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Assignment 4

① $(1-x^2)\delta^2 y/\delta x - 2x\,\delta y/\delta x + 2y = 0$

$(1-x^2)y^{(2)} - 2xy^{(1)} + 2y = 0$

$y^{(n)} = u^n v + nu^{(n-1)}v^{(1)} + \frac{n(n-1)}{2!}u^{(n-2)}v^{(2)} + \ldots\ldots$

$[y^{(2+n)}\cdot(1-x^2] + ny^{(1+n)}\cdot(-2x) + \frac{n(n-1)}{2!}y^{(n)}\cdot(-v)] + [y^{(1+n)} - 2x + ny^{(1)} - 2] + [2y^{(n)}] = 0$

$(1-x^2)y^{(2+n)} - 2xny^{(1+n)} - n(n-1)y^{(n)} - 2xy^{(1+n)} - 2ny^{(n)} + 2y^{(n)} = 0$

Let $x = 0$

$y^{(2+n)} - n(n-1)y^{(n)} - 2ny^{(n)} + 2y^{(n)} = 0$

$y^{(2+n)} + y^{(n)}[-n^2+n-2n+2] = 0$

$y^{(2+n)} + y^{(n)}[-n^2-n+2] = 0$

$\boxed{(y^{(2+n)})_0 = -(y^{(n)})_0\cdot[-n^2-n+2] =}$ reameria rlat

$n=0 \therefore (y^{(2)})_0 = -(y^{(0)})_0\cdot(2) = -2(y^{(0)})_0$

$n=1 \therefore (y^{(3)})_0 = -(y^{(1)})_0\cdot[0] = 0$

$n=2 \therefore (y^{(4)})_0 = -(y^{(2)})_0[-4] = 4(y^{(2)})_0 = (4)(-2)(y^{(0)})_0$

$n=3 \therefore (y^{(5)})_0 = [-y^{(3)}]_0\cdot[-10] = 10(y^{(3)})_0 = (10)(0) = 0$

$n=4 \therefore (y^{(6)}]_0 = -(y^{(4)}]_0(-18) = 18(y^{(4)}]_0 = (18)(4)(-2)(y^{(0)})_0$

$n=5 \therefore (y^{(7)}]_0 = -(y^{(5)})_0[-28) = (28)(0) = 0$

$y = (y^{(0)})_0 + x(y^{(1)})_0 + \frac{x^2}{2!}(y^{(2)})_0 + \frac{x^3}{3!}(y^{(3)})_0 + \ldots$

$y = (y^0)_0 + x(y^{(1)})_0 + \frac{x^2}{2!}(2)(y^{(0)})_0 + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(4)(-2)(y^{(0)})_0 +$

$\ldots + \frac{x^5}{5!}(0) + \frac{x^6}{6!}(18)(4)(-2)(y^{(0)})_0 + \frac{x^7}{7!}(0)$

$y = (y^0)_0 + x(y^{(1)})_0 - x^2(y^{(2)})_0 - \frac{x^4}{3\times1}(y^{(3)})_0 - \frac{x^5}{5}(y^{(3)})_0$

$y = (y^0)[1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5}] + (y^{(1)})_0[x_0]$

(2). (i)

$3e^{-4t} - 5e^{4t} = f(t)$

$L[F(t)] = \frac{3}{s+4} - \frac{5}{s-4} = \frac{3(s-4) - 5(s+4)}{(s+4)(s+4)} = \frac{3s - 12 - 5s - 20}{(s+4)(s-4)}$

$= \frac{-2s - 32}{(s+4)(s+4)}$

(ii) $\sin 4t + \cos 4t = f(x)$

$L[F(t)] = \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2} = \frac{4+s}{(s^2+4^2)}$

(iii) $t^3 + 2t^2 - 6 + 4$

$\frac{n!}{s^{n+1}} = \frac{3!}{s^4} + 2.\frac{2!}{s^3} - \frac{1}{s^2} + \frac{4}{s} = \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

$= \frac{1}{s^4}[6 + 4s - s^2 + 4s^3]$

(v) $t\sin 3t = f(t)$

$L[\sin 3t] = \dfrac{3}{s^2+3^2} = 3/s^2+9$

$L[t\sin 3t] = -\dfrac{\partial}{\partial s} f(x)$

$u = 3 \quad v = s^2+9$

$du = 0 \quad dv = 2s$

$\dfrac{(s^2+9)0 - 3(2s)}{(s^2+9)^2} = \left[\dfrac{-6s}{(s^2+9)^2}\right] = \dfrac{6s}{(s^2+9)}$

(v) $e^{-2t}\cos 5t = f(t)$

$L[f(t)] = L[\cos 5t] = 5/s^2+5^2$

$L[f(t)] = \dfrac{s+2}{(s+2)^2+5^2}$ //

(vi) $\dfrac{e^{-t} - e^{-2t}}{t}$

$\lim_{t\to 0}\left[\dfrac{-e^{-t}+2e^{-2t}}{1}\right] = 2/1 = 2$

$L[e^{-t} - e^{2t}] = 1/s+1 - 1/s+2$

$L\left[\dfrac{e^{-t}-e^{-2t}}{t}\right] = \int_{(r2)}^{\infty} \left(\left(\dfrac{1}{r+1}\right) - \left(\dfrac{1}{r+2}\right)\right) dr$

$$= \int_{r=s}^{\infty} \frac{1}{r+1}\, \delta r - \int_{r=s}^{\infty} \frac{1}{6+2}\, \delta v$$

$$= \left[\ln(r+1) - 2\log(r+2)\right]_s^{\infty}$$

$$= \left[\ln \frac{r+1}{(r+2)^2}\right]_s^{\infty} = \ln\left[\frac{\infty+1}{\infty+2} - \frac{s+1}{(s+2)}\right]$$

$$= -\ln\left[\frac{s+1}{(s+2)^2}\right] = \ln\left[\frac{(s+2)}{s+1}\right]$$

(Vii) $e^{4t}\cos 2t$

$$t[\cos 2t] = \frac{-s}{s^2+2^2} = \frac{s}{s^2+4}$$

$$L[e^{4t}\cos 2t] = \frac{s-4}{(s-4)^2+4}$$

(Viii) $t \sin 2t$

$$L[\sin 2t] = \frac{2}{s^2+2^2}$$

$$L[t \sin 2t] = -\frac{d}{ds}[f(s)]$$

$v = 2 \quad \delta v = 0, \quad v = s^2+4 \quad \delta v = 2s$

$$\frac{(s^2+4)\cdot 0 - 2(2s)}{(s^2+4)^2} = \left(\frac{-4s}{(s^2+4)^2}\right) \cdot - \quad \frac{-4s}{(s^2+4)^2}$$

(ix) $t^3 + 4t^2 + 5 = f(t)$

$$= \frac{3!}{s^4} + \frac{4 \cdot 2!}{s^3} + 5/s = 6/s^4 + 8/s^3 + 5/s$$

$$= 1/s^4 \left[6 + 8s + 5s^3\right]$$

(x) $e^{3t}(t^2+4)$

$$L[t^2+4] = \frac{2!}{s^3} + 4/s = 2/s^3 + 4/s$$

$$L[e^{3t}] = 2/(s-3)^3 + 4/(s-3) = \frac{4s^2 - 24s + 38}{(s-3)^2}$$

(xi) $t^2 \cos t = f(t)$

$$L(\cos t) = s/(s^2+1) \quad \therefore L[t^2 \cos t] = -\delta^2/\delta s^4 \left[s/(s^2+1)\right]$$

$$d/ds \left[s/(s^2+1)\right] = \frac{(s^2+1) \cdot 1 - s(2s)}{(s^2+1)^2} = \frac{1-s^2}{(s^2+1)^2}$$

$$d/\delta s \left[\frac{1-s^2}{(s^2+1)^2}\right] \quad \therefore u = 1-s^2 \quad du = -2s$$

$$v = (s^2+1)^2 \quad \delta v = 4s(s^2+1)$$

$$v = s^2+1 \quad \delta v = 2s$$

$$w = v^2 \quad \delta w = 2v$$

$$dv/\delta s \times \delta w/\delta v = 2s \times 2v$$

$$= 4sv = 4s(s^2+1)$$

$$\frac{(s^2+1)^2 \cdot -2s - (1-s^2)\,4s(s^2+1)}{(s^2+1)^2} = \frac{-2s(s^2+1)^2 - 4s(s^2+1)(1-s^2)}{(s^2+1)}$$

$$= \frac{-2s[s^2+1-2+2s^2]}{(s^2+1)^3}$$

$$= \frac{-2s(3s^2-1)}{(s^2+1)^3} \quad \therefore L[t^2\cos t] = -\frac{d^2}{ds^2}\left[\frac{s}{s^2+1}\right]$$

$$\therefore \frac{d^2}{ds^2}\left[\frac{s}{s^2+1}\right] = \frac{2s(3s^2-1)}{(s^2+1)^3}$$

(ii) $\frac{\sinh 2t}{t} = f(x)$

$$\lim_{t \to 0}\left[\frac{\sinh 2t}{t}\right] = \frac{2\cosh 2t}{1} = 2/1 = 2$$

$$L\left[\frac{\sinh 2t}{t}\right] = L[\sinh 2t] = \frac{2}{s^2-2^2} = \frac{2}{s^2-4}$$

$$L\left[\frac{\sinh 2t}{t}\right] = \int_{r=s}^{\infty} \frac{2}{r^2-4}\,dr = 2\int_{r=s}^{\infty} \frac{1}{r^2-4}\,dr$$

(3). $\frac{s-5}{(s-3)(s-4)} = f(s) = \frac{A}{s-3} + \frac{B}{s-4}$

$$A: (s-3)f(s) = \frac{s-5}{\cancel{(s-3)}(s-4)}\Bigg|_{s=3} = \frac{(3-5)}{(3-4)} = \underline{\underline{2}}$$

B. $s-4\,f(s) = \dfrac{s-5}{(s-3)(s-4)}\Bigg|_{s=4} = \dfrac{4-5}{4-3} = -1$

$f(s) = \dfrac{2}{s-3} - \dfrac{1}{s-4}$

$f(t) = 2e^{3t} - e^{4t}$

(4) $\dfrac{2s-6}{(s-2)(s-4)} = f(s) = \dfrac{A}{s-2} + \dfrac{B}{s-4}$

A) $\dfrac{2s-6}{s-4}\Bigg|_{s=2} = \dfrac{2(2)-6}{2-4} = 1$

(B) $\dfrac{2s-6}{s-2}\Bigg|_{s=4} = \dfrac{2(4)-6}{4-2} = 1$

$f(s) = \dfrac{1}{s-2} + \dfrac{1}{s-4}$

$f(t) = e^{2t} + e^{4t}$

(iii) $\dfrac{5s-8}{s(s-4)} = f(s) = \dfrac{A}{s} + \dfrac{B}{s-4}$

A. $\dfrac{5s-8}{s-4}\Bigg|_{s=0} = \dfrac{5(0)-8}{0-4} = \dfrac{8}{4} = 2$

B $\dfrac{5s-8}{s}\Bigg|_{s=4} = \dfrac{5(4)-8}{4} = 3$

$f(s) = \dfrac{2}{s} + \dfrac{3}{s-4}$

$f(t) = 2 + 3e^{4t}$

(iv). $\dfrac{s^2-3s-4}{(s-3)(s-1)^2} = F(s) = A/_{s-3} + B/_{(s-1)} + C/_{(s-1)}$

$A: \dfrac{s^2-3s-4}{(s-1)^2}\Big|_{s=3} = \dfrac{3^2-3(3)-4}{(3-1)^2} = -1$

$B = \dfrac{s^2-3s-4}{s-3}\Big|_{s=1} = \dfrac{1^2-3(1)-4}{1-3} = 3$

$C = d/_{ds}\left[\dfrac{s^2-3s-4}{s-3}\right]_{s=1} = \dfrac{(s-3)(2s-3)-[s^2-3s-4]}{(s-3)^2}$

at $s=1$

$\dfrac{(1-3)(2(1)-3)-[1^2-3(1)-4]}{(1-3)^2} = 0$

$F(s) = -1/_{s-3} + 3/_{(s-1)^2} + 2/_{s-1}$

$f(t) = -e^{3t} + 3te^{t} + 2e^{t}$

(v) $\dfrac{s-9}{s^2+4s+20} = f(t)$

$$F(s) = \frac{s-5}{s^2+4s+4+16} = \frac{s}{(s+2)^2+16} - \frac{5}{(s+2)^2+16}$$

$$= \frac{s+2}{(s+2)^2+4^2} - \frac{7 \sim 4/4}{(s+2)^2+4^2}$$

$$= \frac{s+2}{(s+2)^2+4^2} - \frac{7}{4} \cdot \frac{4}{(s+2)^2+4^2}$$

$$f(t) = e^{-2t}\left[\cos 4t - \frac{7}{4}\sin 4t\right]$$