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ENG 381 ASSIGNMENT IV

1. $(1-x^2)\frac{dy}{dx} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)y^2 - 2xy' + 2y = 0$

$y^n = u^n v + nu^{n-1}v' + \frac{n(n-1)}{2!}u^{n-2}v^2 + \cdots$

$\left[y^{(2+n)} - (1-x^2) + ny^{(1+n)} + (-2x) + \frac{n(n-1)}{2!}y^n + (-2)\right] + \left[y^{(1+n)} \cdot (-2x)\right.$

$+ ny^n \cdot -2 + [2y^n] = 0$

$(1-x^2)y^{(2+n)} - 2xny^{(1+n)} - n(n-1)y^{(n)} - 2xy^{(1+n)} - 2ny^{(n)} + 2y^{(n)}$

Let $x = 0$

$y^{(2+n)} - n(n-1)y^{(n)} - 2ny^{(n)} + 2y^n = 0$

$y^{n+2} + y^n[-n(n-1) - 2n + 2] = 0$

$y^{n+2} + y^n[-n^2 + n - 2n + 2] = 0$

$y^{n+2} + y^n[-n^2 - n + 2] = 0$

$y^{n+2} = -(y^n)[-n^2 - n + 2]$

$n = 0$:- $y^2 = -y^0 \cdot 2 = -2y^0$

$n = 1$ $\quad y^3 = -y^1[0] = 0$

$n = 2$ $\quad y^4 = -y^2[-4] = 4y^2 = 4(-2y^0) = -3y^0$

$n = 3$ $\quad y^5 = -y^3 \cdot [-10] = 10y^3 = 10 \cdot 0 = 0$

$n = 4$ $\quad y^6 = -y^4 \cdot [-18] = 18y^4 = 18 \cdot 4 = -2y^0$

$n = 5$ $\quad y^7 = -y^5 \cdot [28] = 28(y^5)_0 = 28 \cdot 0 = 0$

$y = y^0 + xy' + \frac{x^2}{2!}y^2 + \frac{x^3}{3!}y^3 + \cdots$

$y = y^0 + xy' + \frac{x^2}{2!}(-2)y^2 + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(4)(-2)y^0 + \cdots$

$+ \frac{x^5}{5!}(0) + \frac{x^6}{6!}(18)4(-2)y^0 + \frac{x^7}{7!}(0)$

$$y = y^0 + xy' - x^2 y^0 - \frac{x^4}{3x!} y^0 - \frac{x^6}{5} y^0$$

$$y = y^0 \left[1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5}\right] + y_0' [x]$$

2. Transform each of the following function into laplace (s) domain

i) $3e^{-4t} - 5e^{4t}$

Recall $L[F(t) \pm G(t)\} = L\{f(t) + L\{G(t)\}$

$= L\{3e^{-4t} - 5e^{4t}\} \Rightarrow L\{3e^{-4t}\} - L\{5e^{4t}\} \Rightarrow 3L\{e^{-4t}\}$

$$= 3 \cdot \left\{\frac{1}{s-a}\right\} - 5\left\{\frac{1}{s-a}\right\}$$

$$= 3 \cdot \left\{\frac{1}{s-(-4)}\right\} - 5\left\{\frac{1}{s-4}\right\}$$

$$= \frac{3}{s+4} - \frac{5}{s-4}$$

2. $\sin 4t + \cos 4t$

$\Rightarrow L\{\sin 4t + \cos 4t\} = L\{\sin 4t\} + L\{\cos 4t\}$

$$= \frac{a}{s^2+a^2} + \frac{s}{s^2+a^2}$$

$$= \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2}$$

$$= \frac{4}{s^2+16} + \frac{s}{s^2+16} = \frac{4+s}{(s^2+16)}$$

iii) $t^3 + 2t^2 - t + 4$

$\Rightarrow L\{t^2 + 2t^2 - t - 4\} - L\{t^3\} + L\{2t^2\} - L\{t\} + L\{4\}$

$$t^n = \frac{n}{s^{n+1}}$$

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$$= \frac{3!}{s^{3+1}} + 2\left\{\frac{2!}{s^{2+1}}\right\} - \left\{\frac{1}{s^{1+1}}\right\} + \frac{4}{s}$$

$$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

IV. $e^{-5t}\cos 5t$

Recall the first shift theorem

$$L\{\cos 5t\} = \frac{s}{s^2+a^2}$$

$$= \frac{s}{s^2+5^2} = \frac{s}{s^2+25}$$

replacing s by a shift of e^{-2t} :- $s+2$

$$\therefore L\{e^{-2t}\cos 5t\} = \frac{st^2}{[s+2]^2+25}$$

V. $t\sin 3t$

$$L\{\sin 3t\} = \frac{a}{s^2+a^2}$$

$$= \frac{3}{s^2+3^2}$$

$$F(s) = \frac{3}{s^2+9}$$

$$L\{t\sin 3t\} = -f'(s)$$

$f'(s)$ = using quotient rule

$u = 3$ $\quad du/ds = 0$

$v = s^2+9$ $\quad dv/ds = 25$

$$= \frac{v\frac{du}{ds} - u\frac{dv}{ds}}{v^2}$$

$$= \frac{[s^2+9]\,[0] - 3[2s]}{[s^2+9]^2}$$

$$= \frac{-6s}{[s^2+9]^2}$$

$$-F'(s) = -1 \cdot \left\{ \frac{-6s}{[s^2+9]^2} \right\}$$

$$= \frac{6s}{[s^2+9]^2}$$

VI $\dfrac{e^{-t} - e^{-2t}}{t}$

$$L\left\{ \frac{e^{-t} - e^{-2t}}{t} \right\}$$

$$\lim_{t \to 0} \left\{ \frac{e^{-t} - e^{-2t}}{t} \right\} = \frac{1-1}{0} = \frac{0}{0} = \text{indeterminate}$$

Applying L'hospital's rule

$$\lim_{t \to 0} \left\{ \frac{-e^{-t} - (-2)e^{-2t}}{1} \right\} = \left\{ \frac{-1+2}{1} \right\} = \frac{1}{1} = 1 \text{ \{determinate\}}$$

$$L\left\{ \frac{f(t)}{t} \right\} = \int_{\sigma = s}^{\infty} f\sigma \, d\sigma$$

$$F\sigma = L\{f(t)\}$$

$$L\{f(t)\} = e^{-t} - e^{-2t} = \frac{1}{s+a} - \frac{1}{s+a}$$

$$= \frac{1}{s+1} - \frac{1}{s+2}$$

$$F\,6 = \frac{1}{6+1} - \frac{1}{6+2}$$

$$\int F6 = L\left\{\frac{F(t)}{t}\right\} = \int_{6=5}^{\infty} \frac{1}{6+1} - \frac{1}{6+2}\, d6$$

$$= \int_{6+5}^{\infty} \frac{1}{6+1}\, d6 - \int_{6=5}^{\infty} \frac{1}{6+2}\, d6$$

$$= \left[\ln(6+1) - \ln(6+2)\right]_S^{\infty}$$

$$= \left[\ln(6+1) - \ln(6+2)\right]_S^{\infty}$$

$$= \left[\ln\frac{(6+1)}{(6+2)}\right]_S^{\infty} = \ln\left[\frac{\infty+1}{(\infty+2)} - \frac{3+1}{(s+2)}\right)$$

$$= -\ln\left[\frac{s+1}{s+2}\right] = \ln\left[\frac{(s+2)}{s+1}\right]$$

VII $e^{4t}\cos 2t$

$$L\{e^{4t}\cos 2t\} = e^{4t}\, L\{\cos 2t\}$$

$$L\{\cos 2t\} = \frac{s}{s^2+2^2}$$

$$= \frac{s}{s^2+4}$$

replacing s by a shift of e^{4t} ; $-s-4$

$$L\{e^{4t}\cos 2t\} = \frac{s-4}{[s-4]^2+4}$$

VIII $t\sin 2t$

$$= L\{t\sin 2t\} = -\frac{d}{ds}\left[F(s)\right\}$$

$$F(s) = L\{\sin 2t) = \frac{2}{s^2+2^2}$$

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$$F(s) = \frac{2}{s^2+4}$$

$F'(s)$ using quotient rule

$u = 2$ $\quad du/ds = 0$

$V = s^2+4$ $\quad dv/ds = 2s$

$$\frac{V\frac{du}{ds} - u\frac{dv}{ds}}{V^2}$$

$$= \frac{(s^2+4)\cdot 0 - 2(2s)}{[s^2+4]^2}$$

$$= \frac{-4s}{[s^2+4]^2}$$

$$\therefore L\{t \sin 2t\} = -F'(s)$$

$$= -L - \left\{\frac{-4s}{[s^2+4]^2}\right\}$$

$$= \frac{4s}{[s^2+4]^2}$$

ix $t^2 + 4t^2 + 5$

$$L\{t^2+4t^2+5\} = L\{t^2\} + 4L\{t^2\} + L\{5\}$$

$$= \frac{3!}{s^{3+1}} + 4\left\{\frac{2!}{s^{2+1}}\right\} + \frac{5}{3}$$

$$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{3}$$

x

x $e^{3t}(t^2+4)$

$\Rightarrow$ Let $x = t^2+4$

$L\{e^{3t}x\}$

$\therefore L\{x\} = L\{t^2+4\}$

$= L\{t^2\} + L\{4)$

$= \frac{2!}{s^{2+1}} + \frac{4}{s}$

$= \frac{2}{3} + \frac{4}{s}$

replacing s by a shift of s − 3

$L\{e^{3t}x\} = \frac{2}{[s-3]^3} + \frac{4}{[s-3]}$

x) $t^2 \cos t$

$L\{t^2 \cos 3t\} = t^3 L\{\cos t\}$

$F(s) = L\{\cos t\} = \frac{s}{s^2+1^2}$

$F(s) = \frac{s}{s^2+1^2}$

$F'(s)$ using quotient rule

$u = s \qquad du/ds = 1$

$v = s^2+1 \qquad dv/ds = 2s$

$$\frac{v\frac{du}{ds} - u\frac{dv}{ds}}{v^2}$$

$= \frac{[s^2+1]-1 - 2s[s]}{[s^2+1^2]^2}$

$= \frac{s^2+1^2-2s^2}{[s^2+1^2]^2}$

$= \frac{-s^2+1}{[s^2+1^2)^2}$

Recall

$$-f''(s) = -\frac{d}{ds}\left\{\frac{s^2-1}{[s^2+1^2]^2}\right.$$

using quotient rule

$$\frac{V\frac{du}{ds} - U\frac{dv}{ds}}{V^2}$$

$u = s^2-1$ $\quad du/ds = 2s$

$v = [s^2+1]^2$ $\quad dv/ds = 4s[s^2+1]$

$$\frac{[s^2+1]^2 \cdot 2s - [s^2-1][4s^2+4s]}{[[s^2+1]^2]}$$

$$= \frac{[2s^2-4s^2+2s]-[4s^2-4s]}{[(s^2+1)^2]}$$

$$= \frac{2s^2-4s^3+2s-4s^2+4s}{[(s^2+1)^2]}$$

$$= \frac{-2s^5-4s^3+6s}{[(s^2+1)^2]}$$

$$= \frac{-2s^5-4s^3+6s}{s^4+2s^2+1}$$

$$f''(s) = -\frac{d}{ds}\left\{\frac{-2s^5-4s^3+6s}{s^4+2s^2+1}\right\}$$

$$f''(s) = \frac{2s^5+4s^3-6s}{s^4+2s^2+1}$$

xii $\sinh 2t$

$$= L\left\{\frac{\sinh 2t}{t}\right\}$$

3. Convert the following function to time domains

1) $\frac{s-5}{(s-3)(s-4)}$

$$L^{-1}\left\{\frac{s-5}{(s-3)(s-4)}\right\} = \frac{A}{(s-3)} + \frac{B}{(s-4)}$$

$$\frac{s-5}{(s-3)(s-4)} = \frac{A(s-4) + B(s-3)}{(s-3)(s-4)}$$

Assuming $s-4$

$$4 - 5 = A(4-4) + B(4-3)$$

$$-1 = B(1)$$

$$B = -1$$

Assuming $s-3$

$$s - 3 = A(3-4) + B(3-3)$$

$$-2 = A(-1)$$

$$A = 2$$

$$\frac{s-5}{(s-3)(s-4)} = \frac{2}{(s-3)} + \frac{-1}{(s-4)}$$

$$= \frac{2}{s-3} - \frac{1}{s-4}$$

$$= 2\left\{\frac{1}{s-3}\right\} - \left\{\frac{1}{(s-4)}\right\}$$

$$= 2e^{3t} - e^{4t}$$

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ii) $\frac{2s-6}{(s-2)(s-4)}$

$$L^{-1}\left\{\frac{2s-6}{(s-2)(s-4)}\right\}$$

$2s-6 = A(s-4) + B(s-2)$

Assuming $s-4$

$(2(4)-6) = A(4-4) + B(2-2)$

$8-6 = A(0) + B(2)$

$2 = 2B$

$B = 1$

Assuming $s-2$

$2(2)-6 = A(2-4) + B(2-2)$

$4-6 = -2A + 0$

$-2 = -2A$

$A = 1$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{1}{s-2} + \frac{1}{s-4}$$

$$L^{-1}\left\{\frac{2s-6}{(s-2)(s-4}\right\} = e^{2t} + 4^{4t}$$

iii) $\frac{5s-8}{s(s-4)}$

$$L^{-1}\left\{\frac{5s-s}{s(s-4)}\right\} = \frac{A}{s} + \frac{B}{s-4}$$

$5s-8 = A(s-4) + B(s)$

Assuming $s = 4$

$5(4)-8 = A(4-4) + B(4)$

$20-8 = 4B$

$12 = 4B$

$B = 3$

Assuming $s = 0$

$s(0) - 8 = A(0-4) + B(0)$

$-8 = -4A$

$A = 2$

$$L^{-1}\left\{\frac{5s-8}{s(s-4)}\right\} = \frac{2}{s} + \frac{3}{s-4}$$

$$= \frac{2}{s} + 3\left\{\frac{1}{s-4}\right\}$$

$$= 2 + 3e^{4t}$$

$$\frac{s-5}{s^2+4s+20}$$

$$L\left\{\frac{s-5}{s^2+4s+20}\right\} =$$

$$F(s) = \frac{s-5}{s^2+4s+4+16} = \frac{s}{(s+2)^2+16} - \frac{5}{(s+2)^2+16}$$

$$= \frac{s+2-2}{(s+2)^2+16} - \frac{5}{(s+2)^2+16} = \frac{s+2}{(s+2)^2+16} - \frac{2}{(s+2)^2+16}$$

$$= \frac{s+2}{(s+2)^2+4^2} - \frac{7 \times 4/4}{(s+2)^2+4^2}$$

$$= \frac{s+2}{(s+2)^2+4^2} - \frac{7}{4} \cdot \frac{4}{(s+2)^2+4^2}$$

$$F(t) = e^{-2t}\cos 4t - \frac{7}{4}e^{-2t}\sin 4t$$

$$= e^{-2t}\left[\cos 4t - \frac{7}{4}\sin 4t\right]$$

$$\left\{ \frac{\quad}{s^2+4s+20} \right\}$$

$$F(s) = s - 5$$

V. $$\frac{s^2-3s-4}{(s-3)(s-1)^2}$$

$$= F(s) = \frac{A}{s+3} + \frac{B}{(s-1)^2} + \frac{C}{s-1}$$

$$A: \left.\frac{s^2-3s-4}{(s-1)^2}\right|_{s=3} = \frac{3^2-3(3)-4}{(3s-1)^2} = -1$$

$$B: \left.\frac{s^2-3s-4}{s-3}\right|_{s=1} = \frac{1^2-3(1)-4}{1-3} = 3$$

$$C: \frac{d}{ds}\left[\frac{s^2-3s-4}{s-3}\right]_{s=1} = \frac{(s-3)(2s-3)-[s^2-3s-4]}{(s-3)^2}$$

$$\frac{(1-3)(2(1)-3)-[1^2-3(1)-4]}{(1-3)^2} = 2$$

$$F(s) = \frac{-1}{s-3} + \frac{3}{(s-1)^2} + \frac{s}{s-1}$$

$$F(t) = -e^{-3t} + 3te^{t} + 2e^{t}$$

$$= e^{t}[3t+2] - e^{3t}$$