Name: MELLY MIRACLE U.
Matric No: 15/ENG07/027
Petroleum Engineering

Assignment IV

Transform each of the functions into laplace transforms

$L\{f(t)\} \rightarrow f(s) = \int_0^\infty e^{-st} f(t)\, dt$

(I) $3e^{-4t} - 5e^{4t}$

$L\{3e^{-4t} - 5e^{4t}\}$

$L\{3e^{-4t}\} - L\{5e^{4t}\}$

$\frac{3}{s+4} - \frac{5}{s-4}$

(II) $\sin 4t + \cos 4t$

$L\{\sin 4t + \cos 4t\}$

$L\{\sin 4t\} + L\{\cos 4t\}$

$\frac{4}{s^2+16} + \frac{s}{s^2+16}$

(III) $t^3 + 2t^2 - t + 4$

$L\{t^3 + 2t^2 - t + 4\}$

$L\{t^3\} + L\{2t^2\} - L\{t\} + L\{4\}$

$\frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

IV) $e^{-3t} \cos t$

$L\{e^{-2t} \cos t\}$

$= \frac{s-2}{(s-2)^2 + 25}$

V $t \sin 3t$

$L\{t \sin 3t\}$

$\frac{3}{s^2+9}$

VI) $\frac{e^{-t} - e^{-2t}}{t}$

$L\{e^{-t} - e^{-2t}\}$

$L\{e^{-t}\} - L\{e^{-2t}\}$

$\frac{1}{s+1} - \frac{1}{s+2}$

$= \int_s^\infty \left(\frac{1}{s+1} - \frac{1}{s+2}\right) ds$

$\int_s^\infty \frac{1}{s+1}\, ds - \int_s^\infty \frac{1}{s+2}\, ds$

$[\ln(s+1) - \ln(s+2)]_s^\infty$

$\left[\frac{\ln(s+1)}{s+2}\right]_s^\infty$

$\left[\ln\frac{(\infty+1)}{(\infty+2)} - \frac{\ln(s+1)}{(s+2)}\right]$

$0 - \frac{\ln(s+1)}{s+2}$

$\ln\frac{s+2}{s+3}$

VII) $e^{4t} \cos 2t$

$L\{e^{4t} \cos 2t\}$

$\frac{s-4}{(s-4)^2+4}$

VIII) $t \sin t$

$\frac{2}{s^2+4}$

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t^3+4t^2+5

$L\{t^3+4t^2+5\}$

$L\{t^3\}+L\{4t^2\}+L\{5\}$

$\frac{6}{s^4}+\frac{8}{s^3}+\frac{5}{s}$

$e^{3t}(t^2+4)$

$L\left(\frac{2}{s^3}+\frac{4}{s}\right)$ $s-3$

$t^3\cos t$

$\frac{s-1}{(s-1)^2+1}$

$\frac{\sinh 2t}{t}=L\left\{\frac{\sinh 2t}{t}\right\}$

$=\frac{\tan^{-1}(2)}{s}$

Convert the terms to partial fraction

$\frac{s-5}{(s-3)(s-4)}=\frac{A}{(s-3)}+\frac{B}{(s-4)}$

$\frac{A(s-4)+B(s-3)}{(s-3)(s-4)}$

$s-5=As-4A+Bs-3B$

$s-5=As+Bs-4A-3B$

$A+B=1 \times -4$

$-4A-3B=-5 \times 1$

$-4A-3B=-5$

$-B=1$

$B=-1$

$A+B=1$

$A-1=1$

$A=2$

$\frac{2}{(s-3)}-\frac{1}{(s-4)}$

$L^{-1}\left[\frac{2}{(s-3)}-\frac{1}{(s-4)}\right]$

$2e^{3t}-e^{4t}$

2.) $\frac{2s-6}{(s-2)(s-4)}$

$\frac{A}{(s-2)}+\frac{B}{(s-4)}$

$\frac{A(s-4)+B(s-2)}{(s-2)(s-4)}$

$2s-6=As-4A+Bs-2B$

$A+B=2 \times -4$

$-4A-2B=-6\times 1$

$-4A+4B=-8$

$-4A-2B=-6$

$-2B=2$

$B=1$

$A+B=2$

$A+1=2$

$A=1$

$\frac{1}{(s-2)}+\frac{1}{(s-4)}$

$L^{-1}\left(\frac{1}{(s-2)}+\frac{1}{(s-4)}\right)$ $e^{2t}+4^{t}$

3. $\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$\frac{5s-8}{s(s-4)} = \frac{A(s-4)+Bs}{s(s-4)}$

$5s-8 = A(s-4)+Bs$

$5s-8 = As-4A+Bs$

$+4A = +8$

$A = 2$

$A+B = 5$

$2+B=5$

$B=3$

$L^{-1}\left(\frac{2}{s}+\frac{B}{(s-3)}\right)$

$2+e^{3t}$

1.) $\frac{s^2-3s-4}{(s-3)(s-1)^2}$

$\frac{A}{(s-3)} + \frac{B}{(s-1)} + \frac{C}{(s-1)^2}$

$s^2-3s-4 = A(s-1)^2 + B(s-3)(s-1) + C(s-3)$

$(s-3)=0$

$C=-2 \quad A=1$

$B=-3$

$\frac{1}{(s-3)} - \frac{3}{(s-1)} - \frac{2}{(s-1)^2} = L^{-1}\left(\frac{1}{s-3}\right) - L^{-1}\left(\frac{3}{s-1}\right)$

$e^{3t} - 3e^{t}$

1. $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)y'' - 2xy' + 2y = 0$

$W_1 (1-x^2)y^n$

$U^n = y^{n+2} \qquad V = (1-x^2)$

$V' = -2x$

$V'' = -2$

$W_2 = 2xy'$

$y^{n+1} = U \qquad V = 2x \quad V' = 2$

$W_3 = 2y$

$U^n = 2y^n$

From Leibnitz

$y^n = U^nV^0 + nU^{n-1}V' + \frac{n(n-1)}{2!}U^{n-2}V^2$

from W_1

$y^{n+2}(1-x^2) + n(y^{n+2})2x + \frac{n(n-1)}{2!}y$

$y^{n+2}(1-x^2) - 2xn(y^{n+2}) - n(n-1)y^n$

From W_2

$y^{n+1}2x + ny^n 2$

From W_3

$2y^n$

$y^n = (1-x^2)y^{n+2} - 2xn(y^{n+2}) - n(n-1)y^n$
$- y^{n+1}2x + 2ny^n + 2y^n$

When $x=0$

$y^n = y^{n+2} - (n^2+n)y^n + 2ny^n + 2y^n$

$y^n = y^{n+2} - (n^2+n)y^n + y^n(2n-2) = 0$

$y^n = y^{n+2} - (y^n n^2 + y^n n) + (2ny^n - 2y^n)$

$y^n = y^{n+2} - y^n n^2 - y^n n + 2ny^n + 2y^n$

$y^n = y^{n+2} - y^n(n^2 - n + 2) = 0$

$y^{n+2} - y^n(-n^2+n-2)$

$n=0 \; (y^{(2)})_0 = y_0 \; (-2)$

$n=1, \; (y^{(3)})_0 = (y^{(1)})_0 \; (0) = 0$

$n=2 \; (y^{(4)})_0 = (y^{(2)})_0 \; (4)(-2)$

$n=3 \; (y^{(5)})_0 = (y^{(3)})_0 = 0$

$n=4 \; (y^{(6)})_0 = (y^{(4)})_0 = (18)(4)(-2)$

$n=5 \; (y^{(7)})_0 = (y^{(5)})_0 = 0$

$n=6 \; (y^{(8)})_0 = (y^{(6)})_0 (18)(4)(-2)(40)$

$n=7 \; (y^{(9)})_0 = (y^{(9)})_0 = 0$

$y = (y^{2})_0 \frac{x^2}{2!} + (y^{(3)})_0 \frac{x^3}{3!} + (y^{(4)}) \frac{x^4}{4!} + (y^5) \frac{x^5}{5!}$

$+ (y^6) \frac{x^6}{6!} + (y^{(7)}) \frac{x^7}{7!} + (y^{(8)}) \frac{x^8}{8!}$

$+ (y^9) \frac{x^9}{9!}$

$y = (y^{0})_0 (-2) \frac{x^2}{2!} + (y^{(2)})_0 (-2) \frac{x^4}{4!} + \frac{x^6}{6!}$

$y (y^{(4)})_0 \; (18)(-4)(-2) \; y^8$

$(y^8)_0 \; (18)(-4)(-2)(40) \frac{x^8}{8!}$