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15/ENG01/002

ENG 381 ASSIGNMENT 4

1) $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)y'' - 2xy' + 2y = 0$

Let $w_1 = (1-x^2)y''$

$u = y^{(2)}$ $\quad v = (1-x^2)$

$u^n = y^{n+2}$ $\quad v' = -2x$

$u^{n-1} = y^{n+2}$ $\quad v'' = -2$

$u^{n-2} = y^n$

Let $w_2 = 2xy'$

$u = 2y'$ $\quad v = 2x$

$u^n = y^{n+1}$ $\quad v' = 2$

$u^{n-1} = y^n$

Let $w_3 = 2y$

$u^n = 2y^n$

From Leibnitz

$y = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v''$

From w_1

$y^{n+2}(1-x^2) + n(y^{n+2}) - 2x + \frac{n(n-1)}{2!}$

$y^n - 2$

$y^{n+2}(1-x^2) - 2xn(y^{n+2})$

$- n(n-1)y^n$

From w_2

$y^{n+1}2x + ny^n 2$

For w_3

$2y^n$

$y^n = (1-x^2)y^{n+2} - 2xn$

$(y^{n+2}) - n(n-1)y^n -$

$y^{n+1} - 2x + 2ny^n +$

$2y^n$

$y^n = y^{n+2} - x^2y^{n+2} - 2xn(y^{n+2})$

$-(n^2+n)y^n - y^{n+1}2x$

$+ 2ny^n + 2y^n$

when $x = 0$

$y^n = y^{n+2} - (n^2+n)y^n + 2ny^n$

$+ 2y^n$

$y^n = y^{n+2} - (n^2+n)y^n + y(2n-2) = 0$

$$y^n = y^{n+2} - (y^n n^2 + y^n n) + (2ny^n - 2y^n)$$

$$y^n = y^{n+2} - y^n n^2 - y^n n + 2ny^n + 2y^n$$

$$y^n = y^{n+2} - y^n (n^2 - n + 2) = 0$$

$$(y^{n+2})_0 = y^n(-n^2 + n - 2)$$

$n = 0$, $(y^2)_0 = (y^{(0)})_0 (-2)$

$n = 1$, $(y^{(3)})_0 = (y^{(1)})_0 (0) = 0$

$n = 2$, $(y^{(4)})_0 = (y^{(2)})_0 (4)(-2)$

$n = 3$, $(y^{(5)})_0 = (y^{(3)})_0 = 0$

$n = 4$, $(y^{(6)})_0 = (y^4)_0 = (18)(4)(-2)$

$n = 5$, $(y^{(7)})_0 = (y^{(5)})_0 = 0$

$n = 6$, $(y^{(8)})_0 = (y^{(6)})_0 = (18)(4)(-2)(-)$

$n = 7$, $(y^{(9)})_0 = y^{(7)})_0 = 0$

$$y = (y^{(2)})\frac{x^2}{2!} + (y^{(3)})\frac{x^3}{3!} + (y^{(4)})\frac{x^4}{4!} + (y^{(5)})\frac{x^5}{5!}$$

$$+ (y^{(6)})_0 \frac{x^6}{6!} + (y^{(7)})_0 \frac{x^7}{7!} + (y^{(8)})\frac{x^8}{8!} + (y^{(9)})\frac{x^9}{9!}$$

$$y = (y^{(0)})(-2)\frac{x^2}{2!} + (y^{(2)})_0 (40)(-2)\frac{x^4}{4!} \; \frac{x^6}{6!} (y^{(4)})_0 (18)$$

$$(4)(-2) + (y^{(8)})_0 (18)(-4)(-2)(40)\frac{x^8}{8!}$$

2 (i) $3e^{-4t} - 5e^{4t}$

$\mathcal{L}[3e^{-4t} - 5e^{4t}]$

$\mathcal{L}\{3e^{-4t}\} - \mathcal{L}\{5e^{4t}\}$

$\frac{3}{s+4} - \frac{5}{s-4}$

(ii) $\sin 4t + \cos 4t$

$\mathcal{L}\{\sin 4t + \cos 4t\}$

$\mathcal{L}\{\sin 4t\} + \mathcal{L}\{\sin 4t\}$

$\frac{4}{s^2+16} + \frac{s}{s^2+16}$

(iii) $t^3 + 2t^2 - t + 4$

$\mathcal{L}[t^3 + 2t^2 - t - 4]$

$\mathcal{L}\{t^3\} + \mathcal{L}\{2t^2\} - \mathcal{L}\{t\} + \mathcal{L}[4]$

$\frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

(iv) $e^{-2t}\cos 5t$

$\mathcal{L}\{e^{-2t}\}\{\cos 5t\}$

$= \frac{s-2}{(s-2)^2+25}$

(v) $t \sin 3t$

$\mathcal{L}\{t \sin 3t\}$

$\frac{3}{s^2+9}$

(vi) $\frac{e^{-t} - e^{-2t}}{t}$

$\mathcal{L}\{e^{-t} - e^{-2t}\}$

$\mathcal{L}\{e^{-t}\} - \mathcal{L}\{e^{-2t}\}$

$\frac{1}{s+1} - \frac{1}{s+2}$

$= \int_s^\infty \frac{1}{\sigma+1} - \frac{1}{\sigma+2}) \, d\sigma$

$\int_s^\infty \frac{1}{\sigma+1} \, ds - \int_s^\infty \frac{1}{\sigma+2} \, ds$

$\left[\ln(\sigma+1) - \ln \sigma+2\right]_s^\infty$

$\left[\ln \frac{(\sigma+1)}{(\sigma+2)}\right]_s^\infty$

$\left[\ln \frac{(\infty+1)}{(\infty+2)} - \ln \frac{(s+1)}{(s+2)}\right]$

$0 - \ln \frac{(s+1)}{(s+2)}$

$\ln \frac{s+2}{s+3}$

(vii) $e^{4t} \cos 2t$

$\mathcal{L}\left[e^{4t} \cos 2t\right\}$

$\frac{s-4}{(s-4)^2+4}$

(viii) $t \sin 2t$

$\frac{2}{s^2+4}$

(ix) $t^3 + t^2 + 5$

$\mathcal{L}\{t^3 + 4t^2 + 5\}$

$\mathcal{L}\{t^3 + 4t^2 + 5\}$

$= [t^3] + \mathcal{L}[4t^2] + \mathcal{L}[5]$

$\frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$

x) $e^{3t}(t^4 + t)$

$\frac{1}{s-3}\left(\frac{3}{s^3} + \frac{4}{c}\right)$

xi) $t^2 \cos t$

$\frac{s-1}{(s-1)^2+1}$

(xii) $\frac{\sinh 2t}{t} = L\left\{\frac{\sinh 2t}{t}\right\}$

$= \frac{\tan^{-1}(2)}{s}$

(c)(i) $\frac{s-2}{(s-3)(s-4)} = \frac{A}{(s-3)} + \frac{B}{(s-4)}$

$\frac{A(s-4) + B(s-3)}{(s-3)(s-4)}$

$s-5 = As - 4A + Bs - 3B$

$s-5 = As + Bs - 4A - 3B$

$s-5 = s(A+B) - 4A - 3B$

$A + B = 1 \times -4$

$-4A - 3B = -5 \times 1$

$-4A - 4B = -4$

$-4A - 3B = -5$

$B = 1$

$B = -1$

$A + B = 1$

$A - 1 = 1$

$A = 2$

$\frac{2}{(s-3)} - \frac{1}{(s-4)}$

$L\left[\frac{2}{(s-3)} - \frac{1}{(s-4)}\right]$

$2e^{3t} - e^{4t}$

(ii). $\frac{2s-6}{(s-2)(s-4)}$

$\frac{A}{(s-2)} + \frac{B}{(s-4)}$

$\frac{A(s-4) + B(s-2)}{(s-2)(s-4)}$

$2s - 6 = As - 4A + sB - 2B$

$2s - 6 = As + sB - 4A - 2B$

$2s - 6 = s(A+B) - 4A - 2B$

$A + B = 2 \quad \times -4$

$-4A - 2B = -6 \quad \times 1$

$-4A - 4B = -8$

$-4A - 2B = -6$

$-2B = -2$

$B = 1$

$A + B = 2$

$A + 1 = 2 \qquad A = 1$

$A = 2 - 1 = 1. \qquad B = 1$

$\frac{1}{(s-2)} + \frac{1}{(s-4)}$

$L^{-1}\left[\frac{1}{s-2} + \frac{1}{s-4}\right] = e^{2t} + e^{4t}$

(iii) $\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$\frac{5s-8}{s(s-4)} = \frac{A(s-4) + Bs}{s(s-4)}$

$5s - 8 = A(s-4) + Bs$

$5s - 8 = As - 4A + Bs$

$5s - 8 = As - Bs - 4A$

$5s - 8 = s(A + B) - 4A$

$-4A = -8$

$A = 2$