\left\{\frac{1}{s+4}\right\} - 5\left\{\frac{1}{s-4}\right\}$$

$$= \frac{3}{s+4} - \frac{5}{s-4}$$

ii) $\sin 4t + \cos 4t$

$$\Rightarrow L\{\sin 4t + \cos 4t\} = L\{\sin 4t\} + L\{\cos 4t\}$$

$$= \frac{a}{s^2+a^2} + \frac{s}{s^2+a^2}$$

$$= \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2}$$

$$= \frac{4}{s^2+16} + \frac{s}{s^2+16}$$

$$= \frac{4+5}{(s^2+16)}$$

iii) $t^3 + 2t^2 - t + 4$

$\to \mathcal{L}\{t^3 + 2t^2 - 4\} = \mathcal{L}\{t^3\} + \mathcal{L}\{2t^3\} - \mathcal{L}\{t\}$
$+ 2[4]$

$$t^n = \frac{n!}{s^{n+1}}$$

$$= \frac{3!}{s^{3+1}} + 2\left\{\frac{2!}{s^{2+1}}\right\} - \frac{1!}{s^{1+1}} + \frac{4}{s}$$

$$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} \to \frac{4}{s}$$

iv) $e^{-2t} \cos 5t$

$$\mathcal{L}\{\cos 5t\} = \frac{s}{s^2+a^2}$$

$$= \frac{3}{s^2+5^2}$$

$$= \frac{3}{s^2+25}$$

replacing s by a shift of e^{-2t} i.e. $s+2$

$$\therefore \mathcal{L}\{e^{-2t} \cdot \cos 5t\} = \frac{s+2}{[s+2]^2+25}$$

v) $t - \sin 3t$

$$\mathcal{L}\{\sin 3t\} = \frac{a}{s^2+a^2}$$

$$= \frac{3}{s^2+3^2}$$

$$f(s) = \frac{3}{s^2+9}$$

$$L\{t\sin 3t\} = -f'(s)$$

$$u = 3 \quad du/ds = 0$$

$$v = s^2+9 \quad \frac{dv}{ds} = 2s$$

$$\therefore \frac{v\frac{du}{ds} - u\frac{dv}{ds}}{v^2}$$

$$\therefore \frac{[s^2+9]\cdot 0 - 3\cdot 2s}{[s^2+9]^2}$$

$$= \frac{-6s}{[s^2+9]^2}$$

$$-f'(s) = -1\cdot\left\{\frac{-6s}{[s^2+9]^2}\right\}$$

$$= \frac{6s}{[s^2+9]^2}$$

vi $\frac{e^{-t} - e^{-2t}}{t}$

$$L\left\{\frac{e^{-t} - e^{-2t}}{t}\right\}$$

$$\lim_{t\to 0}\left\{\frac{e^{-t} - e^{-2t}}{t}\right\} = \frac{1-1}{0} = \frac{0}{0} \Rightarrow \text{Indeterminate}$$

Apply L'Hospital rule

$$\lim_{t\to 0}\left\{\frac{-1e^{-t} - (-2)e^{-2t}}{1}\right\} = \left\{\frac{-1+2}{1}\right\} = \frac{1}{1} = 1 \text{ determinate}$$

$$L\left\{\frac{f(t)}{t}\right\} = \int_{\sigma=s}^{\infty} F\sigma \, ds$$

$$F\sigma = L\{f(t)\}$$

$$L\{f(t)\} = e^{-t} - e^{-2t} = \frac{1}{s+a} - \frac{1}{s+a}$$

$$= \frac{1}{s+1} - \frac{1}{s+2}$$

$$F\sigma = \frac{1}{\sigma+1} - \frac{1}{\sigma+2}$$

$$\int f\sigma = L\left\{\frac{f(t)}{t}\right\} = \int_{\sigma=s}^{\sigma} \frac{1}{\sigma+2} - \frac{1}{\sigma+2} \, d\sigma$$

$$= \int_{\sigma=s}^{\infty} \frac{1}{s+1} \, \delta s - \int_{\sigma=s}^{\infty} \frac{1}{\sigma+2} \, \delta s$$

$$= \left[\ln(s+1) - \ln(s+2)\right]_s^{\infty}$$

$$= \left[\ln(s+1) - \ln(s+2)\right]_s^{\infty}$$

$$= \left[\ln \frac{\sigma+1}{(\sigma+2)}\right]_s^{\infty} - \ln\left[\frac{\infty+1}{(\infty+2)} - \frac{s+1}{(s+2)}\right]$$

$$= -\ln\left[\frac{s+1}{s+2}\right] = \ln\left[\frac{(s+2)}{s+1}\right]$$

Vu) $e^{4t} \cdot \cos 2t$

$$L\{e^{4t} \cdot \cos 2t\} = e^{4t} \cdot L\{\cos 2t\}$$

$$L\{\cos 2t\} = \frac{s}{s^2+2^2}$$

$$= \frac{s}{s^2+4}$$

replacing s by a shift of e^{4t} ∴ s-4

$L\{e^{4t}\cos 2t\} = \frac{s-4}{[s-4]^2+4}$

vi) $t\sin 2t$

$= L\{t\sin 2t\} = \frac{-d}{ds}\{f(s)\}$

$F(s) = L\{\sin 2t\} = \frac{2}{s^2+2^2}$

$\bar{F}(s)$ = using quotient rule.

$u = 2$ $\qquad du/ds = 0$

$v = s^2+4$ $\qquad dv/ds = 2s$

$\frac{v\,du/ds - u\,dv/ds}{v^2}$

$= \frac{-4s}{[s^2+4]^2}$

∴ $L\{t\sin 2t\} = -f'(s)$

$= -1 \cdot \left\{\frac{-4s}{[s^2+4]^2}\right\}$

$= \frac{4s}{[s^2+4]^2}$

ix t^3+4t^2+5

$L\{t^3+4t^2+5\} = L\{t^3\} + 4L\{t^2\} + L\{5\}$

$= \frac{3!}{s^{3+1}} + 4\left\{\frac{2!}{s^{2+1}}\right\} + \frac{5}{s}$

x) $e^{3t}(t^2+4)$

$\rightarrow$ Let $x = t^2+4$

$L(x) = L\{t^2+4\}$

$= \frac{2b}{s^{2+1}} + \frac{4}{s}$

$= \frac{2}{s^3} + \frac{4}{s}$

replacing $L\{e^{3t}x\} = \frac{2}{[s-3]^3} + \frac{4}{[s-3]}$

x: $t^2 \cos t$

$L\{t^2\cos t\} = t^2 \, L\{\cos 3t\}$

$F(s) = L\{\cos 3t\} = \frac{s}{s^2+1^2}$

$f(s) = \frac{s}{s^2+1^2}$

$F'(s) =$ using quotient rule.

$u = s \quad \frac{du}{ds} = 1$

$v = s^2+1^2 \quad \frac{dv}{ds} = 2s$

$\frac{v\frac{du}{ds} - u\frac{dv}{ds}}{v^2}$

$\frac{[s^2+1^2]\cdot 1 - 2s[s]}{[s^2+1^2]^2}$

$= \frac{-s^2+1}{[s^2+1^2]^2}$

Recall $-f^4(s) = \frac{-d}{ds}\left[\frac{s^2-1}{[s^2+1]^2}\right]$

Using quotient rule
$\frac{v\frac{du}{ds} - u\frac{dv}{ds}}{v^2}$

$u = s^2-1$ $\quad$ $\frac{du}{ds} = 2s$

$v = [s^2+1]^2$ $\quad$ $\frac{dv}{ds} = 4s[s^2+1]$

$$\frac{[s^2+1]^2 \cdot 2s - [s^2-1][4s^2+s]}{[[s^2+1]^2]}$$

$$= \frac{-2s^3 - 4s^3 + 6s}{[(s^2+1)^2]}$$

$$= \frac{-2s^5 - 4s^5 + 6s}{s^4+2s^2-4}$$

$f^4(s) = \frac{-d}{ds}\left\{\frac{-2s^2-4s^2+6}{s^4+2s^2+1}\right\}$

$F^4(s) = \frac{2s^2+4s^3-6s}{s^4+2s^2+1}$

3) Convert the following to time domain

1. $\frac{s-5}{(s-3)(s-4)}$

$L^{-1}\left\{\frac{s-5}{(s-3)(s-4)}\right\} = \frac{A}{(s-3)} + \frac{B}{(s-4)}$

$\frac{s-5}{(s-3)(s-4)} = A(s-4) + B(s-3)$

Assuming $s=4$

$A = 2 \quad B = -1$

$$\therefore \frac{s-5}{(s-3)(s-4)} = \frac{2}{s-3} + \frac{-1}{(s-4)}$$

$$= \frac{2}{s-3} - \frac{1}{s-4}$$

$$= 2e^{3t} - e^{4t}$$

v) $\frac{2s-6}{(s-2)(s-4)}$ $\quad L^{-1}\left\{\frac{2s-6}{(s-2)(s-4)}\right\}$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{A}{(s-2)} + \frac{B}{s+4}$$

$$2s-6 = A(s-4) + B(s-2)$$

Assuming $s=4$

$$8-6 = A(0) + B(2)$$

$$B = 1$$

Assuming $s=2$

$$4-6 = A(2-4) + B(0)$$

$$A = 1$$

$$2s-6 = \frac{1}{s-2} + \frac{1}{s-4}$$

$$L\left\{\frac{2s-6}{(s-2)(s-4)}\right\} = e^{2t} + e^{4t}$$

iii $\dfrac{5s-8}{s(s-4)} = L^{-1}\left\{\dfrac{5s-8}{s(s-4)}\right\} = \dfrac{A}{s} + \dfrac{B}{s-4}$

$5s-8 = A(s-4) + B(s)$

Assuming $s=4$

$5(4) - 8 = A(4-4) + B(4)$

$20-8 = 4B$

$B = 3$

Assuming $s=0$

$5 \times 0 - 8 = A(0-4) + B(0)$

$-8 = -4A$

$A = 2$

$L^{-1}\left\{\dfrac{5s-8}{s(s)}\right\} = \dfrac{2}{s} + \dfrac{3}{s-4}$

$= \dfrac{2}{s} + 3\left\{\dfrac{1}{s-4}\right\}$

$= 2 + 3e^{4t}$

$$\frac{s^2 - 3s - 4}{(s-3)(s-1)^2}$$

$$= f(s) = \frac{A}{s+2} + \frac{B}{(s+1)^2} + \frac{C}{s-1}$$

$$A = \frac{s^2 - 3s - 4}{(s-1)^2} \quad s = 3 \qquad = \frac{3^2 - 3(3) - 4}{(3-1)^2} = -1$$

$$B = \frac{1^2 - (5 \times 1) - 4}{1-3} = 3$$

$$C = \frac{(s-3)(2s-3) - [s^2 - 3s - 4]}{[s-3^2)}$$

$$= \frac{(\cancel{1} - 3(2 \times 3) - [1^2 - 3 - 4]}{(-3^2)} = 2$$

$$f(s) = \frac{-1}{s-3} + \frac{3}{(s-\cancel{7})^2} + \frac{2}{s-1}$$

$$f(t) = e^{-3t} + 3te^t + 2e^t$$

$$= e^t [3t+2] e^{3t}$$