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15/ENG02/019
COMPUTER ENGINEERING
ASSIGNMENT 4
ENG381

1. $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)y^{(2)} - 2xy^{(1)} + 2y = 0$

For $(1-x^2)y^2$

$(1-x^2)$ let $v = 1-x^2$ $\quad u = y^2$

$v' = -2x \quad u^n = y^{(n+2)}$

$v'' = -2 \quad u^{(n-1)} = y^{(n+1)}$

$v''' = 0 \quad u^{(n-2)} = y^n$

$y^n = y^{(n+2)}(1-x^2) + ny^{(n+1)}(-2x) + \frac{n(n-1)}{2!}y^n(-2)$

$-2xy^{(1)}$

$V = -2x \quad u = y^{(1)}$

$V' = -2 \quad u^{(n)} = y^{(n+1)}$

$u^{(n-1)} = y^n$

$y^{(n+1)}(-2x) + ny^{(n)}(-2)$

$2y$

$V = 2 \quad u = y$

$V' = 0 \quad u^{(n)} = y^{(n)}$

$= y^{(n)}(2)$

$y^{(n)} = y^{(n+2)}(1-x^2) + ny^{(n+1)}(-2x) + \frac{n(n-1)}{2!}y^n(-2) + y^{(n+1)}(-2x) + ny^{(n)}(-2) + y^{(n)}(2)$

at $x = 0$

$y^{(n)} = y^{(n+2)}(1) + ny^{(n+1)}(0) + \frac{n(n-1)}{2!}y^n(-2) + y^{(n+1)}(0) + ny^{(n)}(-2) + y^{(n)}(2)$

$y^{(n)} = y^{(n+2)} - \frac{2n(n-1)}{2}y^n + ny^{(n)}(-2) + 2y^{(n)}$

$y^{(n)} = y^{(n+2)} - n(n-1)y^n + ny^n(-2) + 2y^{(n)}$

$ny^{(n)} + y^{(n)} = 0$

$] = 0$

$] = 0$

$y^{(n)} = y^{(n+2)} + y^{(n)}(1-n^2-2n+2)$

$y^{(n)} = y^{(n+2)} + y^{(n)}(-n^2-2n+3)$

$y^{(n)} = y^{(n+2)} + y^{(n)}(-n^2-2n+3) = 0$

$y^{(n+2)} = -y^{(n)}(-n^2-2n+3)$

$y^{(n+2)} = y^{(n)}(n^2+2n-3)$

when $n = 0$

$y^{(2)} = y^{(0)}(-3) = -3y^0$

when $n = 1$

$y^{(3)} = y^{(1)}(1^2+2-3)$

$y^{(3)} = 0$

when $n = 2$

$y^{(4)} = y^{(2)}(4+4-3)$

$y^{(4)} = 5y^2 = (5)(-3)y^0$

when $n = 3$

$y^{(5)} = y^{(3)}(9+6-3)$

$y^{(5)} = y^{(3)}(12) = 0$

when $n = 4$

$y^{(6)} = y^{(4)}(16+8-3)$

$y^{(6)} = y^{(4)}(21) = (21)(5)(-3)y^{(0)}$

when $n = 5$

$y^{(7)} = y^{(5)}(25+10-3)$

$y^{(7)} = y^{(5)}(32) = 0$

when $n = 6$

$y^{(8)} = y^{(6)}(36+12-3)$

$y^{(8)} = y^{(6)}(45) = (45)(21)(5)(-3)y^{(0)}$

For the Maclaurin Series,

$y = y_0 + x(y')_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^{(4)})$

$y_0 + xy' + \frac{x^2}{2!}(-3y^{(0)}) + \frac{x^4}{4!}(5)(-3)y^{(0)} + \frac{x^6}{6!}(21)(5)(-3)y^{(0)}$

$y = y_0 + xy' + \frac{x^2}{2}$

2. $3e^{-4t} - 5e^{4t}$

$= \frac{3}{s+4} - \frac{5}{s-4}$

ii. $\sin 4t + \cos 4t$

$= \frac{4}{s^2+4^2} + \frac{s}{s^2+}$

$= \frac{4}{s^2+16} +$

iii. $t^2 + 2t^2 - t$

$\frac{2}{s^3} + \frac{4}{s^3}$

iv. $e^{-2t}\cos$

$\frac{(s+}{(s+2)}$

$= \frac{(s}{(s+}$

$= \frac{(s}{(s+}$

v. $t\sin 3t$

$L(\sin 3t$

$L\{t\sin$

$y = y_0 + xy_0' + \frac{x^2}{2}(-3y_0) + \frac{x^4}{8}(-5)y_0 + \frac{x^6}{16} + \frac{x^8}{128}(-45y_0)\ldots$

2) $3e^{-4t} - 5e^{4t}$

$= \frac{3}{s+4} - \frac{5}{s-4}$

ii $\sin 4t + \cos 4t$

$= \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2}$

$= \frac{4}{s^2+16} + \frac{s}{s^2+16}$

iii $t^2 + 2t^2 - t + 4$

$\frac{2}{s^3} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

iv $e^{-2t}\cos 5t$

$\frac{(s+2)}{(s+2)^2+5^2}$

$= \frac{(s+2)}{(s+2)^2+5^2}$

$= \frac{(s+2)}{(s+2)^2+25}$

v $t \sin 3t$

$L(\sin 3t) = \frac{3}{s^2+3^2}$

$L[t\sin 3t] = -\frac{d}{ds}(F(s))$

$= -\frac{d}{ds}\left(\frac{3}{s^2+3^2}\right)$

$= \frac{(s^2+3^2)(0) - (3)(2s)}{(s^2+9)^2}$

$(s)(-s)y_0$

$= \frac{6s}{(s^2+9)^2}$

$= -\left[\frac{-6s}{(s^2+9)^2}\right]$

vi $\frac{e^{-t} - e^{-2t}}{t}$

$L[e^{-t} - e^{-2t}]$

$= \frac{1}{s+1} - \frac{1}{s+2}$

$L\left[\frac{e^{-t} - e^{-2t}}{t}\right] = \int_{\sigma=s}^{\infty}\left(\frac{1}{\sigma+1} - \frac{1}{\sigma+2}\right)d\sigma$

$_{s=\sigma}^{\infty}[\ln(\sigma+1) - \ln(\sigma+2)]d\sigma$

$= \ln\left(\frac{\sigma+1}{\sigma+2} - \ln\left(\frac{s+1}{s+2}\right)\right)$

$= \ln(1) - \ln\left(\frac{s+1}{s+2}\right)$

$= -\ln\left(\frac{s+1}{s+2}\right)$

$= \ln\left(\frac{s+1}{s+2}\right)^{-1}$

$= \ln\left(\frac{s+2}{s+1}\right)$

viii $e^{4t}\cos 2t$

$L[\cos 2t] \frac{s}{s^2+2^2} = \frac{s}{s^2+4}$

$L[e^{4t}\cos 2t] = F($

$=$

ix $t^3 + 4t^2 + 5$

$\frac{6}{s^4} + \frac{6}{s^3} + \frac{5}{s}$

x $e^{3t}(t^2+4)$

$L[t^2+4] =$

$L[e^{3t}(t^2+4)]$

xi $t^2\cos t$

$L(\cos t) =$

$L[t^2\cos t]$

$L[t\cos t]$

$\frac{d}{ds}$

$L[e^{4t}\cos^{2t}] = f(s-a)$

$= \dfrac{(s-4)}{(s-4)^2+4}$

ix $t^3 + 4t^2 + 5$

$\dfrac{6}{s^4} + \dfrac{6}{s^3} + \dfrac{5}{s}$

x $e^{3t}(t^2+4)$

$L[t^2+4) = \dfrac{2}{s^3} + \dfrac{4}{s}$

$L[e^{3t}(t^2+4)] = \dfrac{2}{(s-3)^3} + \dfrac{4}{(s-3)}$

xi $t^2\cos t$

$L(\cos t] = \dfrac{s}{s^2+1}$

$L(t^2\cos t) = \dfrac{d}{ds}\left(-\dfrac{d}{ds}\left(s/s^2+1\right)\right)$

$u = s \quad \dfrac{du}{dx} = 1$

$v = s^2+1 \quad \dfrac{dv}{dx} = 2s$

$L[t\cos t] = \dfrac{\cancel{s}(s^2+1)(1) - s(2s)}{(s^2+1)^2}$

$= \dfrac{(s^2+1) - 2s^2}{(s^2+1)^2}$

$\dfrac{A+B}{A^2}$ $= \dfrac{A(1+B/A)}{A^2}$

$\dfrac{1}{A} + \dfrac{B}{A^2}$

$= \dfrac{(\cancel{s^2+1}) - 2s^2}{(\cancel{s^2+1})(s^2+1)}$

$= \dfrac{-s^2+1}{(s^2+1)(s^2+1)} - \dfrac{2s^2}{(s^2+1)(s^2+1)}$

$-s^2+1$

$= \dfrac{-2s^2}{(s^2+1)}$

$\dfrac{-s^2+1}{(s^2+1)^2}$

$\dfrac{d}{ds}\left(\dfrac{-2s^2}{(s^2+1)}\right)$

$= \dfrac{(s^2+1)(-4s) - (-2s^2)(2s)}{(s^2+1)^2}$

$= \dfrac{-4s^3 - 4s + 4s^3}{(s^2+1)^2}$

3i $\dfrac{s-5}{(s-3)(s-4)} = \dfrac{A}{s-3} + \dfrac{B}{s-4}$

at A, $s=3$, at B, $s=4$

For A

$(\cancel{s-3}) \times \dfrac{s-5}{(\cancel{s-3})(s-4)}$

at $s=3$

$\dfrac{(3-5)}{(3-4)}$

$= \dfrac{+2}{+1}$

$A = 2$

For B

$(\cancel{s-4}) \times \dfrac{s-5}{(s-3)(\cancel{s-4})}$

at $s=4$

$\dfrac{4-5}{4-3}$

$= \dfrac{-1}{1}$

$B = -1$

$F(s) = \dfrac{2}{s-3} + \dfrac{1}{s-4}$

$F(t) = 2e^{3t} + e^{4t}$

3ii $\dfrac{2s-6}{(s-2)(s-4)} = \dfrac{A}{(s-2)} + \dfrac{B}{(s-4)}$

at A, $s=2$, at B , $s=4$

For A

$(\cancel{s-2}) \times \dfrac{2s-6}{(\cancel{s-2})\, s-4)}$

at $s=2$

$\dfrac{2(2)-6}{2-4}$

$= \dfrac{4-6}{-2}$

$= \dfrac{-2}{-2}$

$= 1$

For B

$(\cancel{s-4}) \times \dfrac{2s-6}{(s-2)(\cancel{s-4})}$

at $\dfrac{2(4)-6}{4-2}$

$= \dfrac{8-6}{2}$

$= 2/2$

$= 1$

$= \dfrac{-4s}{(s^2+1)^2}$

$L(t^2\cos t) = \dfrac{-4s}{(s^2+1)^2}$

xii $\dfrac{\sinh 2t}{t}$

$L(\sinh 2t) = \dfrac{2}{s^2-4}$

$= \int_{\sigma=s}^{\infty} \dfrac{2}{\sigma^2-4}\, d\sigma$

$2\int_{\sigma=s}^{\infty} \dfrac{1}{\sigma^2-4}$

$\cancel{x}$ ~~km~~ $2\int_{\sigma=s}^{\infty} \dfrac{-1}{-1}\, \dfrac{1}{\sigma-4}\, d\sigma$

$= -2\int_{\sigma=s}^{\infty} \dfrac{1}{2^2-\sigma^2}\, d\sigma$

$= -2\left(\tfrac{1}{2}\tan^{-1}\dfrac{\sigma}{2}\right)_s^{\infty}$

$F(s) = \frac{1}{s-2} + \frac{2}{s-4}$

$f(t) = e^{2t} + e^{4t}$

iii $\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$5s-8 = A(s-4) + B(s)$

$5s-8 = As - 4A + Bs$

$A + B = 5$

$-4A = -8$

$A = -2$

$2 + B = 5$

$B = 5-2$

$B = 3$

$\frac{2}{s} + \frac{3}{s-4}$

$F(t) = 2 + 3e^{4t}$

iv $\frac{s^2-3s-4}{(s-3)(s-1)^2}$

$\frac{A}{(s-3)} + \frac{B}{(s-1)} + \frac{C}{(s-1)^2}$

$\frac{(s-4)(s+1)}{(s-3)(s-1)^2} = \frac{A}{(s-3)} + \frac{B}{(s-1)} + \frac{C}{(s-1)^2}$

$(s-4)(s+1) = A(s-1)^2 + B(s-1)(s-3) + C(s-1)^2$

$(s-4)(s+1) = A(s^2-2s+1) + B(s^2-4s+3) + C(s-3)$

$(s^2-3s-4) = As^2 - 2As + A + Bs^2 - 4Bs + 3B + Cs - 3C$

$A + B = 1$

$-2A - 4B + C = -3$

$A + 3B - 3C = 1$

$A = 1-B$

$-2(1-B) - 4B + C$

$1-B+3B-3C =$

$-2B + C = -1$

$+ 2B - 3C = 0$

$0 - 2C = 1$

$A + B = 1$

$A = 1-B$

$2(1-B)$

$\frac{3}{4}$ / $s-3$

$f(t) = \frac{3}{4}e$

v $\frac{s-5}{s^2+4s+2}$

$\frac{s-5}{s^2+4s+20}$

$= \frac{s-5}{(s+2)^2}$

$A = 1-B$; $-2B+2B-4B+C = -3$

$-2(1-B) - 4B + C = -3$; $1 + 2B - 3C = 1$

$1 - B + 3B - 3C = 1$

$-2B + C = -1$

$+ 2B - 3C = 0$

$0 - 2C = 1$

$C = -1/2$

$A + B = 1$

$-2A - 4B - 1/2 = -3$

$-2A - 4B = -3 + 1/2$

$-2A - 4B = -5/2$

$A = 1 - B$, $2A + 4B = 5/2$

$2(1-B) + 4B = 5/2$

$\cancel{2A}\ 2 - 2B + 4B = 5/2$

$2B = 5/2 - 2$

$2B = 1/2$

$B = 1/4$

$A + 1/4 = 1$

$A = 3/4$

$\therefore \frac{3/4}{s-3} + \frac{1/4}{s-1} - \frac{1/2}{(s-1)^2}$

$f(t) = \frac{3}{4}e^{3t} + \frac{1}{4}e^{t} - \frac{1}{2}te^{t}$

v $\frac{s-5}{s^2+4s+20}$

$\frac{s-5}{s^2+4s+20}$

$= \frac{s-5}{(s+2)^2+16}$

$$\frac{s}{(s+2)^2+16} - \frac{5}{(s+2)^2+16}$$

$$= \frac{s}{(s+2)^2+4^2} - \frac{5}{(s+2)^2+4^2}$$

$$\frac{s+2-2}{(s+2)^2+4^2} - \frac{5}{(s+2)^2+4^2}$$

$$\frac{s+2}{(s+2)^2+4^2} - \frac{2}{(s+2)^2+4^2} - \frac{5}{(s+2)^2+4^2}$$

$$\frac{s+2}{(s+2)^2+4^2} - \frac{7}{(s+2)^2+4^2} \frac{\times 4}{4}$$

$$\frac{s+2}{(s+2)^2+4^2} - \frac{7}{4}\frac{\cancel{?}\,4}{(s+2)^2 4^2}$$

$$= e^{-2t}\cos 4t - \frac{1}{4} e^{-2t}\sin 4t$$